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THESIS

EFFECTS OF PULSE SHAPING ON CERENKOV RADIATION

by

Kenneth Merritt Stein

June 1986

Thesis Advisor: John B. Neighbours

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Effects of Pulse Shaping on Cerenkov Radiation

by

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Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

Pulse shaping of periodic relativistic electron pulses generates patterns of Cerenkov radiation distinctive of the distribution of charge within a bunch. Computer simulation mapped the radiation pattern of the level, triangular, trapezoidal, rounded, Gaussian, level plus sinusoidal ripple, and multiple hump charge distributions. Each shape, with exception of the level plus sinusoidal ripple, generates a series of radiation patterns unique to that shape. This research provides a basis for determining the shape of a current pulse based on the radiated energy pattern.

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I. INTRODUCTION

A charged particle or a group of charged particles moving at greater than the velocity of light in a medium will interact with the material to generate a continuous spectrum of electromagnetic radiation. Mallet and Cerenkov conducted experiments, in the 1920's and 1930's, which discovered and provided an explanation for the radiation. Frank and Tamm, in 1937, developed the theory for the process [Ref. 1:pp.1-15]. Recent work has concentrated on the radiation from periodic bunches of electrons and effects due to a non-infinite interaction region [Ref. 2] [Ref. 3].

The shape of a group of charges influences the pattern of radiation produced. Mapping the radiation pattern from different charge shapes may provide insight into whether charge pulse shapes can be determined from observed radiation patterns [Ref. 4:pp.1994,1996].

II. BACKGROUND

A. THE CERENKOV EFFECT

The electric field of a bunch of electrons moving slowly through a medium polarizes the atoms of the medium so that the positive nuclei are displaced toward the electron bunch, and the negative electron clouds away. At any particular time, the medium is polarized in the region near the electron bunch, producing a dipole. An electromagnetic pulse is generated by the formation and disappearance of the dipole as the bunch passes. The electromagnetic pulse propagates away from the dipole source at the velocity of light in the medium. The distance between wave fronts is compressed as the velocity of the electron bunch increases. This effect is similar to the doppler effect for sound radiated from a moving source. The electromagnetic field produced is called sub-Cerenkov radiation [Ref. 5:pp.3-4] [Ref. 6].

If the electron bunch is accelerated to a velocity greater than light in the medium, the pulse source velocity is greater than that of the propagating electromagnetic wave which transports the energy. The result is a wavefront where the eminated wavelets bunch together in phase. This shockfront of electromagnetic radiation is Cerenkov

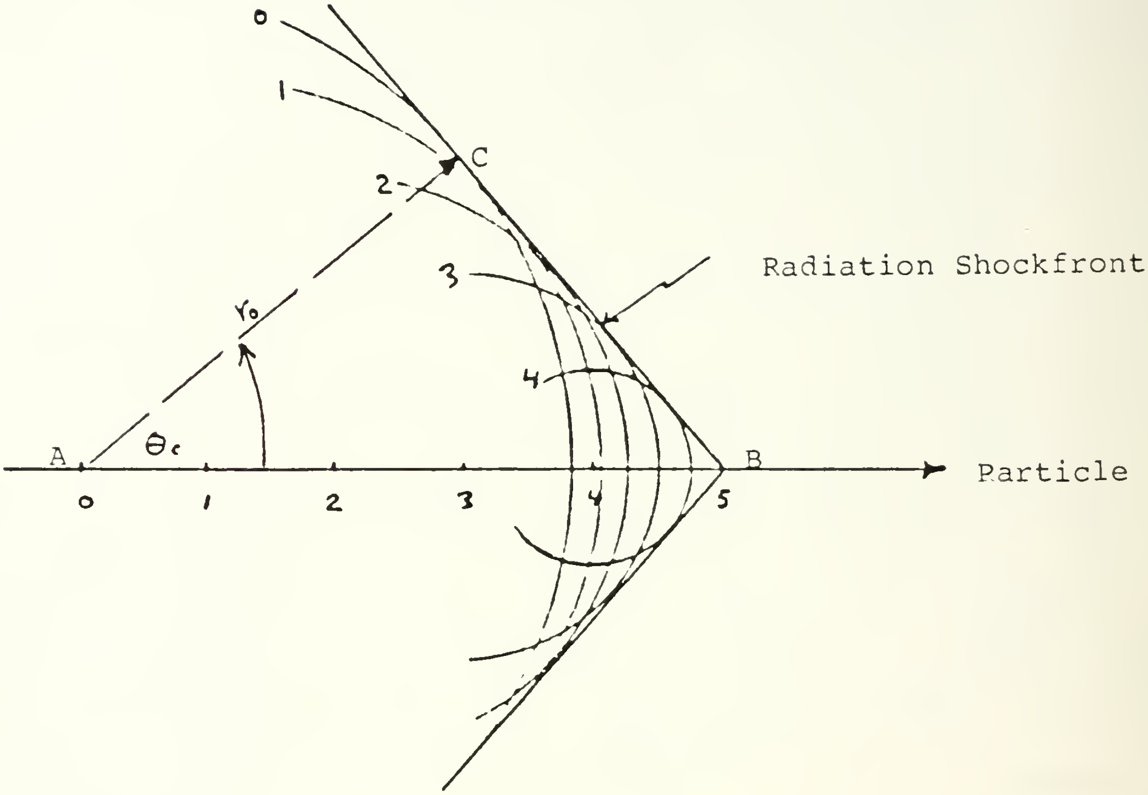


Figure 1. Radiation Shockfront

radiation - that radiation emitted when the velocity of a charged particle exceeds the velocity of light in a medium [Ref. 5:pp.4-5] [Ref. 6].

Figure 1 shows the resultant Cerenkov radiation plane wavefront BC from particles traveling from A to B. Wavelets from points 0 to 5 are coherent along wavefront BC. The angle θc is given by the "Cerenkov relation":

$$\cos \theta c = 1/\beta n \qquad (1)$$

where n is the refractive index of the medium and β is the ratio of the velocity of the particle to the velocity of light in a vacuum [Ref. 5:pp.4-5] [Ref. 6]. From the above, the following is observed:

1. There is a threshold velocity β min = 1/n, below which no radiation appears [Ref. 5:p.5].
2. In the limit β = 1, there is a maximum angle of emission: $\theta max = \cos^{-1} (1/n)$ [Ref. 5:p.6].
3. In a gas, θc has only a slight dependence on electron velocity because β must be close to one in order to obtain the Cerenkov condition [Ref. 3:p.3249].
4. Radiation occurs mainly in the visible and near visible region for which $n > 1$. In the x-ray region, above atomic resonances, n is usually less than 1; therefore Cerenkov radiation in the x ray region is not probable [Ref. 3:p.3252] [Ref. 5:p.6].

Azimuthal symmetry of the wavefront produces a cone of radiation. The distribution in θ of the light intensity approximates to a delta function [Ref. 5:p.6].

B. FINITE INTERACTION LENGTH

Radiation from the polarized molecules along the path of the electron beam is formally equivalent to diffraction from a single slit with plane waves impinging on the slit at angles far from normal incidence. Thus, the previous discussion dealt with the case where the path length is infinitely long. Passing an electron beam through a gas cell of finite length causes the radiation to increase in power and to be spread over a range of emission angles instead of having a sharp Cerenkov angle, θc. This increased power (up to 2 orders of magnitude for a gas medium) and the spreading effect are dependent on the medium length, and independent of bunch structure [Ref. 3:pp.3246, 3251-3252]. The spreading is asymmetric about the Cerenkov cone and the radiated power has interference lobes. The main lobe is peaked at an angle greater than the Cerenkov angle and there is significant power in the other interference lobes [Ref. 7:p.14]. As the finite length increases, less spreading and less power increase occurs, and the position of maximum radiation decreases slowly to the Cerenkov angle. In addition, the resulting radiation intensity is modified by the Fourier transform of the spatial distribution of charge within the bunch. The frequency distribution from a single charge bunch is continuous [Ref. 3:pp.3248-3252].

C. PERIODIC ELECTRON BUNCHING

If periodic bunches of electrons, instead of a single charge distribution, are sent through a medium, Cerenkov radiation at harmonics of the bunch frequency results. At the lower harmonics, such that the wavelength of emitted radiation is larger or on the order of the bunch size, the electrons radiate in phase and cause increased radiated power, even at microwave frequencies [Ref. 2] [Ref. 3:p.3246]. At higher harmonics: [Ref 3:pp.3246, 3249-3250]

1. Destructive interference, described by the Fourier transform of the charge density, decreases intensities with increasing frequency, until incoherent radiation takes over when the wavelength of the radiation is much less than the electron spacing.
2. The observed radiation peaks occur at smaller angles more closely spaced.
3. The angle of peak power decreases.
4. The percentage of total power in the first lobe decreases.

D. CALCULATION OF RADIATED POWER

For periodic bunches of electrons, the power radiated per unit solid angle at frequency (ν) is:

$$W(\nu,\hat{n}) = \frac{\mu\, c\, \nu_0^2\, q^2}{8\,\pi^2} \left| F(\vec{k}) \right|^2 \left[(kL)^2 \operatorname{Sin}^2\theta \; I^2(u) \right] \qquad (2)$$

The parameters describing the radiation are:

$$u = \frac{kL}{2}(\mathrm{Cos}\,\theta_c - \mathrm{Cos}\,\theta) \qquad (3)$$

$$I(u) = \frac{\mathrm{Sin}\,u}{u} \qquad (4)$$

$$\vec{k} = \left(n_x \frac{\omega}{c},\ n_y \frac{\omega}{c},\ n_z \frac{\omega}{v}\right) \qquad (5)$$

$$\rho_o'(\vec{k}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\ dy\ dz\ \exp[-i\,\vec{k}\cdot\vec{r}]\,\rho_o'(\vec{r}) \qquad (6)$$

$$= q\,F(\vec{k}) \qquad (7)$$

where n_x, n_y, and n_z are components of the unit vector $\hat{n}$ in the emission direction, ν is the frequency of the emitted radiation, and L is the length of medium interaction. The total charge of one bunch is q, distributed over a charge distribution $\rho_o'(\vec{r})$ with Fourier transform $\rho_o'(\vec{k})$, and $F(\vec{k})$ is the non-dimensional form factor. The bunch frequency ν_o is equal to the electron velocity divided by the electron bunch spacing, and ν is the frequency of the emitted radiation, a harmonic of ν_o [Ref. 3:p.3248].

An alternative expression for the radiated power is given by the expression:

$$W(\nu,\hat{n}) = \text{constant} \times \left|F(\vec{k})\right|^2 \times G^2(n\beta,\theta) \times \mathrm{Sin}^2\,u \qquad (8)$$

where $$G(n\beta,\theta) = \frac{\sin\theta}{(n\beta)^{-1} - \cos\theta} \qquad (9)$$

and n in the expression for G is the index of refraction. $G^2(n\beta,\theta)$ is the Cerenkov radiation envelope. It is equal to zero at $\theta = 0$ and $\theta = \pi$, and has a pole at θc. The radiation, as described by equation 8, is maintained finite at $\theta = \theta c$ because sin u at $\theta = \theta c$ is identically zero [Ref. 8].

E. BUNCH PROFILE DETERMINATION

The bunch form factors are fourier transforms which differ depending on the charge distribution within a bunch. For a point charge, the form factor is identically one. For any charge distribution of non-zero extent, F(k) must be one for k = 0 and fall off as a function of k. Here we consider only line charges so that $F(k) = F(k_z)$ and k_z goes to zero at $\theta = 90^{\circ}$. Thus there will always be an enhancement of the radiation pattern near 90° as a result of the maximum value of the form factor at that angle. This effect is always present, but is overshadowed at low frequencies by the (broadened and shifted) Cerenkov peaks. As noted previously, at higher harmonics the Cerenkov power radiated in the forward direction decreases as a result of loss of coherence between bunches, and in this situation, the enhancement at $\theta = 90^{\circ}$ from the form factor becomes dominant [Ref. 4:pp.1994-1995].

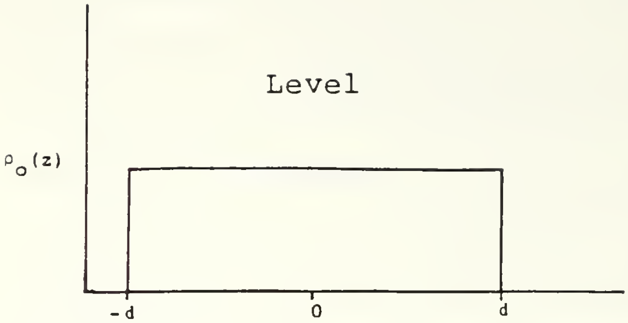
A complete angular map of Cerenkov radiation patterns may be used to characterize the properties of relativistic beam pulses, and may enable one to determine the beam pulse charge distributions from measured radiated patterns [Ref. 4:p.1996]. This work was undertaken to assess the differences in radiated patterns from a number of differently shaped electron beam charge distributions in order to determine some of the problems associated with obtaining electron bunch profiles from radiation pattern measurements.

III. CALCULATIONS AND RESULTS

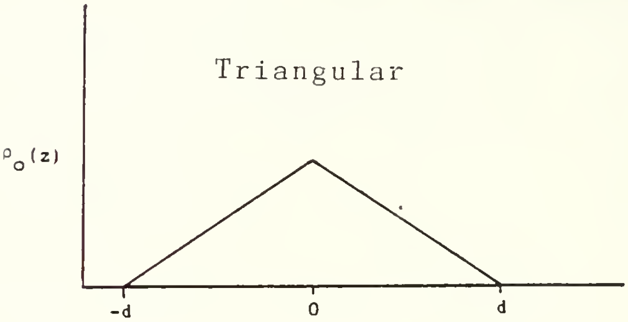
Professors X. K. Maruyama and J. R. Neighbours of the Department of Physics, Naval Postgraduate School, Monterey, California, developed a computer program to calculate and plot angular maps of the radiated energy per unit solid angle for a given form factor. The program considered the following one dimensional (i.e. in z only) beam charge distributions: Gaussian, level (i.e. boxcar), level plus a sinusoidal ripple combination, and a double hump. This author modified the program to perform calculations also for the trapezoidal, rounded, triangular, and multiple hump charge distributions. Figure 2 diagrams the beam pulse shapes and presents the corresponding form factor expressions.

To determine information from each type of charge distribution, calculations of radiated energy were performed using the following parameters:

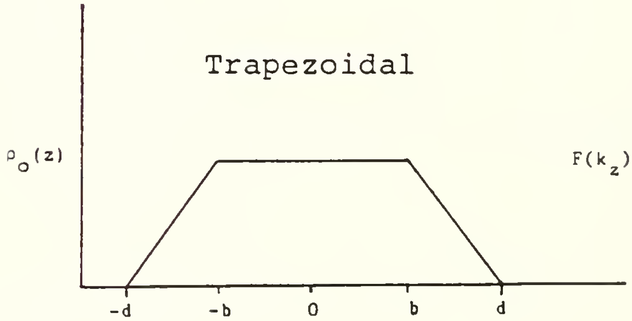
1. Beam Energy: 27 Mev
2. Pulse Frequency: 50 MHz
3. Pulse Length: 3.3 nsec, 1 meter
4. Instantaneous Beam Current: 400 amperes
5. Path Length in Air: 10 meters
6. Cerenkov angle, θc: 0.76^{o} (for n = 1.000268)



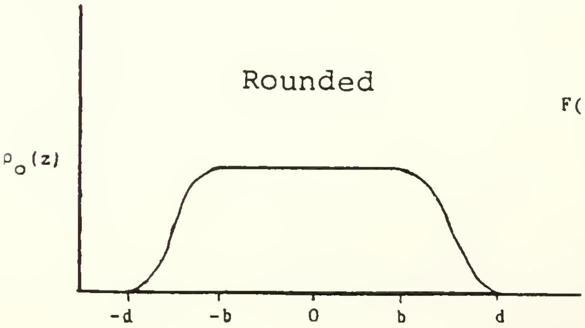
$$F(k_z) = \frac{\mathrm{Sin}\ k_z d}{k_z d}$$



$$F(k_z) = \frac{4}{k_z{}^2 d^2}\ \mathrm{Sin}^2\ \frac{k_z d}{2}$$



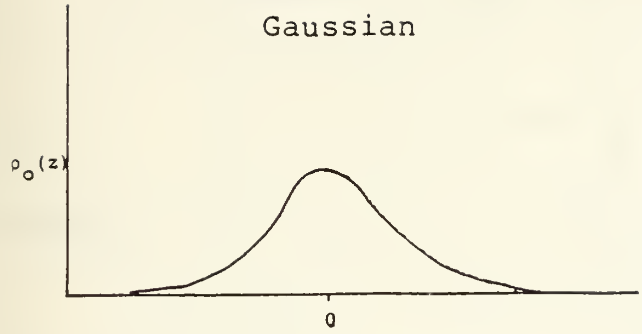
$$F(k_z) = \frac{4}{k_z{}^2(d^2 - b^2)} \left(\mathrm{Sin}^2\ \frac{k_z d}{2} - \mathrm{Sin}^2\ \frac{k_z b}{2}\right)$$



$$F(k_z) = \frac{8}{k_z^3\ (b + d)\ (d - b)^2} \quad \mathrm{X}$$

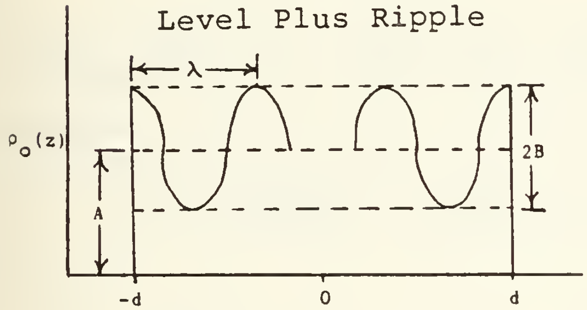
$$\left(2\ \mathrm{Sin}\ \frac{k_z}{2}\ (d + b) - \mathrm{Sin}\ k_z b - \mathrm{Sin}\ k_z d\right)$$

Figure 2. Electron Beam Charge Bunch Shapes and Corresponding Form Factors


Gaussian

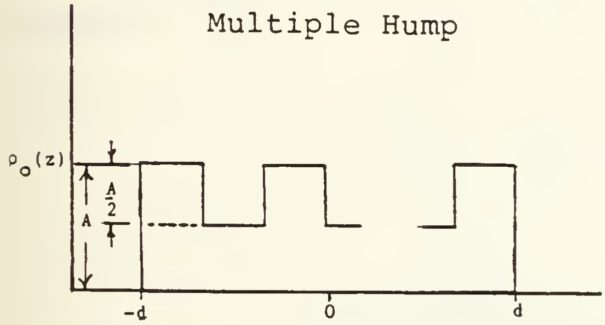
$$F(k_z) = e^{-\left(\frac{k_z b}{2}\right)^2}$$

where b = bunch size


Level Plus Ripple

$$F(k_z) = \frac{\text{Sin } k_z d}{k_z d} + \frac{R}{2}\left(\frac{\text{Sin } (k_z - k_o)\, d}{(k_z - k_o)\, d} + \frac{\text{Sin } (k_z + k_o)\, d}{(k_z + k_o)\, d}\right)$$

where R = B/A $\qquad k_o = 2\pi/\lambda$


Multiple Hump

$$F(k_z) = \frac{2\ \text{Sin } k_z d}{k_z d} - \left.\frac{\text{Sin } k_z d\left(\frac{2N-3}{2N-1}\right)}{k_z d}\right|_{N=2} + \left.\frac{\text{Sin } k_z d\left(\frac{2N-5}{2N-1}\right)}{k_z d}\right|_{N=3} - +$$

where N = number of humps

Figure 2. Electron Beam Charge Bunch Shapes and Corresponding Form Factors

Radiated energy from each pulse shape form was calculated and plotted for the 1st through 12th, 18th, 24th, 30th, 36th, 42nd, 48th, 54th, and 60th harmonics of the 50 Mhz base frequency. The 6th harmonic equated to a radiation wavelength of the same size as the electron bunch, approximately 1 meter. Multiples of the 6th harmonic corresponded to multiple numbers of wavelengths within the 1 meter length (i.e. 60th harmonic equated to a radiation wavelength of 0.1 meters, and a 1 meter electron bunch is 10 radiation wavelengths long). Beam pulse charge distributions of variable geometry were initially restricted as follows:

1. The level plus sinusoidal ripple combination function consisted of a ripple of 3 complete cycles with a maximum amplitude of one half the level function height.

2. The multiple hump function consisted of 2 humps, one at each end of the pulse. The length of the hump for a 2 hump function is one third of the pulse length. (Note: the length of the hump = $(2N-1)^{-1}$ times the pulse length, where N is the number of humps.)

3. The rounded and trapezoidal functions had a top length equal to one half their base length.

Appendices A through G contain the radiation intensity plots. In each case the radiated energy dimensions are watts per unit solid radian angle.

A. LEVEL FUNCTION

Figure 3 shows the radiation intensity lobe development between the 1st and 12th harmonics. For the 1st through 4th

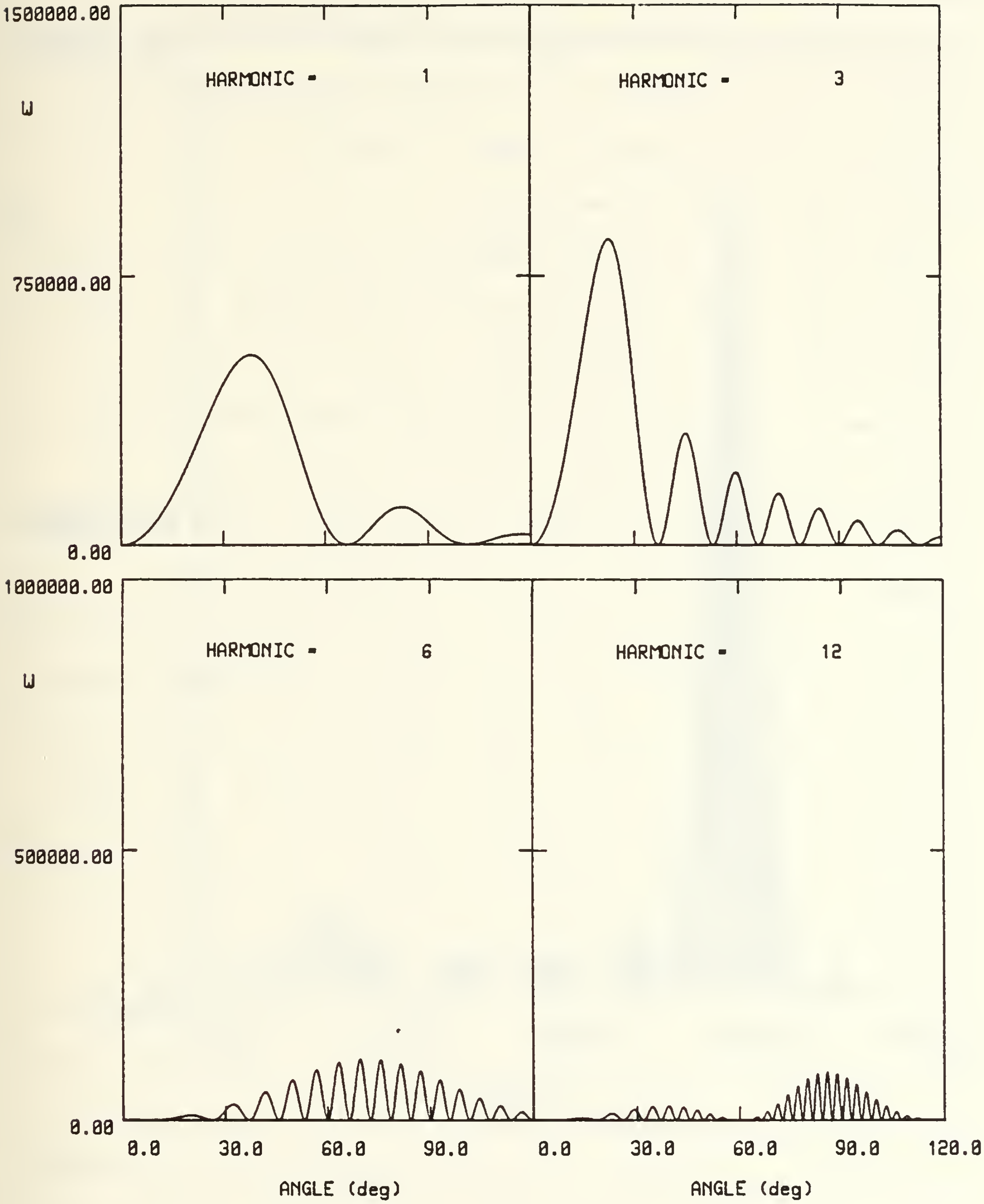


Figure 3. Level Function 1st, 3rd, 6th, and 12th Harmonics

harmonic the radiation patterns are characteristic of point charges; that is, the radiation wavelengths are long compared to the length of the electron bunch. Consequently k_z is small over the entire range of angles and $F(k_z)$ never departs significantly from unity.

Beginning with the 5th harmonic the effect of the form factor becomes apparent, and by the 6th harmonic the form factor pattern is well developed. In general, as the harmonic number increased, the radiation lobes have smaller angular separation and approach the Cerenkov angle. For these parameters, the Cerenkov interference lobes have been replaced by the oscillating pattern whose envelope is related to the form factor at the sixth harmonic. Beginning with the 6th harmonic, with the radiation wavelength equal to the charge bunch length, the radiation pattern takes on the characteristics of the Fourier transform of the bunch shape. The oscillating radiation pattern is contained in an envelope given by the square of the sinc function (sinc x = (sin x/x), centered at $\theta = 90^{\circ}$, and modulated by the Cerenkov radiation envelope. The 6th through 12th harmonic radiation plots show the development of the envelope of the squared sinc function--periodic increase and decrease of the first interference lobe and the production of subsequent lobes. Figure 4 is a radiation intensity plot for the 30th harmonic. It shows the envelope of the square of the sinc

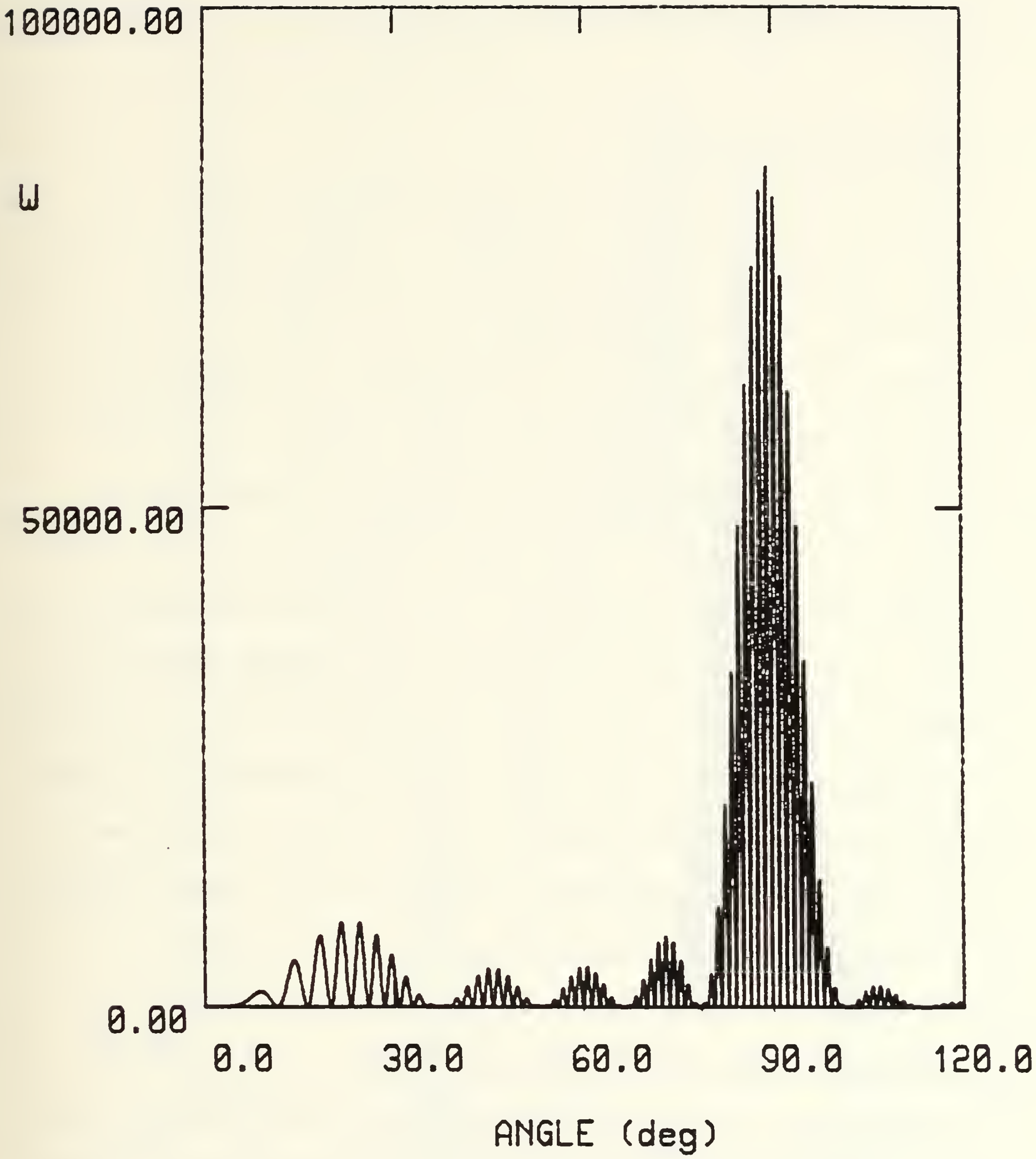


Figure 4. Level Function 30th Harmonic (1.5 GHz)

function modulated on the Cerenkov Radiation envelope. Other level function radiation intensity plots are contained in Appendix A.

B. TRIANGULAR FUNCTION

For the 1st through 11th harmonic the triangular function exhibited the characteristic radiation pattern from point charges traveling through a finite length. For the 1st through 3rd harmonics, as the harmonic increased, the triangular function interference lobes shifted to a lower angle with decreasing angular separation. Because of the increase in the Cerenkov radiation envelope, as θ decreases to θc, the lobe peaks increased. For the 3rd through 5th harmonic, the lobe shift and decreasing angular separation continued, however the first lobe of each harmonic decreased as a consequence of the decrease in the Cerenkov radiation envelope as θ decreases from θc to 0. At the 6th harmonic, with the radiation wavelength the same as the bunch length, the radiation pattern envelope began to shift to a relatively symmetric pattern centered around $\theta = 90^{\circ}$.

Beginning with the 12th harmonic, with the radiation wavelength equal to the average width of the function (i.e. one half the function's base length), the envelope of the radiation pattern takes on the characteristics of the Fourier transform of the charge bunch shape. The interference lobes are contained in an envelope given by the

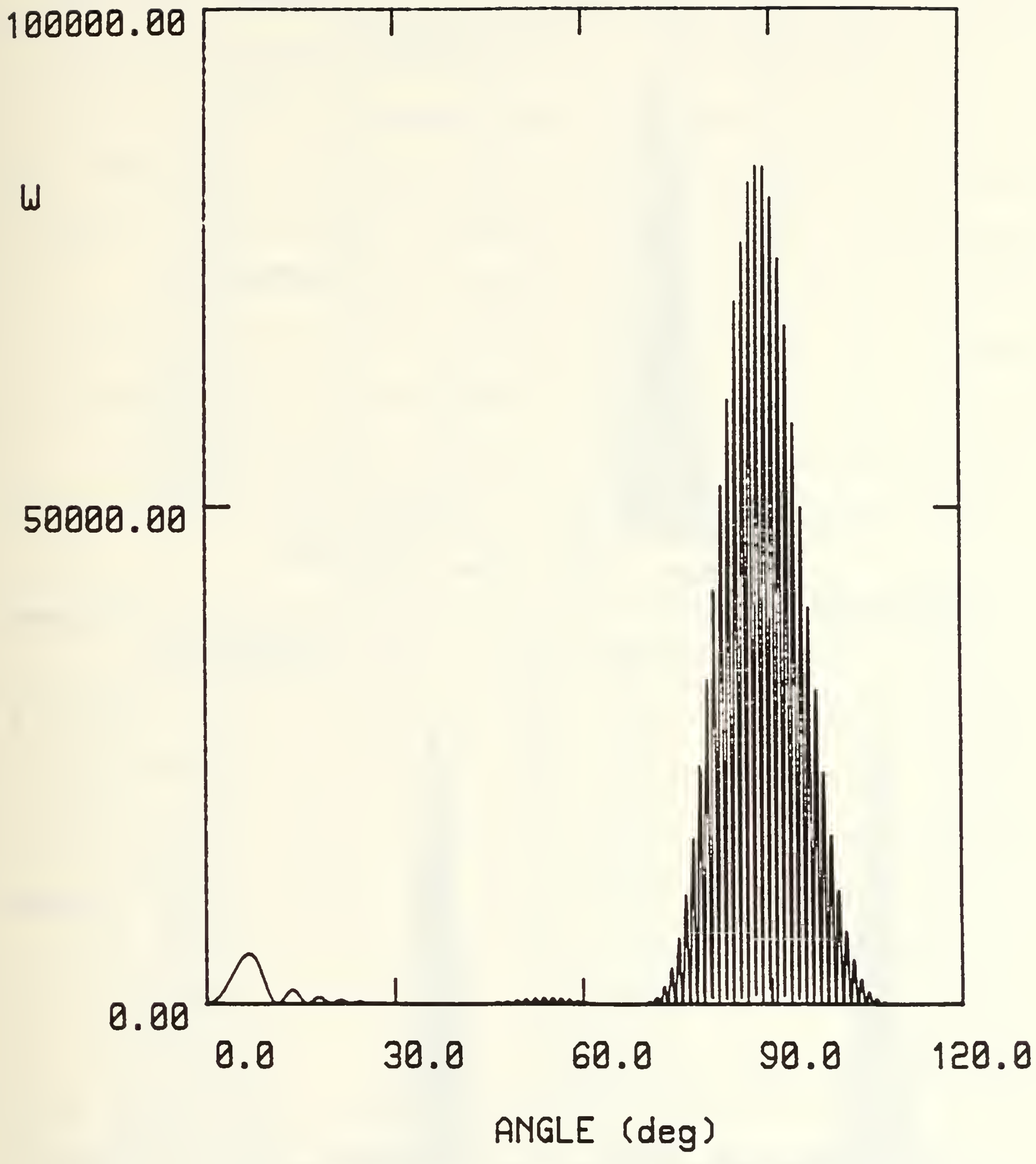


Figure 5. Triangular Function 30th Harmonic (1.5 GHz)

4th power of the sinc function modulated on the Cerenkov radiation envelope. The fourth power of the sinc function greatly reduces the side lobes. Analysis of the zeroes of the lobe envelope indicates the triangular function has an equivalent charge shape length, for radiation wavelength comparison, of one half the triangular function base length. Figure 5 is the radiation intensity plot for the 30th harmonic. It shows the fourth power of the sinc function, with its suppressed sidelobes, modulated on the Cerenkov radiation envelope. Other triangular function radiation intensity plots are contained in Appendix B.

C. TRAPEZOIDAL FUNCTION

For the 1st through 8th harmonic, the trapezoidal function interference lobes closely resemble the triangular function lobes. The difference is that the first peak and subsequent peaks, to a lesser extent, are less intense than the triangular function lobes. As the harmonic number increases, the degree of intensity decline increases. At the 6th harmonic the peak of the first lobe is one half of the triangular function's first peak.

At the 9th harmonic, with the radiation wavelength equal to the average width of the function, the interference lobes take on the characteristic of the Fourier transform of the bunch shape. For the 9th through 60th harmonic, the trapezoidal function interference lobes are between the

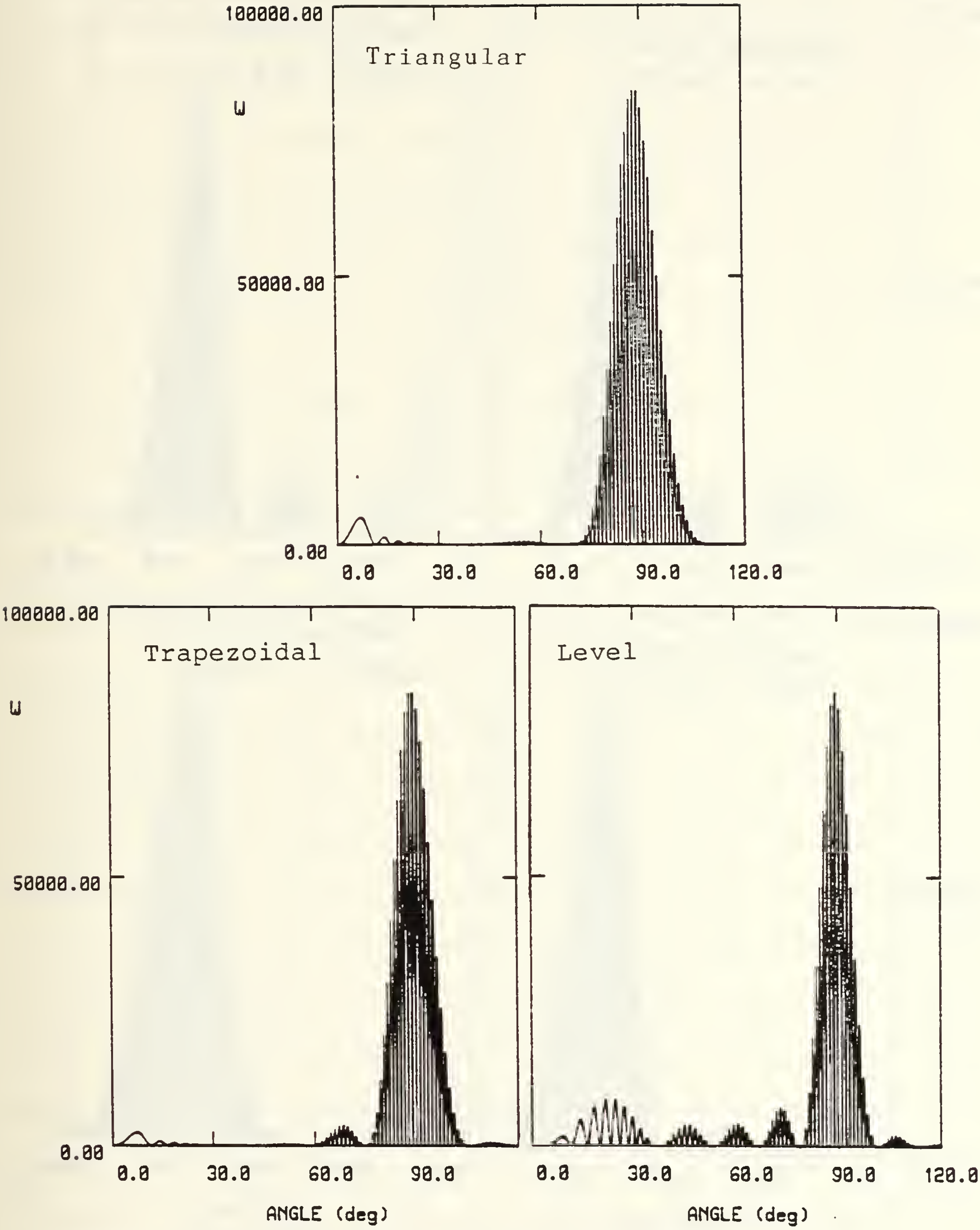


Figure 6. Triangular, Trapezoidal, and Level Functions 30th Harmonic (1.5 GHz)

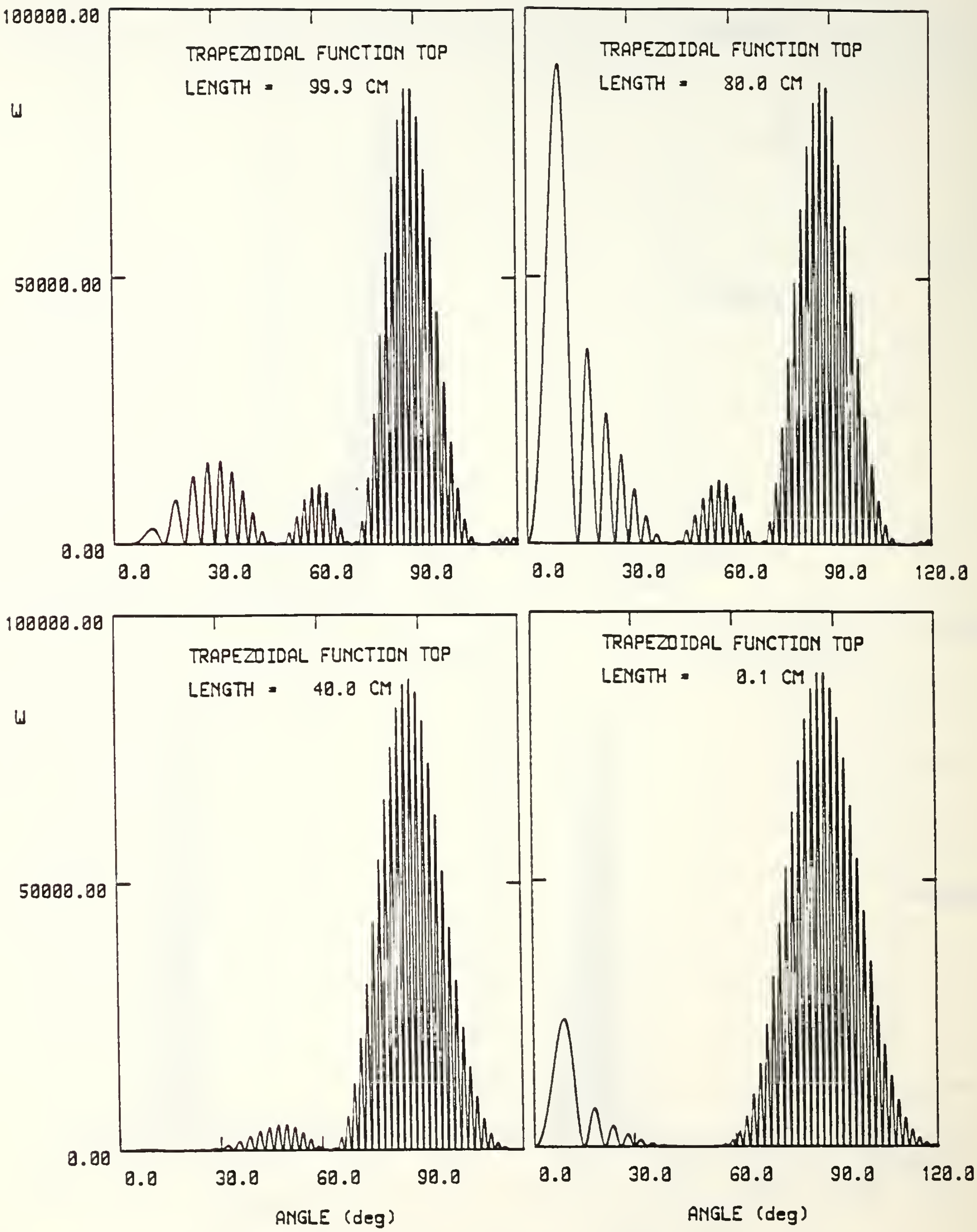


Figure 7. 99.9, 80.0, 40.0, and 0.1 cm Top Length Trapezoidal Functions 18th Harmonic (0.9 GHz)

triangular and level functions previously described. Figure 6 compares the 30th harmonic radiation intensity plots of the triangular, trapezoidal, and level functions. The side lobes of the level and trapezoidal functions are not as supressed as in the triangular function. This is expected since a trapezoid can be considered a triangle with the sides pulled out, or a rectangle with the sides pushed in.

Further analysis was conducted at the 18th harmonic by varying the top length of the function from 99.9 cm to 0.1 cm. Figure 7 compares the 18th harmonic radiation intensity pattern of the 99.9, 80.0, 40.0, and 0.1 cm top length trapezoidal functions. A top length of 99.9 cm approximated a level function and produced a level function radiation pattern. As the top length was shortened to 80 cm, the first lobe envelope significantly changed with a large increase in the first interference lobe and smaller increases in succeeding lobes. The two remaining lobe envelopes were slightly broadened. Further shortening of the top length to 40 cm caused the first lobe envelope to reverse its trend, to shrink and disappear, and the second lobe envelope to decrease to one half its original peak and to broaden. The third envelope lobe, centered near 90°, remained with the same peak and slightly broadened. Continued reduction of the top length to 0.1 cm caused the redevelopment of the first lobe envelope and the disappearance of the second lobe envelope; the result is the triangular function radiation pattern. Other trapezoidal function radiation intensity plots are contained in Appendix C.

D. ROUNDED FUNCTION

For the 1st through 8th harmonic, the rounded function interference lobes closely resemble the triangular function lobes. Again, the difference is that the first peak and subsequent peaks, to a lesser extent, are less intense than the triangular function lobes. As the harmonic increases, the degree of intensity decline increases. At the 6th harmonic the peak of the first lobe is one half of the triangular function's first peak. This is the same result observed with the trapezoidal function.

At the 9th harmonic, with the radiation wavelength the same length as the average width of the function, the interference lobes take on the characteristic of the Fourier transform of the rounded function shape. For the 9th through 60th harmonic, the rounded function interference lobes are between the triangular and level functions previously described. Figure 8 compares the 30th harmonic radiation intensity patterns of the level, rounded, and trapezoidal functions. The side lobes are less supressed than in the similiar trapezoidal function or the triangular function. As in the trapezoidal function case, this is expected since the rounded function can be considered a

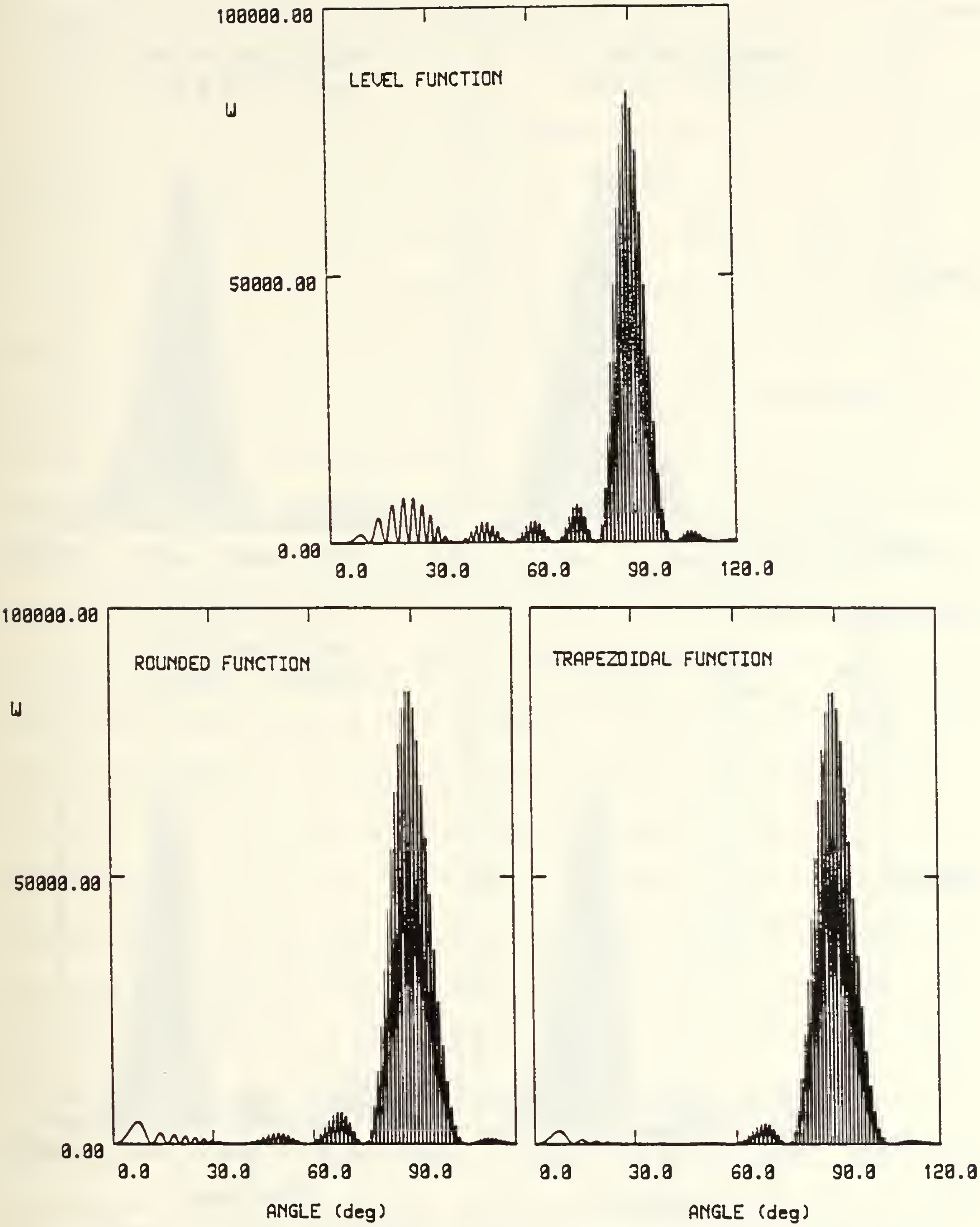


Figure 8. Level, Rounded, and Trapezoidal Functions 30th Harmonic (1.5 GHz)

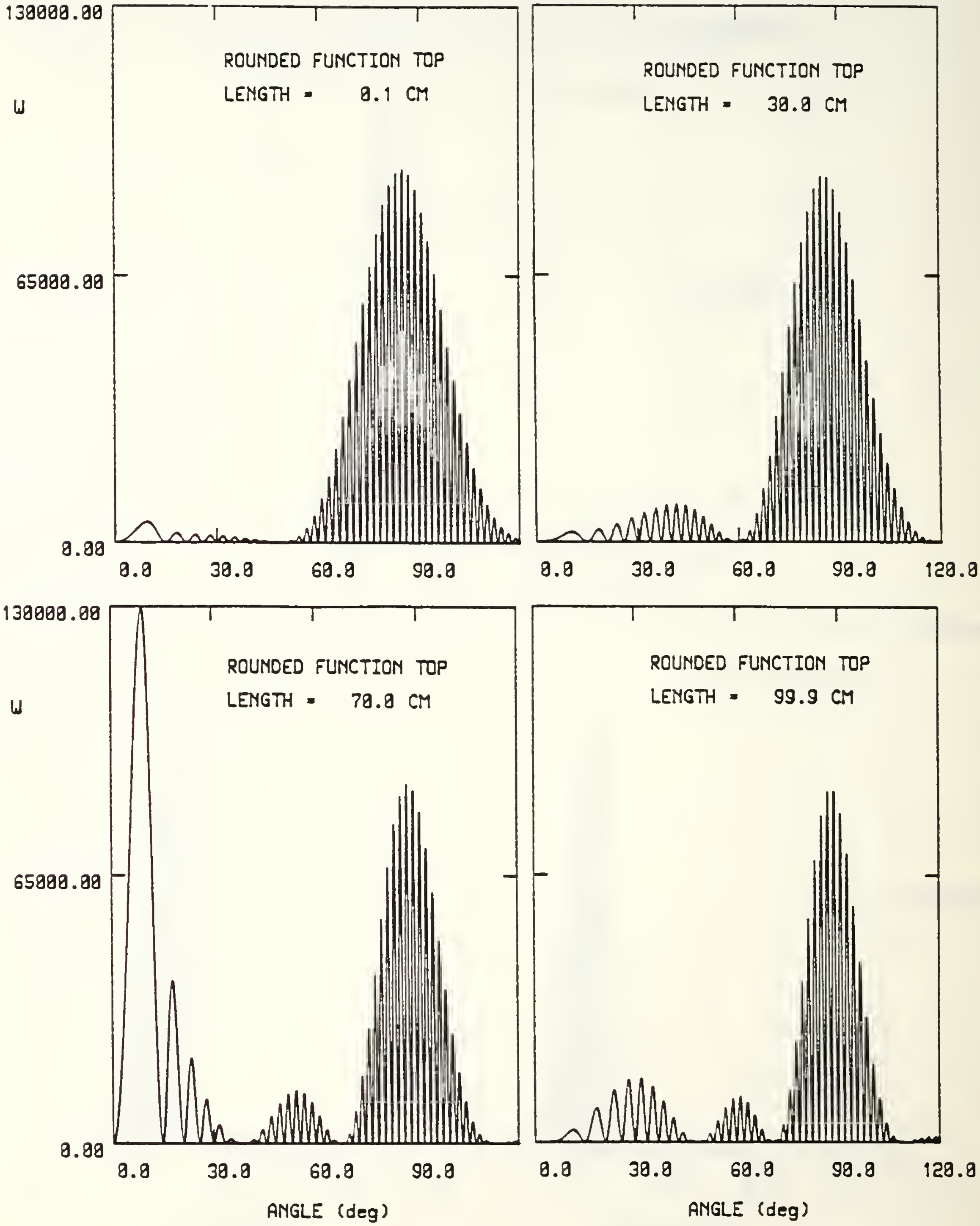


Figure 9. 0.1, 30.0, 70.0, and 99.9 cm Top Length Rounded Functions 18th Harmonic (0.9 GHz)

triangle or gaussian with the sides pulled out, or a rectangle with the sides pushed in.

Further analysis was conducted at the 18th harmonic by varying the top length of the function from 99.9 to 0.1 cm. Figure 9 compares the radiation intensity pattern of the 0.1, 30.0, 70.0, and 99.9 cm top length rounded functions. A top length of 99.9 cm approximated a level function and produced a level function radiation pattern. As the top length was shortened to 70 cm, the first lobe envelope significantly changed with a large increase in the first interference lobe and smaller increases in succeeding lobes. This increase was approximately 40% larger than the increase experienced from the trapezoidal function. The second lobe envelope shifted 10 degrees towards the Cerenkov angle, and the third envelope was not changed. Further shortening of the top length to 30 cm caused the first lobe envelope to reverse its trend, to shrink and be absorbed by the broadening second lobe envelope. The third lobe envelope was slightly broadened and remained centered near 90 degrees. Continued reduction of the top length to 0.1 cm caused suppression of the second lobe envelope and further broadening of the third lobe envelope. The result is a triangular function pattern with a supressed first lobe envelope. Other rounded function radiation intensity plots are contained in Appendix D.

E. GAUSSIAN FUNCTION

For the 1st through 3rd harmonic the Gaussian function exhibited the characteristic geometrical radiation pattern from point charges traveling through a finite path length. As the harmonic increased, interference lobes approached the Cerenkov angle with decreasing angle separation and with decreasing radiation intensity. However, with the 4th harmonic there was a transition to a pattern where the radiation is concentrated in an envelope around 90°. The asymmetric lobe envelope is a narrow Gaussian function modulated on the Cerenkov radiation envelope. Transition was not expected until near the 6th harmonic where the radiation wavelength is approximately the same as the charge bunch length. One possible explanation is that because the Gaussian goes to zero at infinity, the effective charge bunch length appears to be as long as the radiation wavelength of the 4th harmonic. Figure 10 is a radiation intensity plot for the 30th harmonic (1.5 GHz). Here the radiation around 90° is fully developed while that in the forward direction is completely suppressed. The ratio of the peak height for the 30th harmonic compared to that for the fundamental frequency is about 1/5--surprisingly large.

The 54th and 60th harmonics could not be calculated because of computer limitations. Other Gaussian function radiation intensity plots are contained in Appendix E.

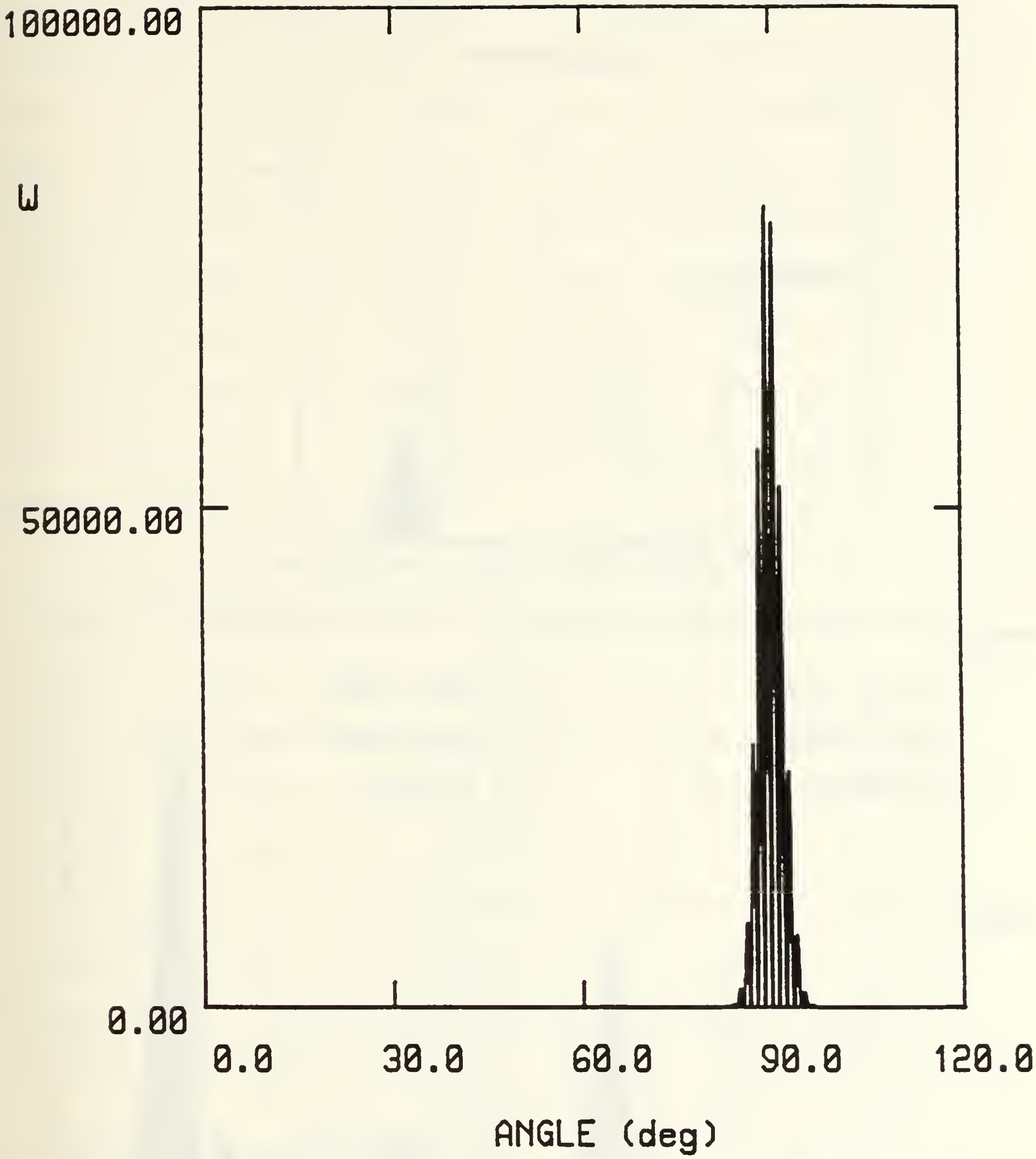


Figure 10. Gaussian Function 30th Harmonic (1.5 GHz)

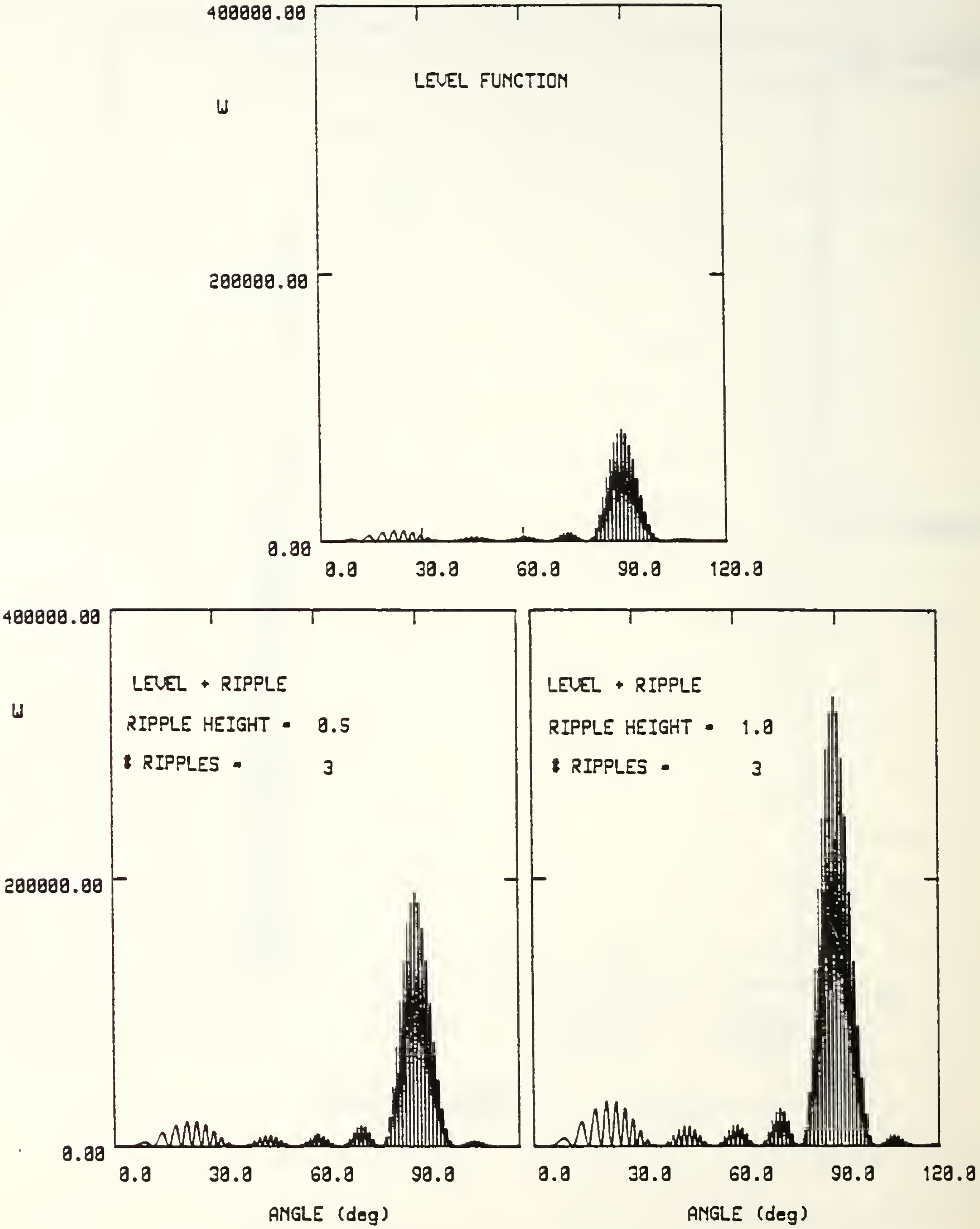


Figure 11. Level and Level Plus Ripple Combination Functions 30th Harmonic (1.5 GHz)

F. LEVEL PLUS RIPPLE COMBINATION FUNCTION

The level plus sinusoidal ripple combination function radiation intensity pattern is an amplification of the level function pattern. The amplification is independent of angle and harmonic number. The degree of amplification depends on the ratio of the ripple amplitude to the level function height. Figure 11 compares the 30th harmonic radiation intensity patterns of the level function, and 0.5 and 1.0 amplitude ratio combination functions. For a ratio of 0.5 the amplification is 2.2, and for a ratio of 1.0 the amplification is 4.0.

Increasing the number of ripple cycles produced puzzling results. The amplification remained constant until the number of cycles reached 16, then the amplification ceased. The resulting pattern was the same as a level function. Shortening the pulse to one half its original 100 cm length changed the transition point to 8 cycles.

There was insufficient time to explore why the above results occurred. The unusual results warrant further study. Other level plus ripple combination function radiation intensity plots are contained in Appendix F.

G. MULTIPLE HUMP FUNCTION

The multiple hump function is a combination of positive and negative level functions. Figure 2 descibes the corresponding form factor. The form factor for a N hump function is the first N terms of the equation presented.

The multiple hump function radiation intensity pattern was a modification of the level function pattern. The fourier transform of the multiple hump function is a combination of sinc functions of a longer scale than the level function transform. Manual calculations could not confirm, but it is believed, that the products of the individual transforms in $F(k)^2$ produce selective amplification of the level function radiation intensity lobe envelopes.

For a two hump function, as the harmonic increased, the the first envelope lobes of the 1st through 5th harmonic were reduced compared to the level function. At the 6th harmonic, with the radiation wavelength the same size as the overall function, this trend reversed. For the 6th through 11th harmonic there is up to 5 times amplification of the first envelope lobes. At the 18th harmonic, with the radiation wavelength the same size as the individual hump width, a pattern unique to the two hump function appears:

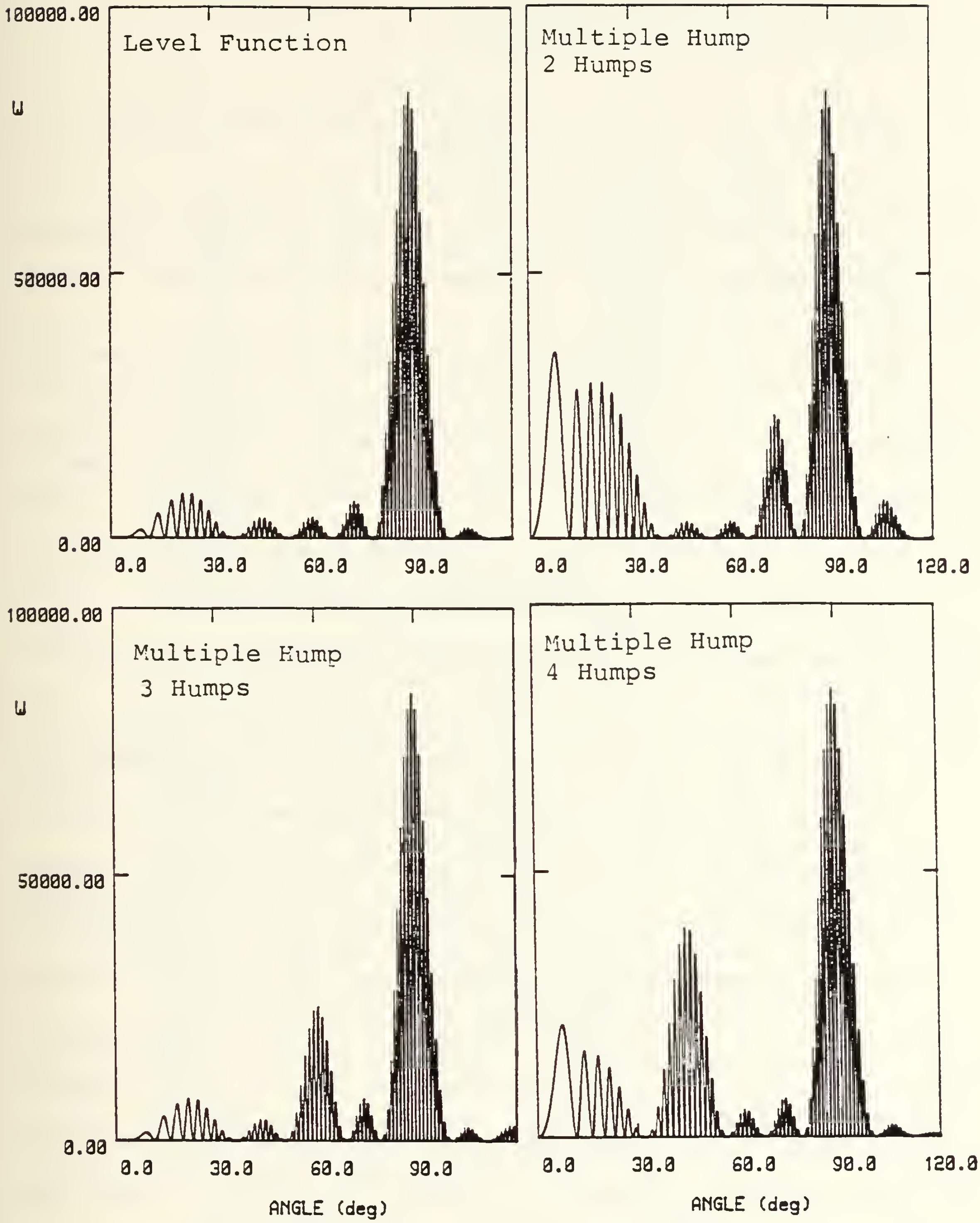


Figure 12. Level and Multiple Hump Functions
30th Harmonic (1.5 GHz)

1. The wave envelope adjacent to the 90° main envelope is always amplified by a factor of approximately 3.

2. In going to $\theta = 0$, every 3rd suceeding envelope is amplified by a factor of approximately 3. Examples are the 30th and 48th harmonic.

3. If the envelope adjacent to $\theta = 0$ is not a multiple of 3, but 2 envelopes away from the last amplified envelope, it is split to form a single lobe and a narrower lobe envelope. Examples are the 21th, 42nd, and 60th harmonic.

Figure 12 compares the 30th harmonic radiation intensity patterns for a level function and multiple hump functions with 2, 3, and 4 humps. Increasing the number of humps has the following effect:

1. The amplification is displaced outward a number of envelopes equal to the number of humps minus 2.

2. As the amplification is displaced outward, the degree of amplification increases significantly.

3. When the amplification is displaced to the envelope adjacent to $\theta = 0$, further increases in the number of humps causes amplification of the first several lobes in the envelope. Maximum amplification of the first lobe occurs when the radiation wavelength is approximately twice the size of the hump width (e.g. 60th harmonic, 10 humps). Continued increases in the number of humps causes a reduction of the envelope to form a level function radiation intensity pattern. As the number of humps goes to infinity the multiple hump function approximates a level function.

Other multiple hump function radiation intensity plots are contained in Appendix G.

IV. DISCUSSION

A. INITIAL COMPARISON

The radiation patterns of the 7 pulse shapes were compared from the perspective of determining pulse shape from a measured radiation pattern. The microwave frequency of 1.5 GHz, corresponding to the 30th harmonic emission, can be readily monitored. At this frequency, the radiation patterns of the 7 pulse shapes can be divided into 4 groups. Figure 13 compares the 4 groups. In each case the pulse shapes have the same charge and length.

1. Gaussian and Triangle Functions

Figure 13 shows the similarity of the two patterns. The Gaussian function radiation pattern is a narrow Gaussian lobe envelope centered near $\theta = 90^{o}$. The triangular function lobe envelope is the fourth power of a sinc function also centered near $\theta = 90^{o}$. Both functions are modulated on the Cerenkov radiation envelope which varies slowly near $\theta = 90^{o}$, therefore what is primarily seen are the effects of $[F(k)]^{2}$. Manual calculations verified the zeroes of the triangular function envelope correspond to a sinc function scaled to the average pulse width (i.e. one half base length for a triangular shape). The fourth power supresses the sinc function sidelobes and dominates over increases in the Cerenkov radiation envelope as θ goes to 0.

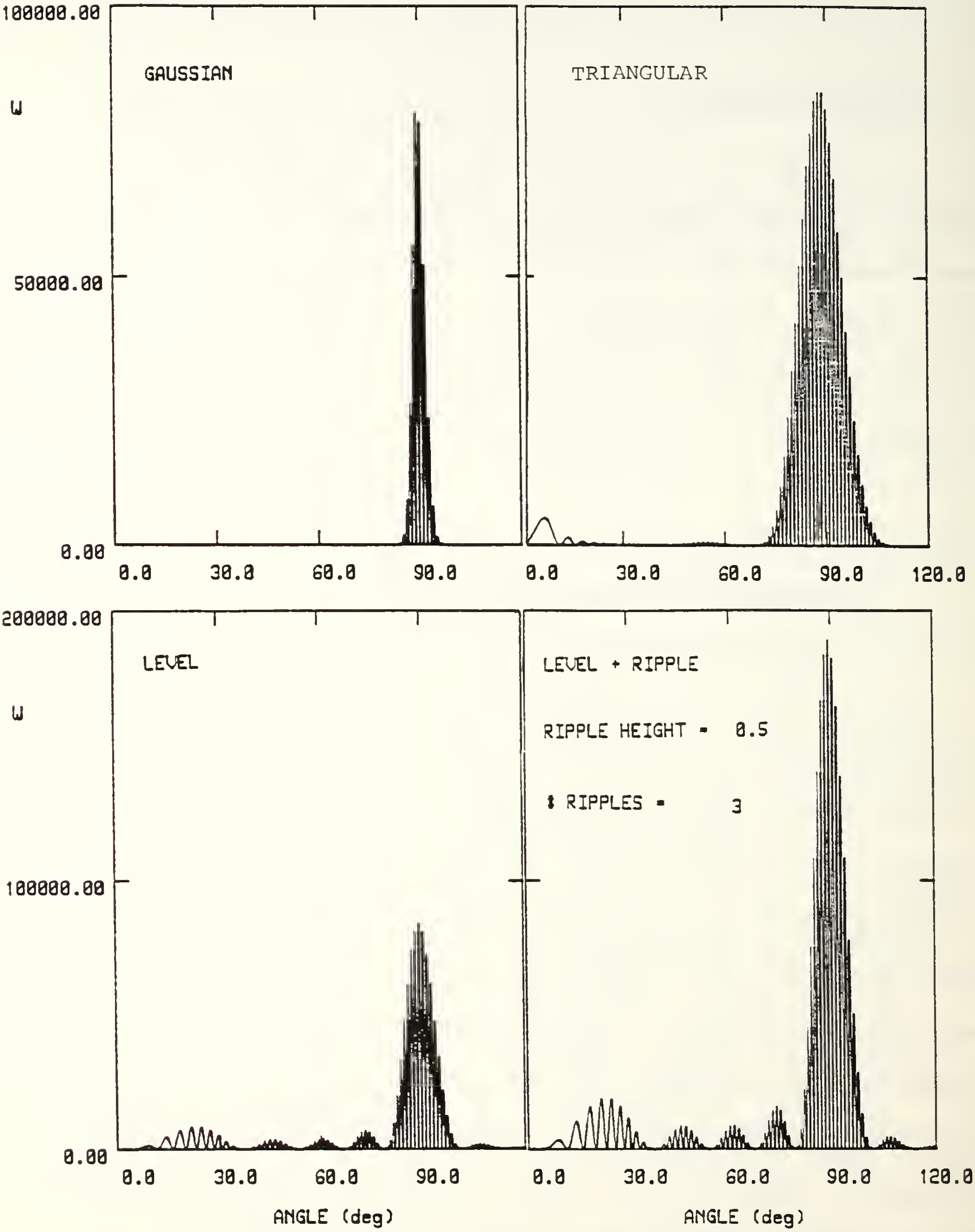


Figure 13. 30th Harmonic (1.5 GHz) Radiation Patterns

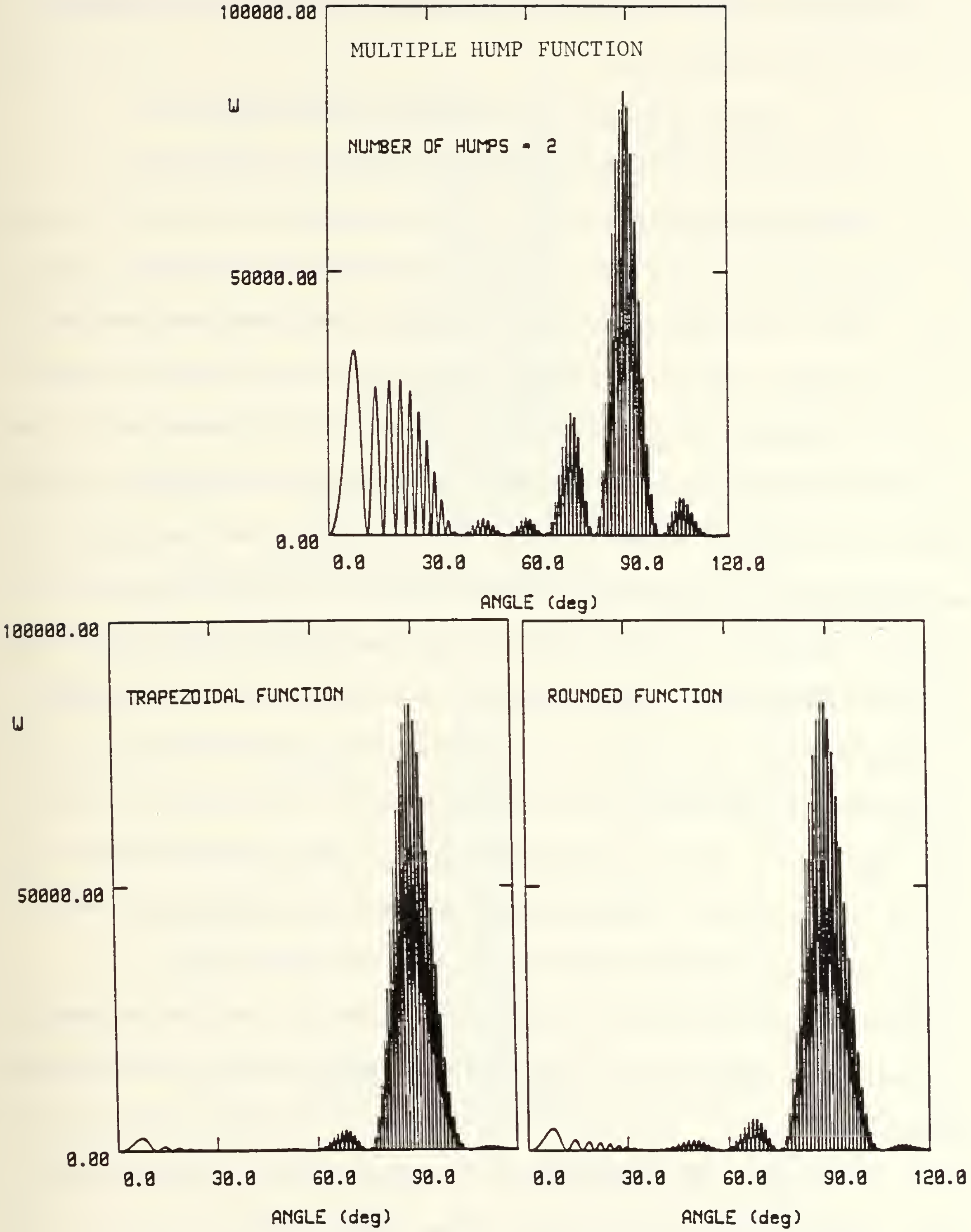


Figure 13. 30th Harmonic (1.5 GHz) Radiation Patterns

The result is that the triangular function lobe envelope is very similiar to the Gaussian function lobe envelope which does not have sidelobes.

2. Level and Level Plus Ripple Functions

The level function radiation pattern lobe envelope is the second power of a sinc function, centered near $\theta = 90^{o}$, and modulated on the Cerenkov radiation envelope. Manual calculations verified the zeroes and peak heights of the envelope. The level plus ripple function lobe envelope is a uniform amplification of the level function pattern. The amplification is independent of harmonic and angle. The degree of amplification depends on the ratio of the ripple amplitude to the level function height. Increasing the ripple frequency does not affect the amplification until a specific number of ripple cycles are within the pulse, then the amplification ceases. The reason for the abrupt ceasation of amplification is not known. From the perspective of determining pulse shape from a measured radiation pattern, the two function pattern shapes appear identical, differing only in the overall intensity. Therefore, one could not determine which of the two pulses caused the radiation pattern without knowledge of the charge density.

Figure 13 compares the two patterns. The level plus ripple is plotted on the same scale as the level function to show the uniform amplification. (Note: the level and level plus ripple patterns are plotted on a different scale than the Gaussian and triangular function patterns.)

3. Multiple Hump Function

The multiple hump function radiation pattern is a selective amplification of the level function pattern. The number of humps determines which of the level function sidelobe envelopes are amplified and by how much. The fourier transforms of the level function combinations in the multiple hump function are sinc functions of longer scale than the level function transform. It is hypothesized that the longer scale sinc^2 function, modulated on the established level function pattern, causes the selective amplification. Figure 13 shows the radiation pattern of a multiple hump function with 2 humps. The first and fourth sidelobes are amplified compared to the level function. (Note: Figure 13 level function plot is scaled differently than the multiple hump function plot.)

4. Trapezoidal and Rounded Functions

Figure 13 shows the similarity between the trapezoidal and rounded function radiation patterns. The two patterns are sinc function variations which are in between the level and triangular function patterns. Manual calculations of the zeroes of a standard sinc function could not find a scale length to fit the patterns; therefore, the patterns are not readily described by powers of the standard

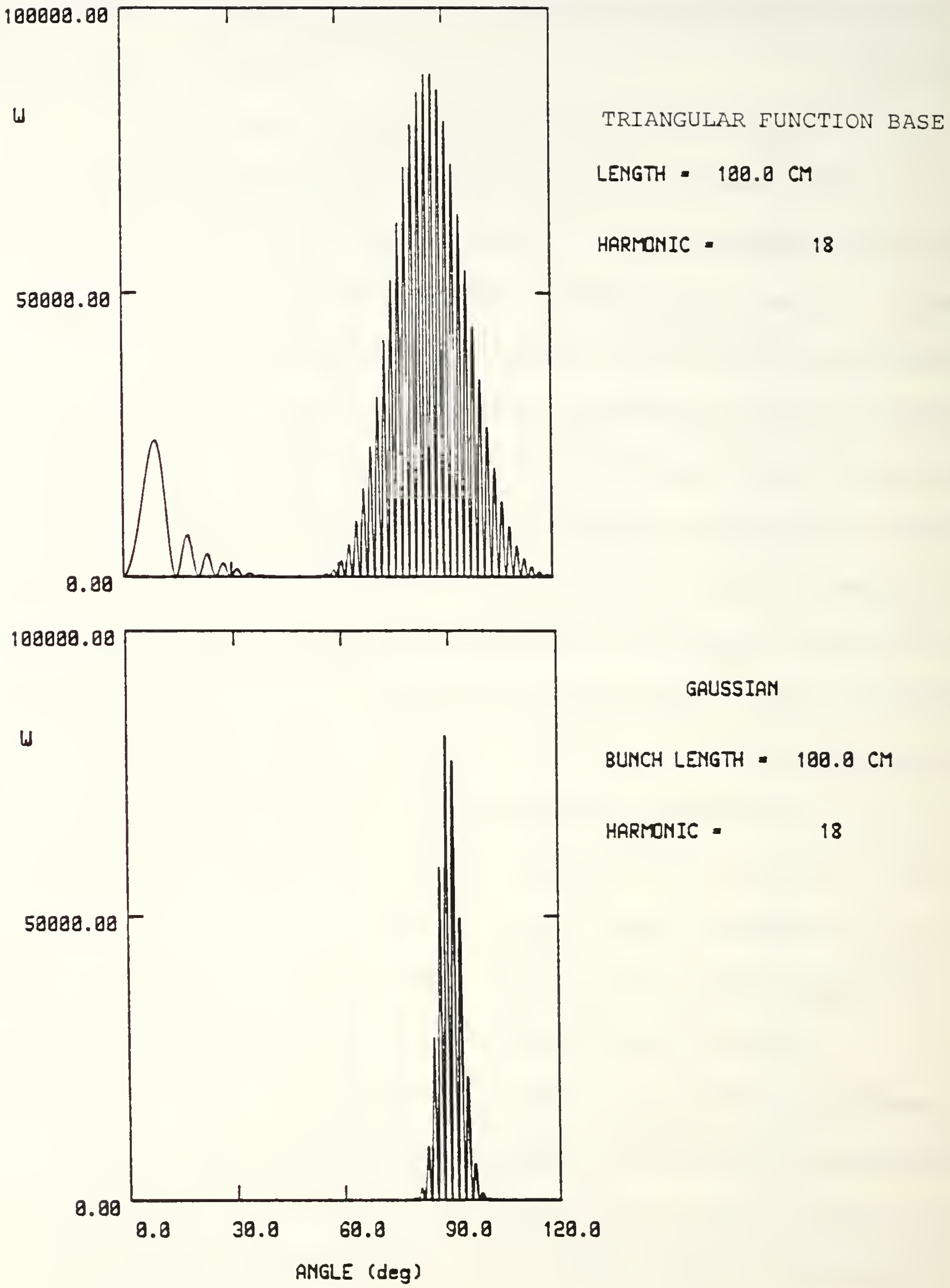


Figure 14. Triangular and Gaussian Functions
18th Harmonic (0.9 GHz)

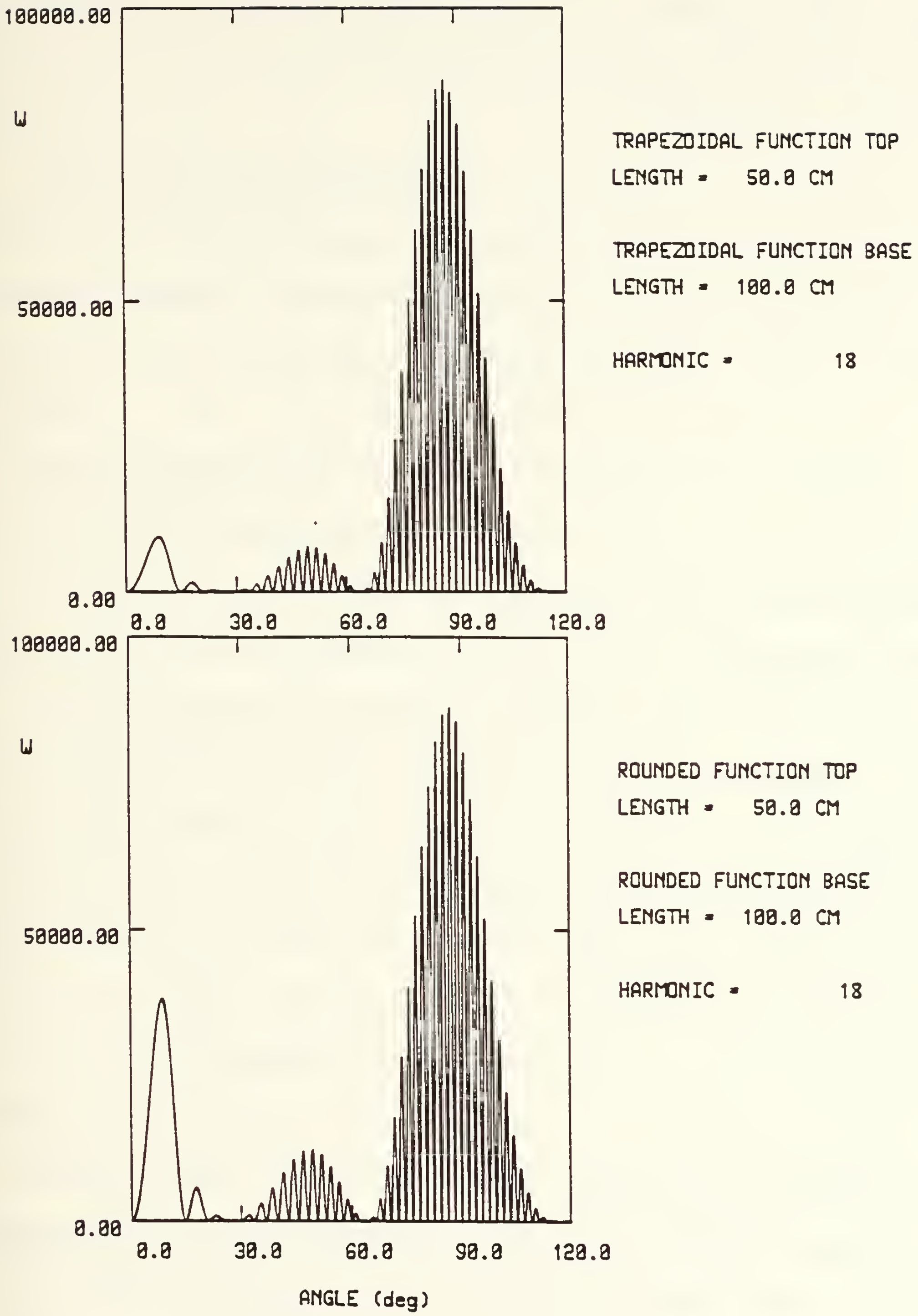


Figure 15. Trapezoidal and Rounded Functions 18th Harmonic (0.9 GHz)

sinc function. At 1.5 GHz, the two functions have similiar sidelobe suppression which makes a pulse shape determination between the two difficult.

B. FURTHER COMPARISON

In order to discriminate between the Gaussian and triangular, and the trapezoidal and rounded functions, the radiation patterns generated at a different frequency were examined. Figure 14 contrasts the Gaussian and triangular function patterns at the 18th harmonic, 0.9 GHz. At the 18th harmonic, the Gaussian function retained its narrow envelope centered near $\theta = 90^{\circ}$. The triangular function envelope became very broad and developed a second lobe envelope adjacent to $\theta = 0^{\circ}$. The change in the triangular function pattern reflects the change in sinc function scaling, modulated on the Cerenkov radiation envelope.

Figure 15 contrasts the trapezoidal and rounded functions. At the 18th harmonic, the difference in sidelobe magnitude between the two functions is more apparent. The rounded function sidelobes are taller than those for the trapezoidal. It is hypothesized the rounded fuction is a sinc function variant of the same scaling, but of a smaller power than the trapezoidal function. This smaller power value causes taller sidelobes. In addition, the placement of the initial sidelobe nearer to the maximum in the Cerenkov radiation envelope causes a taller sidelobe, and magnifies the difference between the two patterns. From the perspective of determining the pulse shape, the second lobe envelope of the rounded function is 40% of the intensity of the main lobe envelope, while the trapezoidal sidelobe is only 10%.

C. ADDITIONAL OBSERVATIONS

For each bunch shape, once the radiation pattern which is characteristic of the Fourier transform of the bunch shape is established, the 90° envelope peak height is nearly constant.

Table I identifies for each charge bunch shape the harmonic which has the highest lobe peak, and the ratio of the 90° envelope peak height to the fundamental frequency peak lobe height. The level function and its derivatives have the same harmonic with the highest peak (i.e. 2nd); likewise for the triangular function and its derivatives (i.e. 3rd). However, the Gaussian function is the only function where the 1st harmonic has the highest peak value. Each charge bunch shape produces significant radiated energy at 90°. With the exception of the Gaussian, this energy is 15-16% of the fundamental frequency peak; for the Gaussian, an even larger 20%.

Figures 7 and 9 show that varying the top length of the trapezoidal and rounded functions causes changes in the intensity of the forward direction lobe envelope. With the top length equal to 80% of the trapezoidal function base length, or 70% of the rounded function base length, the

TABLE 1

ANALYSIS OF ENVELOPE PEAKS

Charge Bunch Shape	Harmonic with Highest Peak	Ratio of 90° Peak Height to Fundamental Frequency Peak
Level	2	.16
Triangular	3	.15
Trapezoidal	3	.15
Rounded	3	.15
Gaussian	1	.20
Level Plus Ripple	2	.16
Multiple Hump	2	.16

forward facing lobe greatly increases and exceeds the previously predominant 90° intensity envelope.

With the exception of the Gaussian function, each charge bunch's radiation pattern takes on the characteristic of the Fourier transform of the bunch shape when the radiation wavelength equals the average width of the shape. It is hypothesized that because the Gaussian function goes to zero at infinity, its radiation pattern acquires the characteric of the Fourier transform of the pulse at a radiation wavelength longer than the charge bunch.

The Cerenkov radiation envelope, which decreases as θ increases beyond θc, causes normally symmetric envelopes to be asymmetric. Near $\theta = 90^{\circ}$, the Cerenkov radiation envelope varies slowly, therefore what is seen are primarily the effects from $|F(k)|^2$.

V. CONCLUSIONS AND RECOMMENDATIONS

This work contains a large library of patterns in the 90° region from various charge distributions that might be encountered. Near $\theta = 90^{\circ}$ the radiation intensity envelope represents the square of the bunch form factor F(k). With the exception of the level plus ripple, each bunch shape causes an unique radiation pattern at specific frequencies which should permit identification of the bunch shape through sidelobe analysis. The level plus ripple bunch shape produces the same radiation pattern form as the level bunch shape; therefore, the radiation pattern could not be used to differentiate between the two. Several important results were:

1. Varying the top lengths of the trapezoidal and rounded pulse shapes causes significantly large enhancements of the forward lobe envelope.
2. Each bunch shape produces significant radiated energy 90° to the side of the beam. The 90° peak of the energy is nearly constant for each harmonic of a particular bunch shape.

To augment this preliminary research the following is recommended:

1. Determine why the level plus ripple combination function behaves so unusally. Why is the amplification uniform, independent of harmonic and angle, and yet abruptly disappears at a specific number of cycles?

2. Confirm that sinc functions of different scale cause the multihump function's selective and non-uniform lobe envelope amplification.

3. Determine why varying the top lengths of the trapezoidal and rounded pulse shapes causes significant enhancement of the forward lobe envelope.

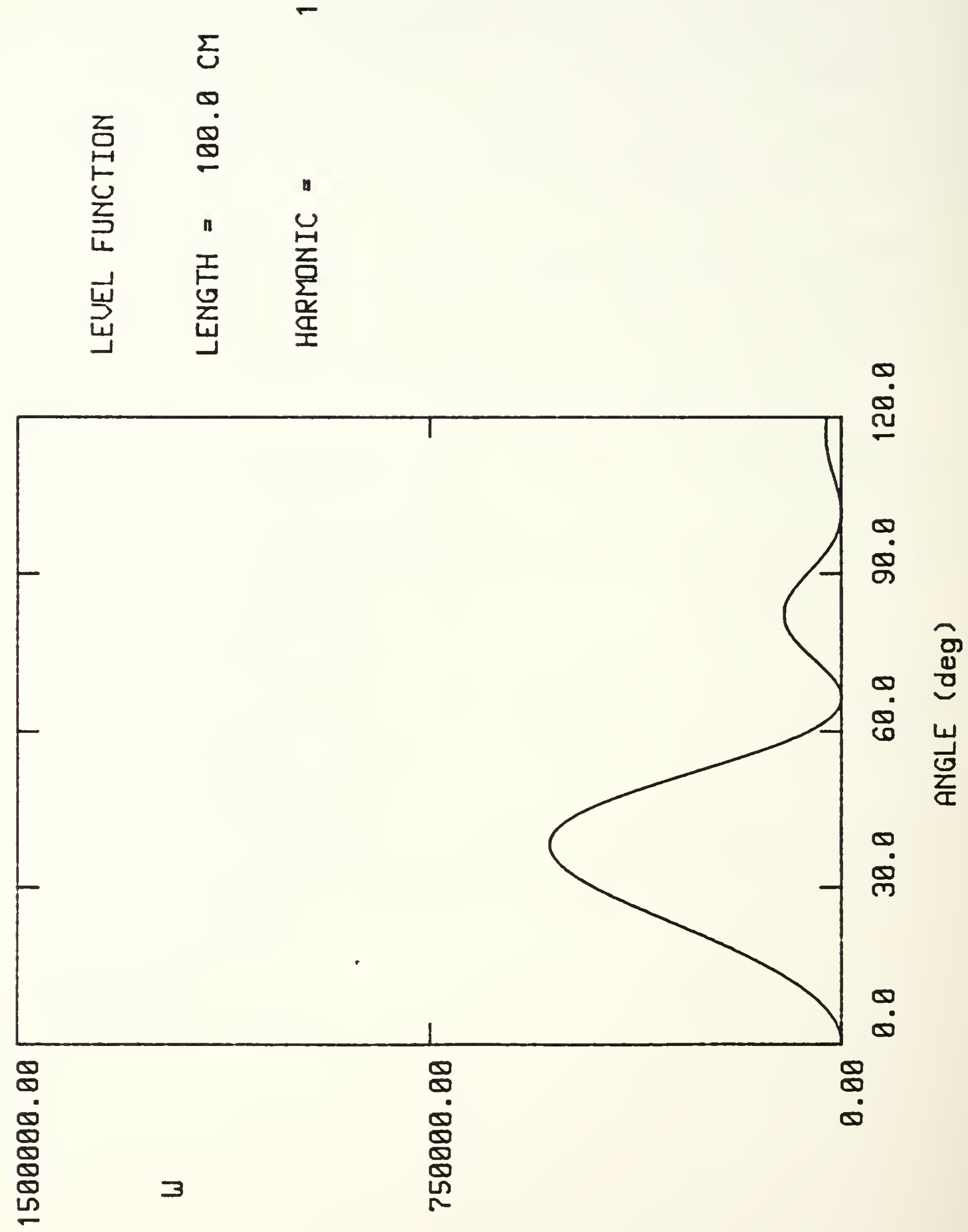
4. Determine why each pulse shape produces significantly large radiation energy 90° to the side of the beam.

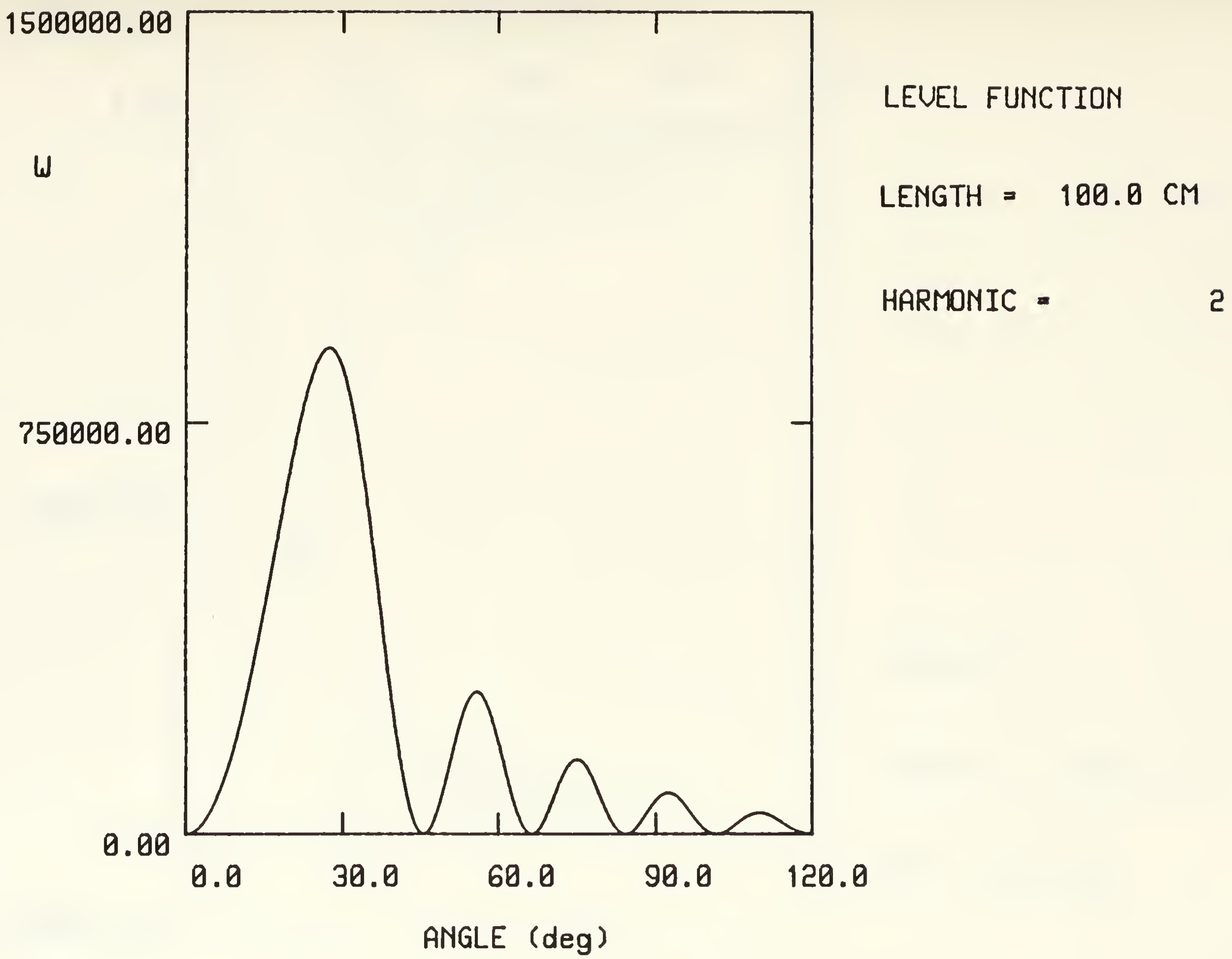
5. Determine the effects of changes in beam energy, pulse width, pulse frequency, and path length.

6. Confirm the results found by mapping the microwave radiation measured at a comparable particle accelerator.

The results presented did not take into account atmospheric attenuation and ground plane reflection [Ref. 4:p.1996]. However, they do provide an excellent reference from which to begin to establish a method of determining the pulse shape of relativistic electron beams based on Cerenkov radiation patterns.

APPENDIX A: LEVEL FUNCTION

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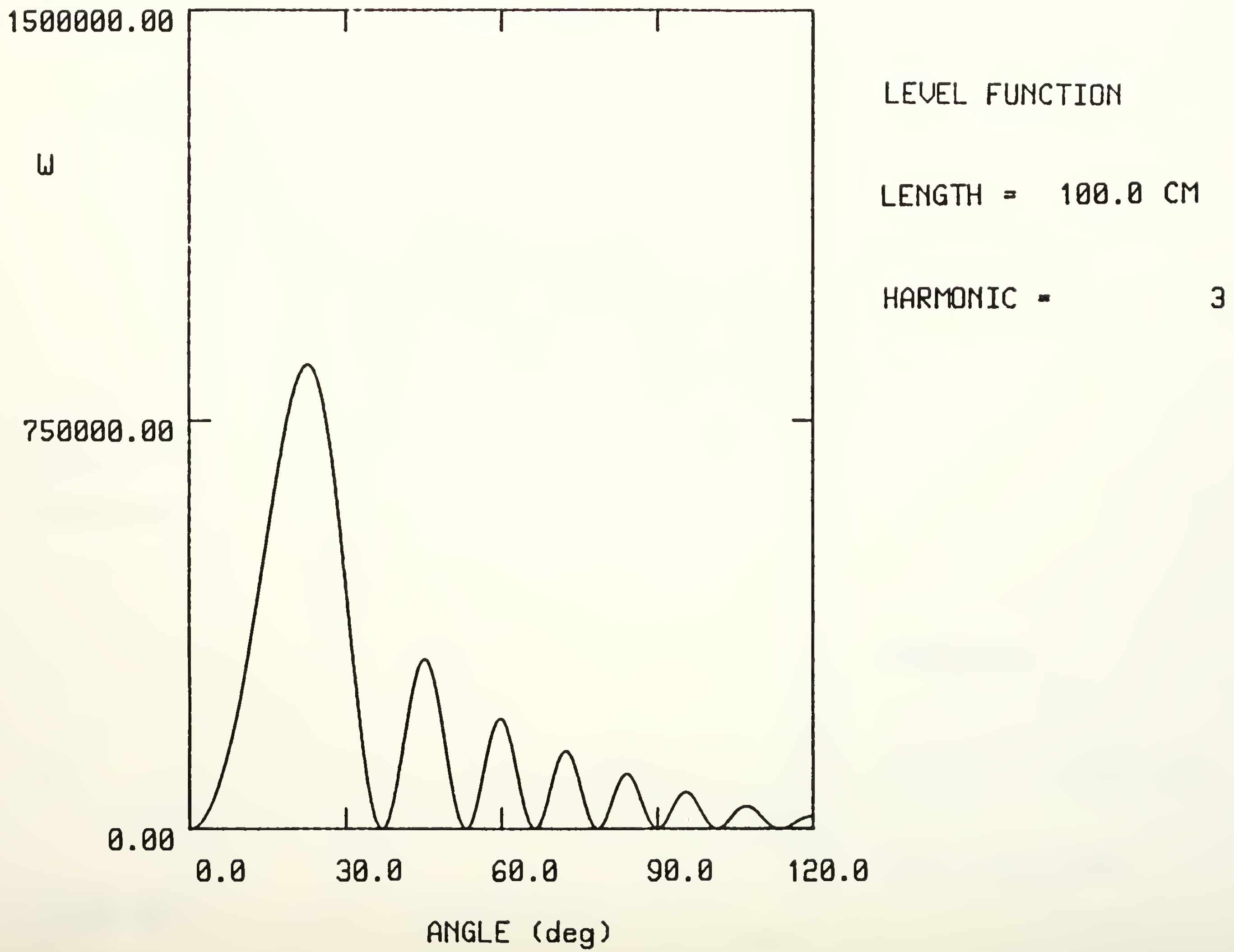
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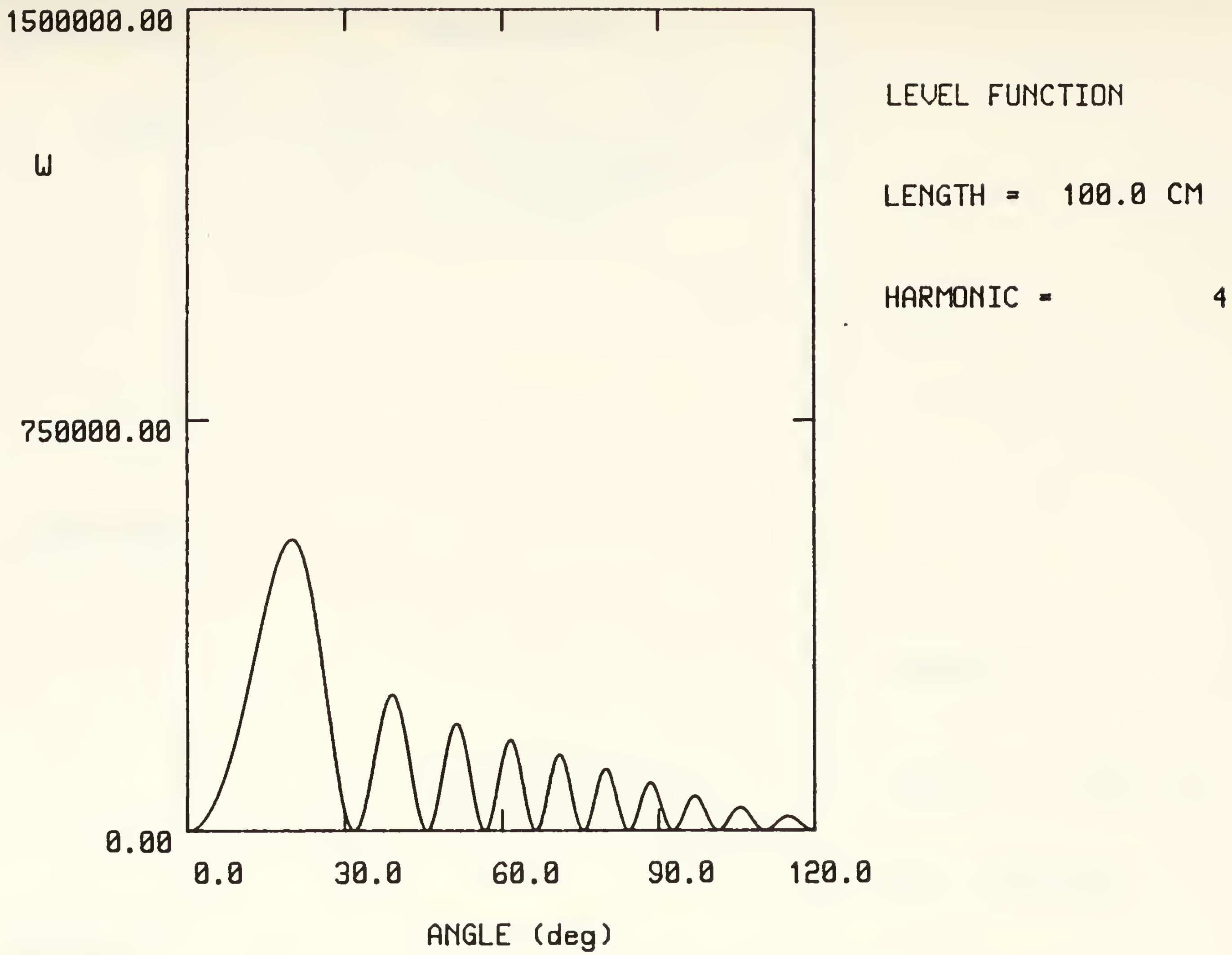
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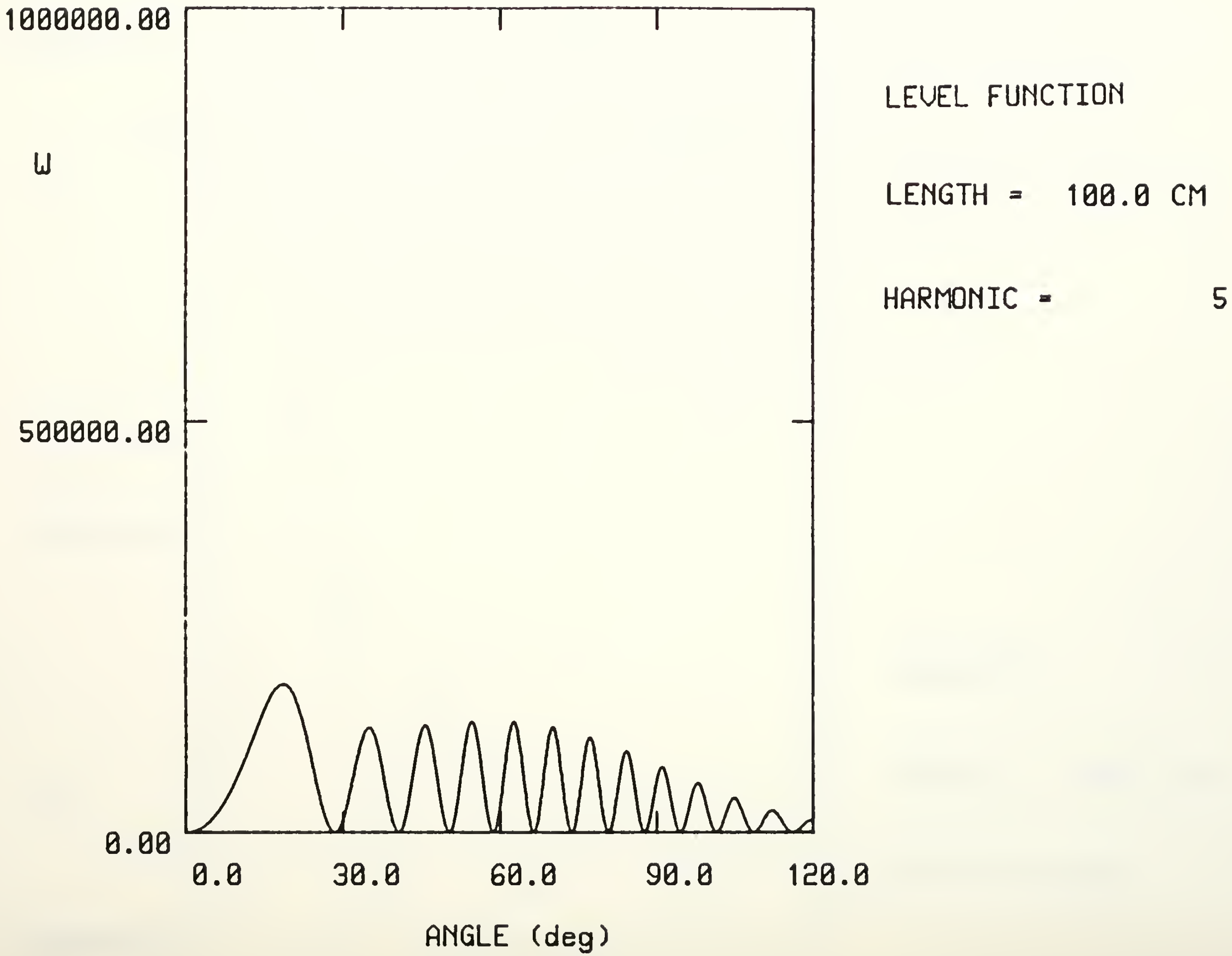
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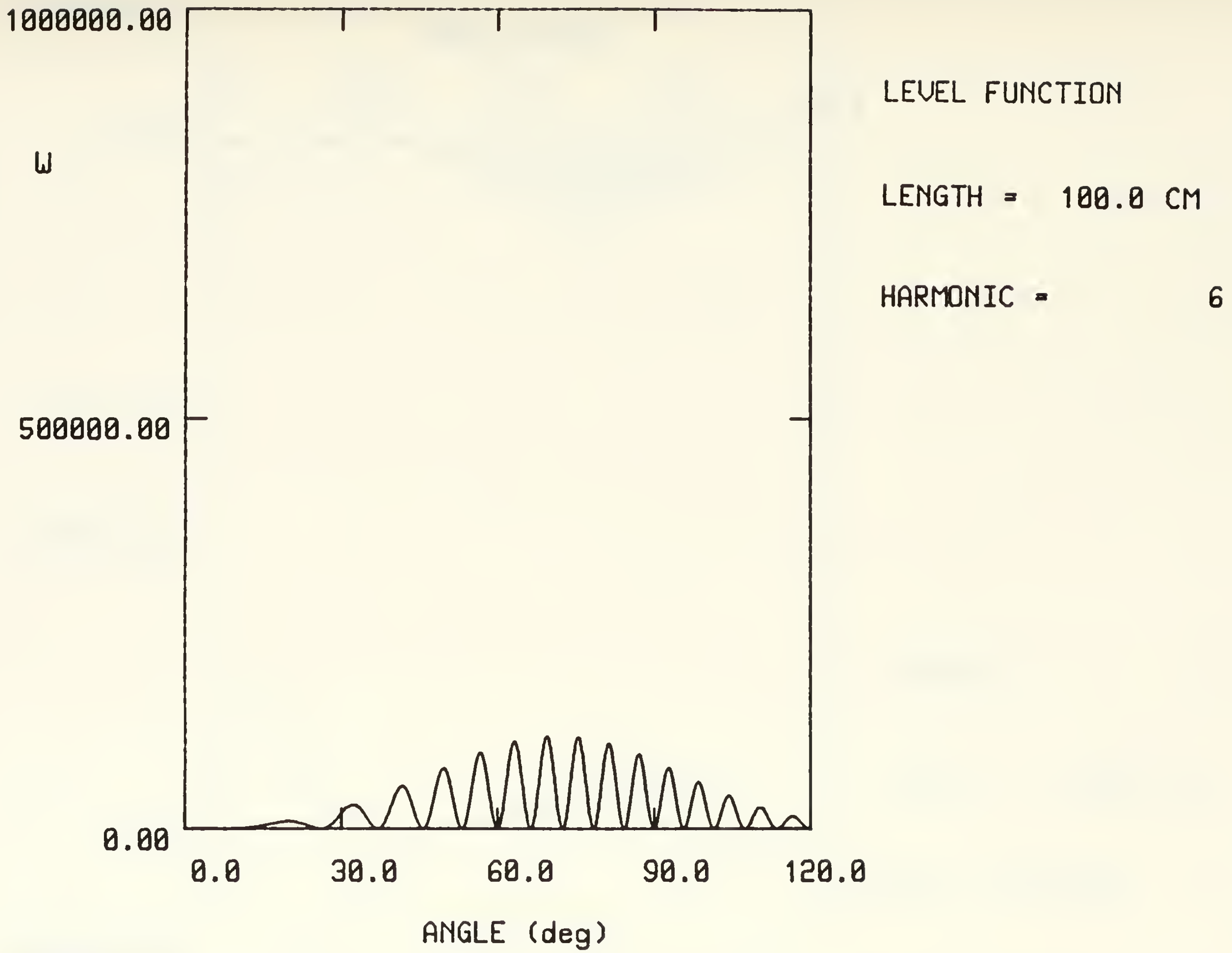
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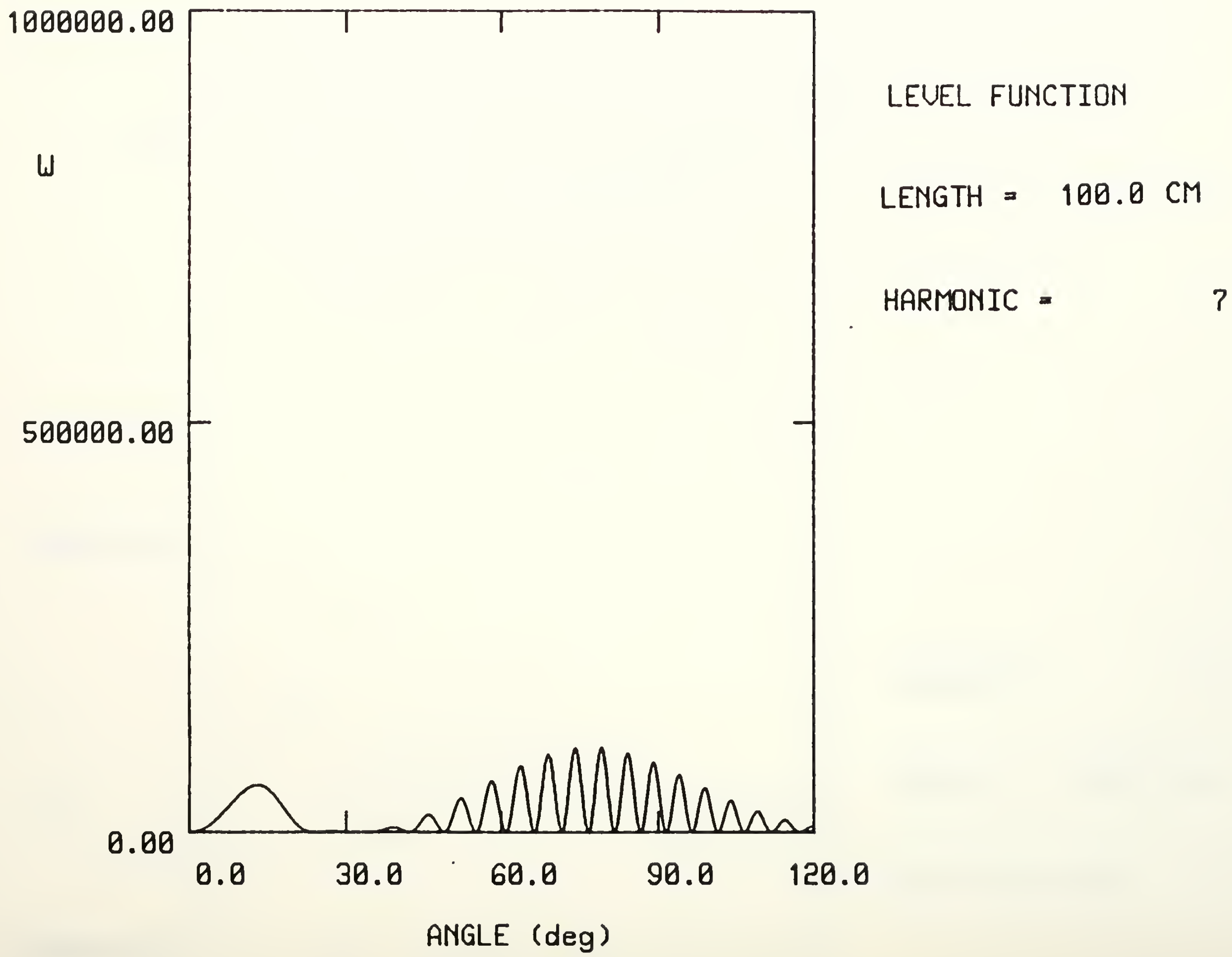
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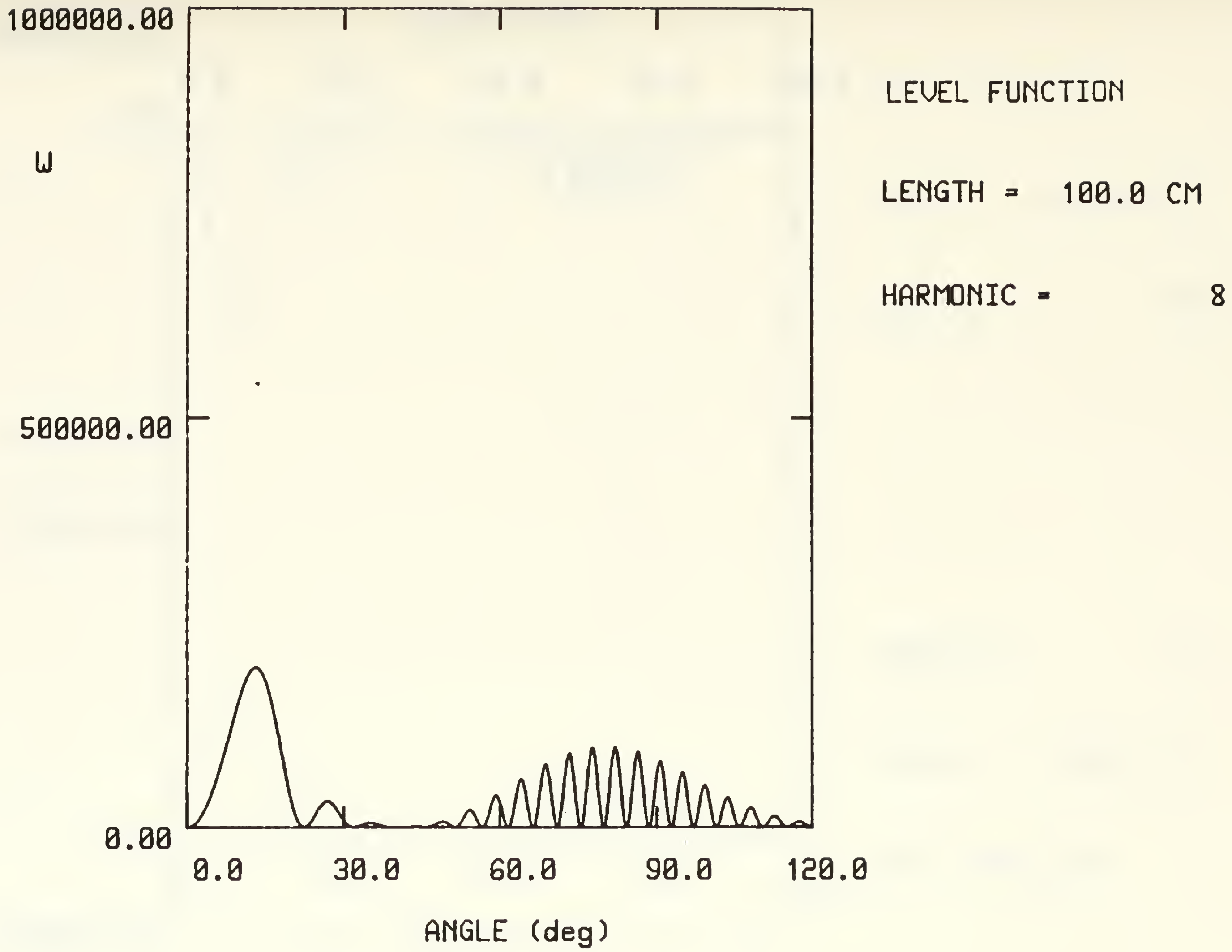
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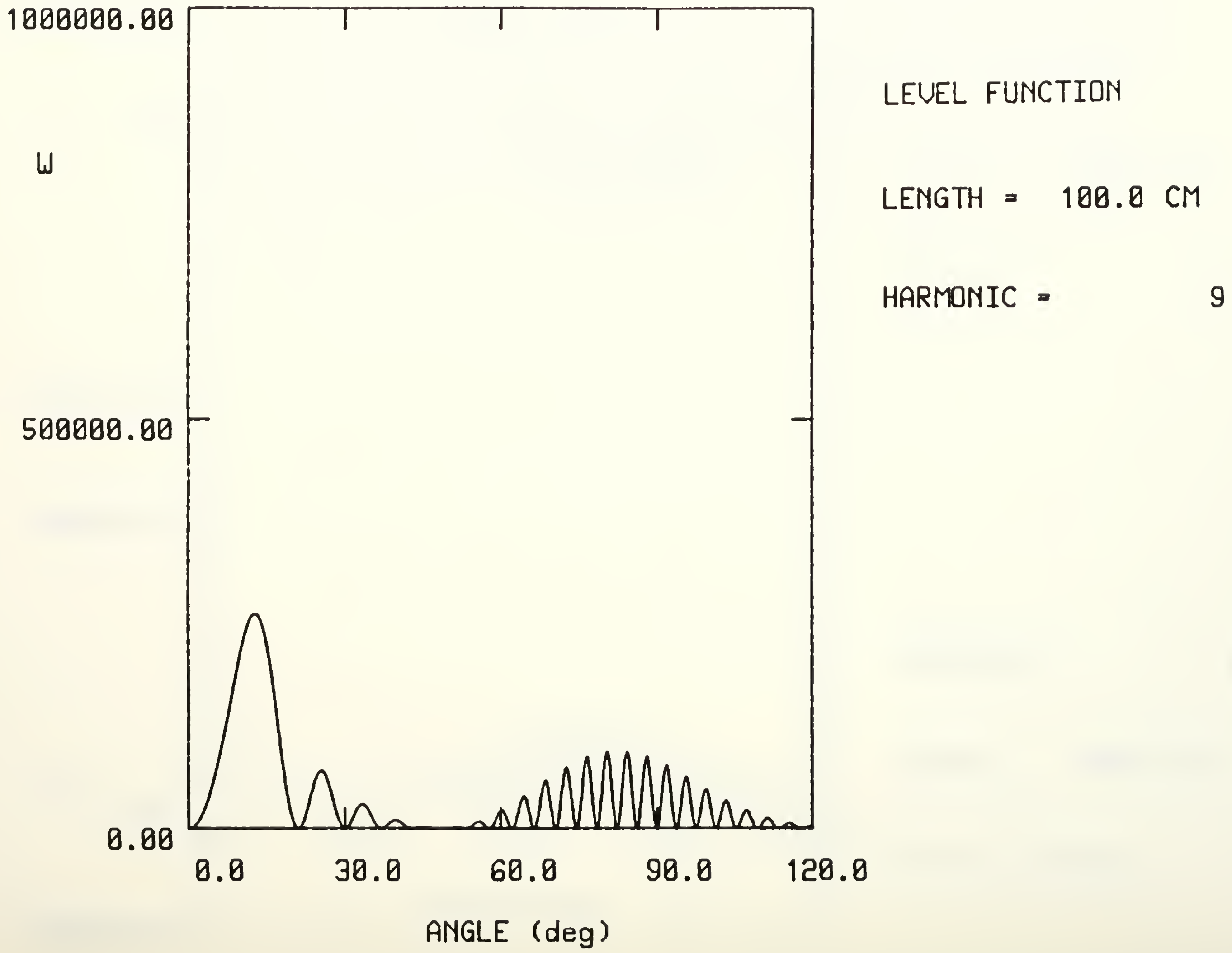
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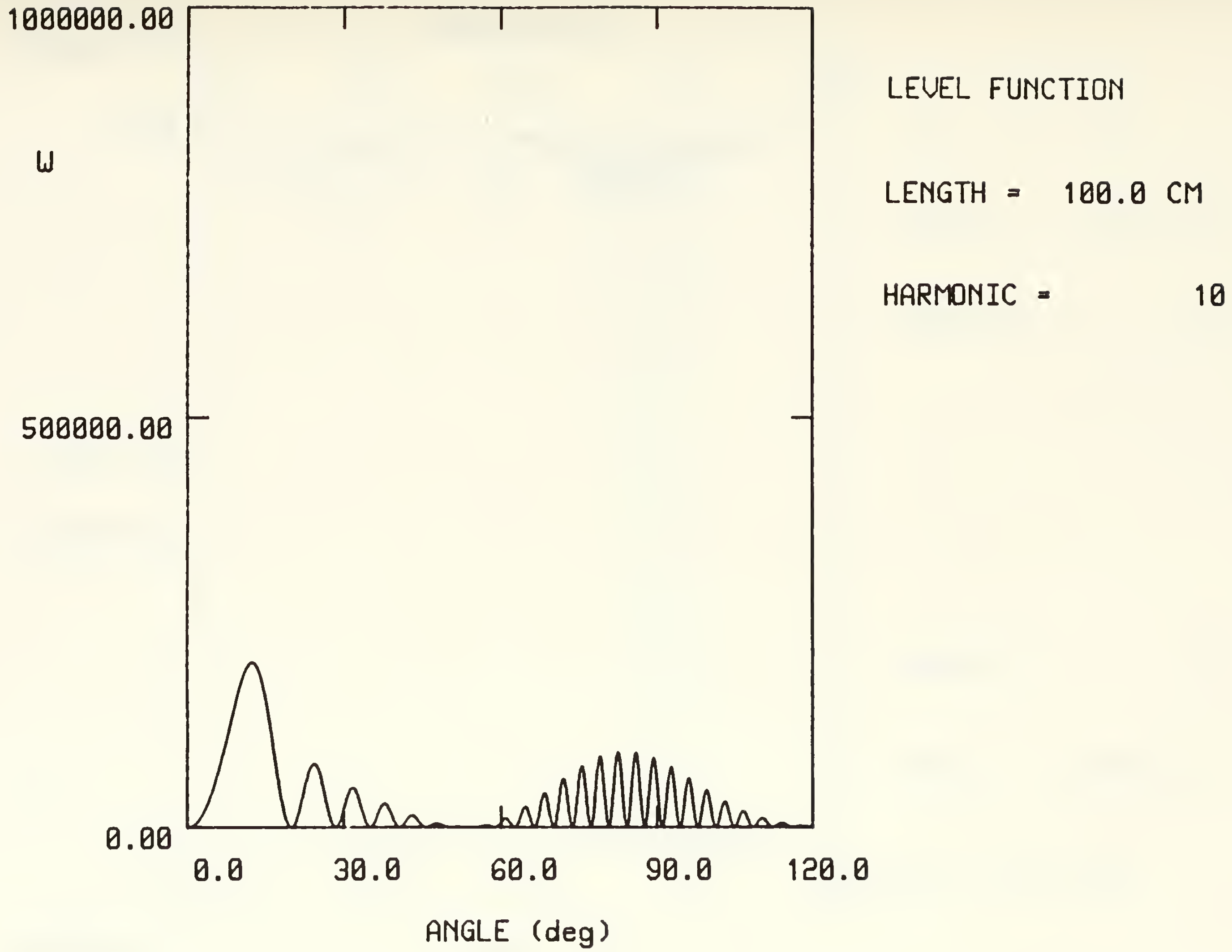
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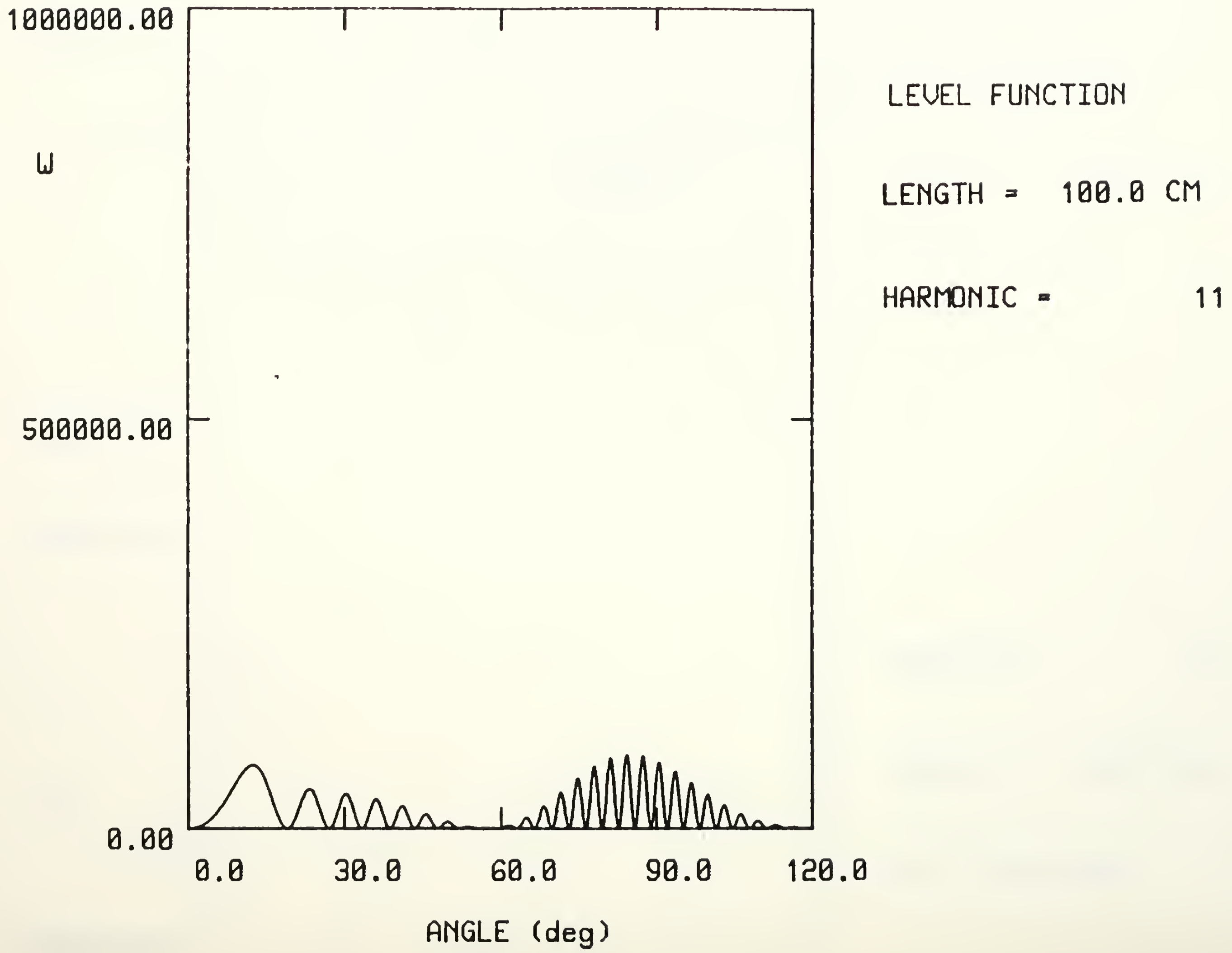
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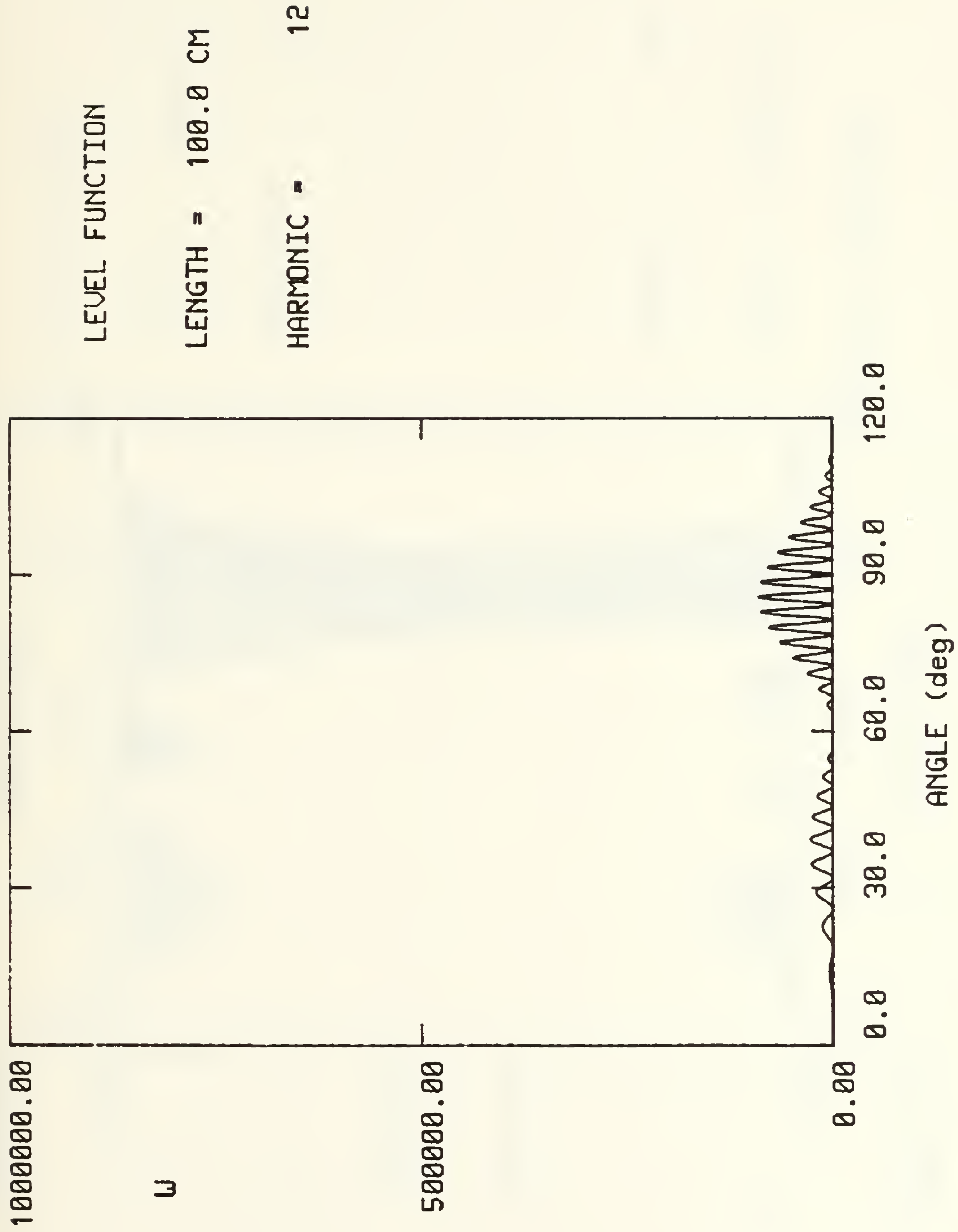
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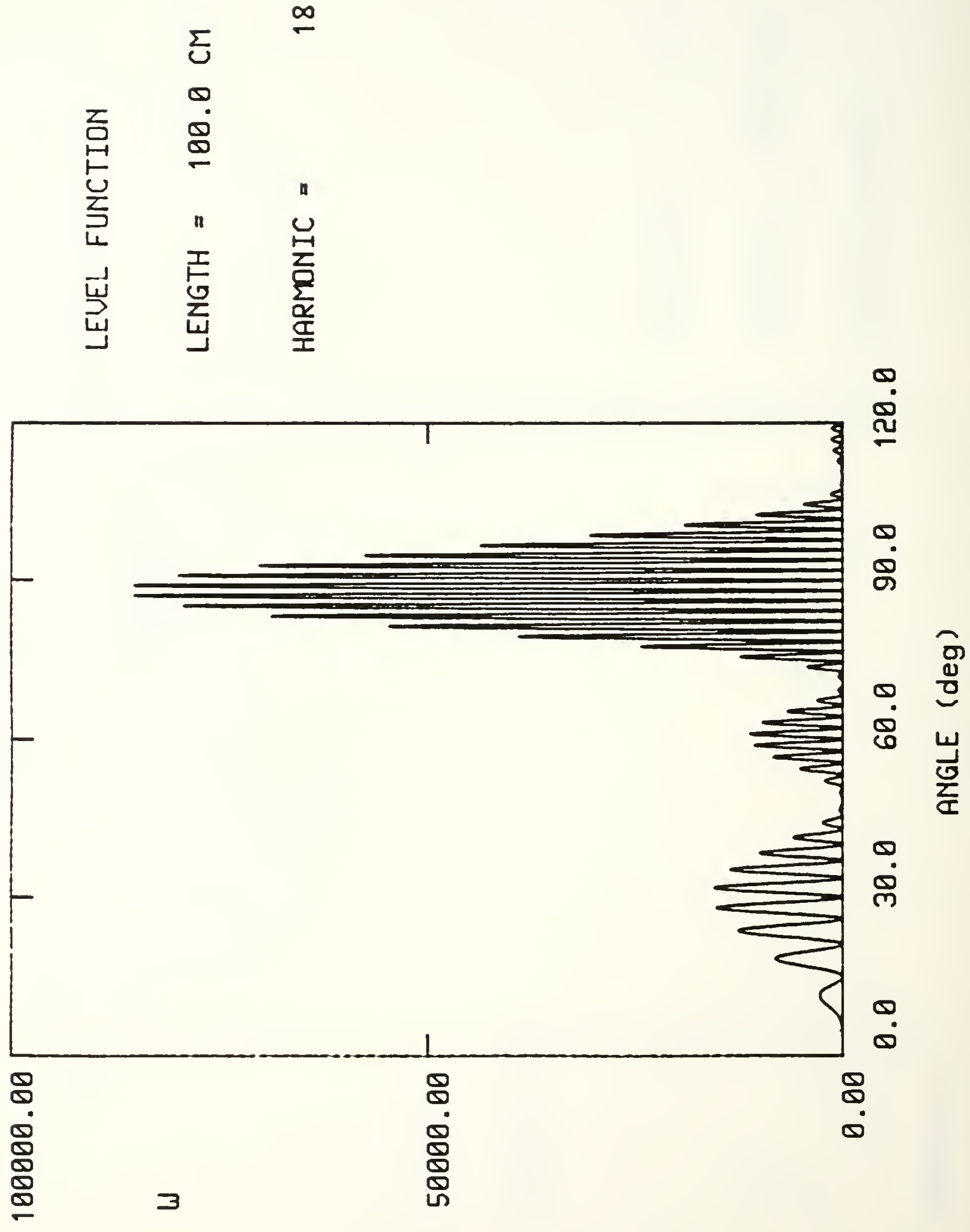
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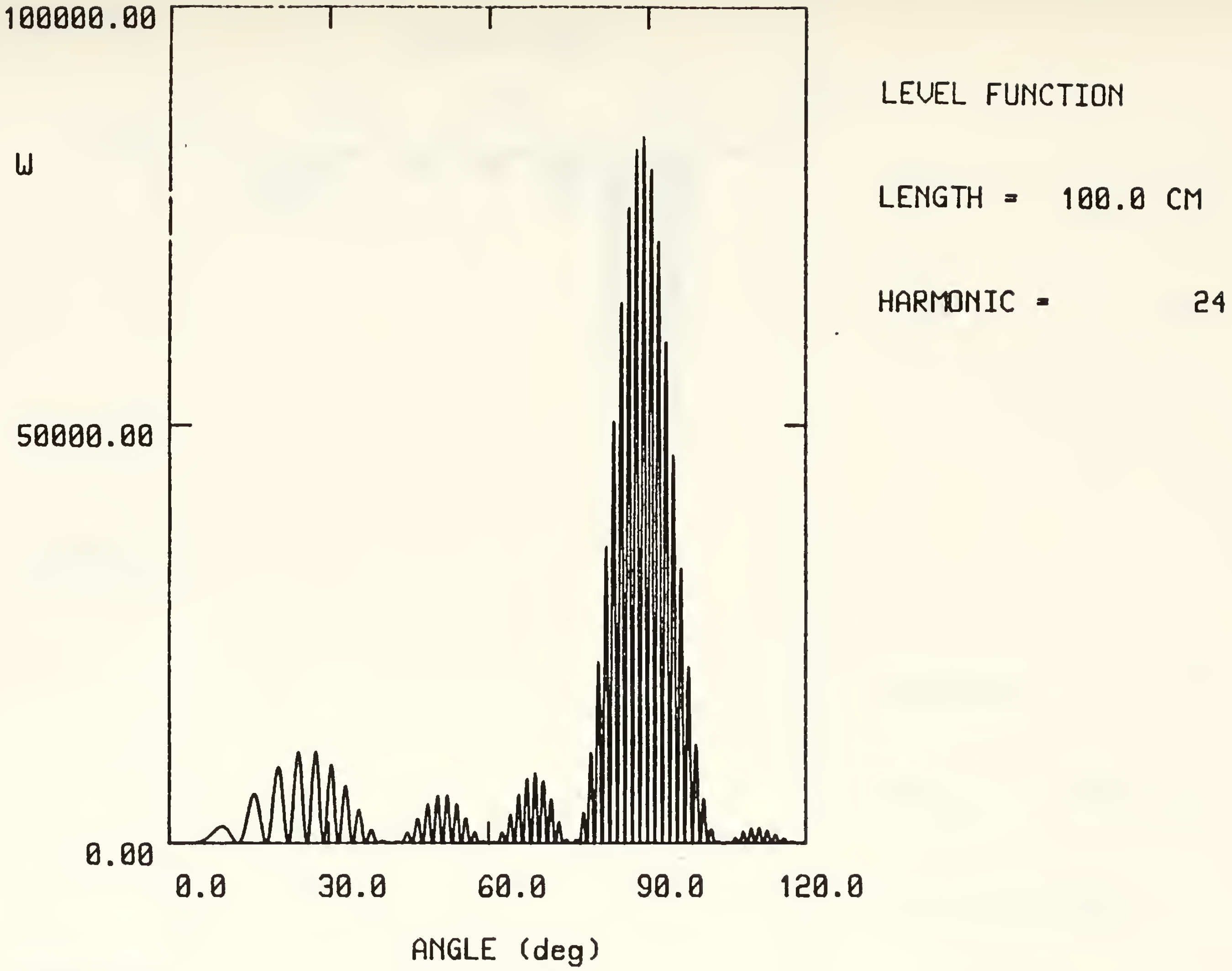
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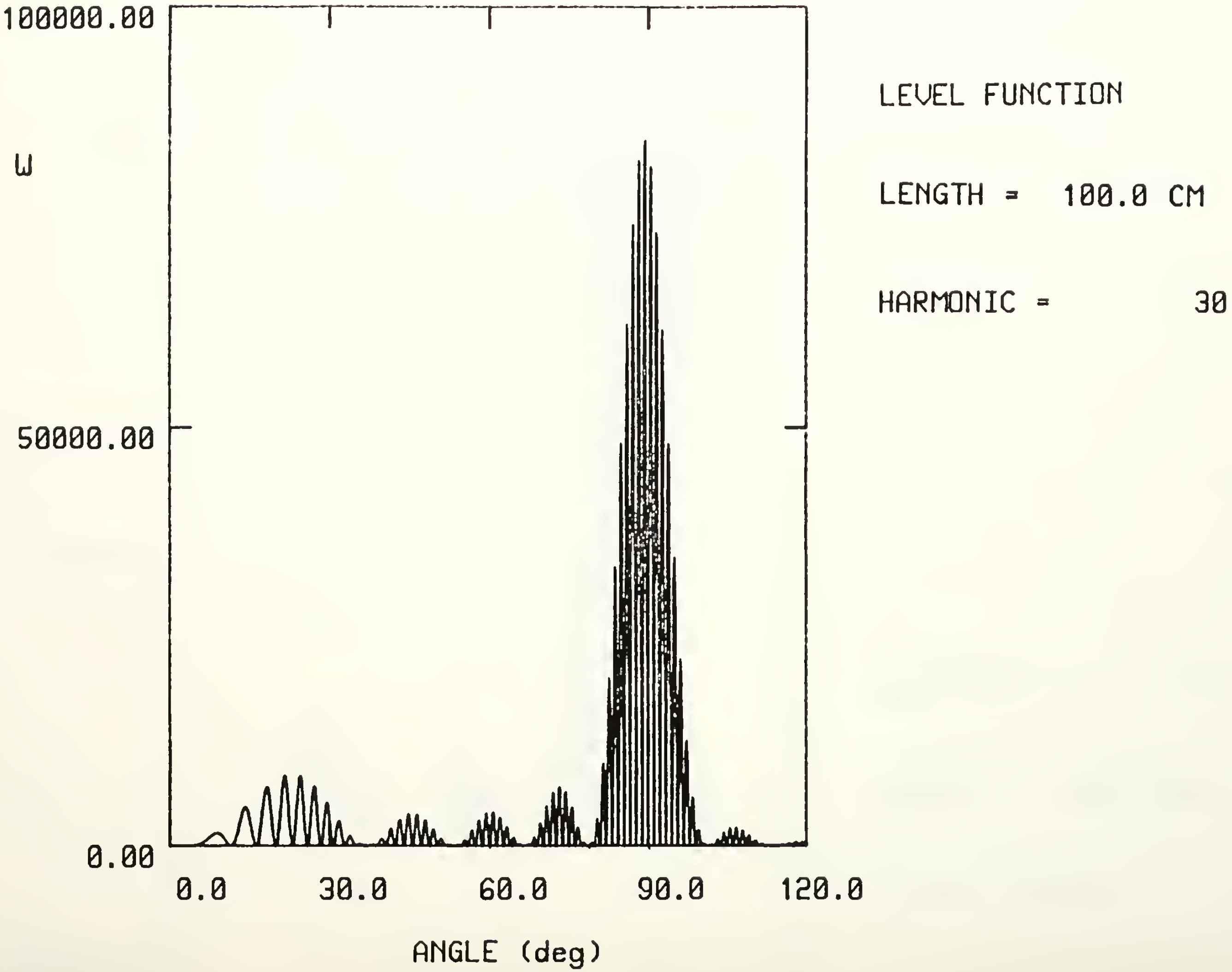
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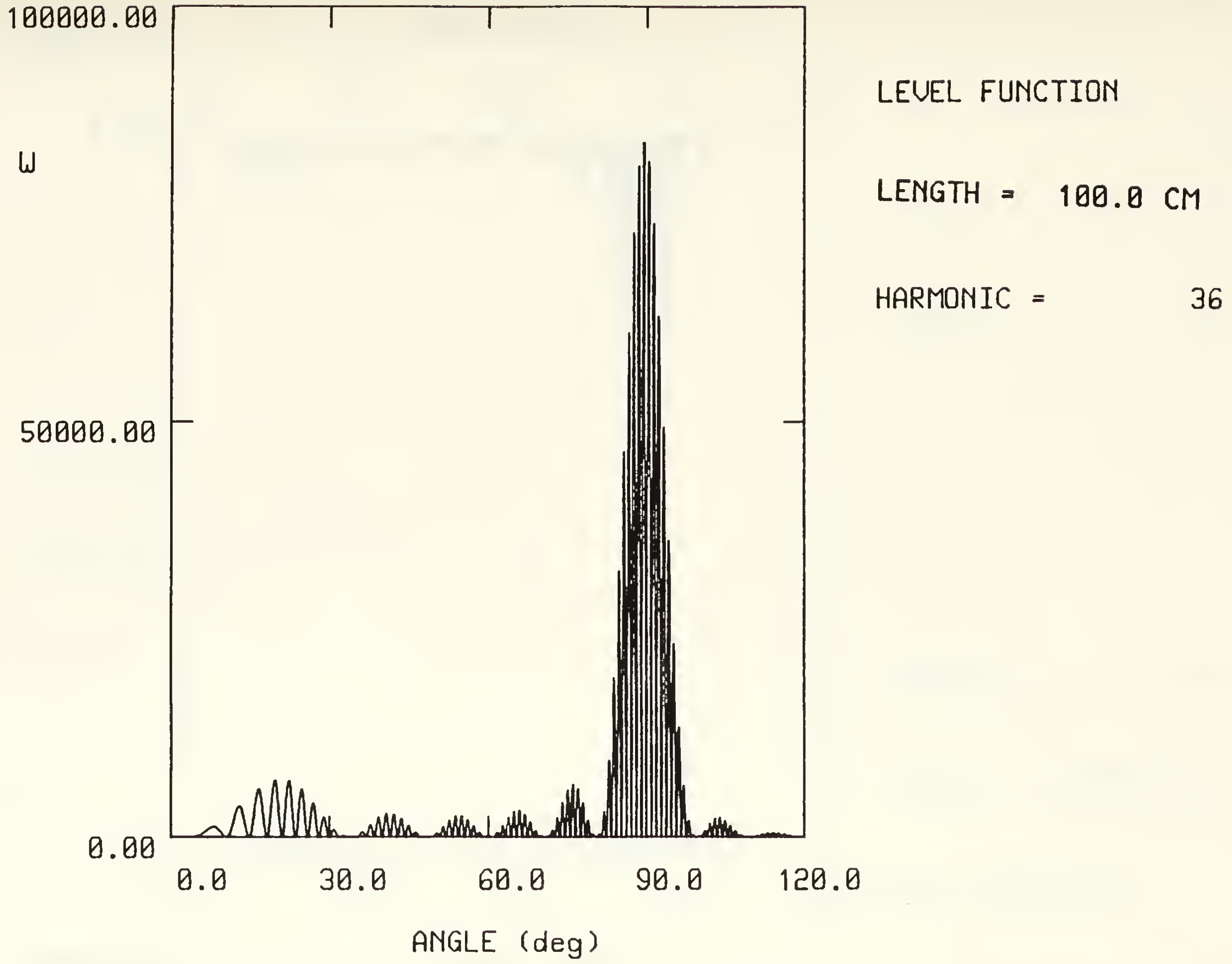
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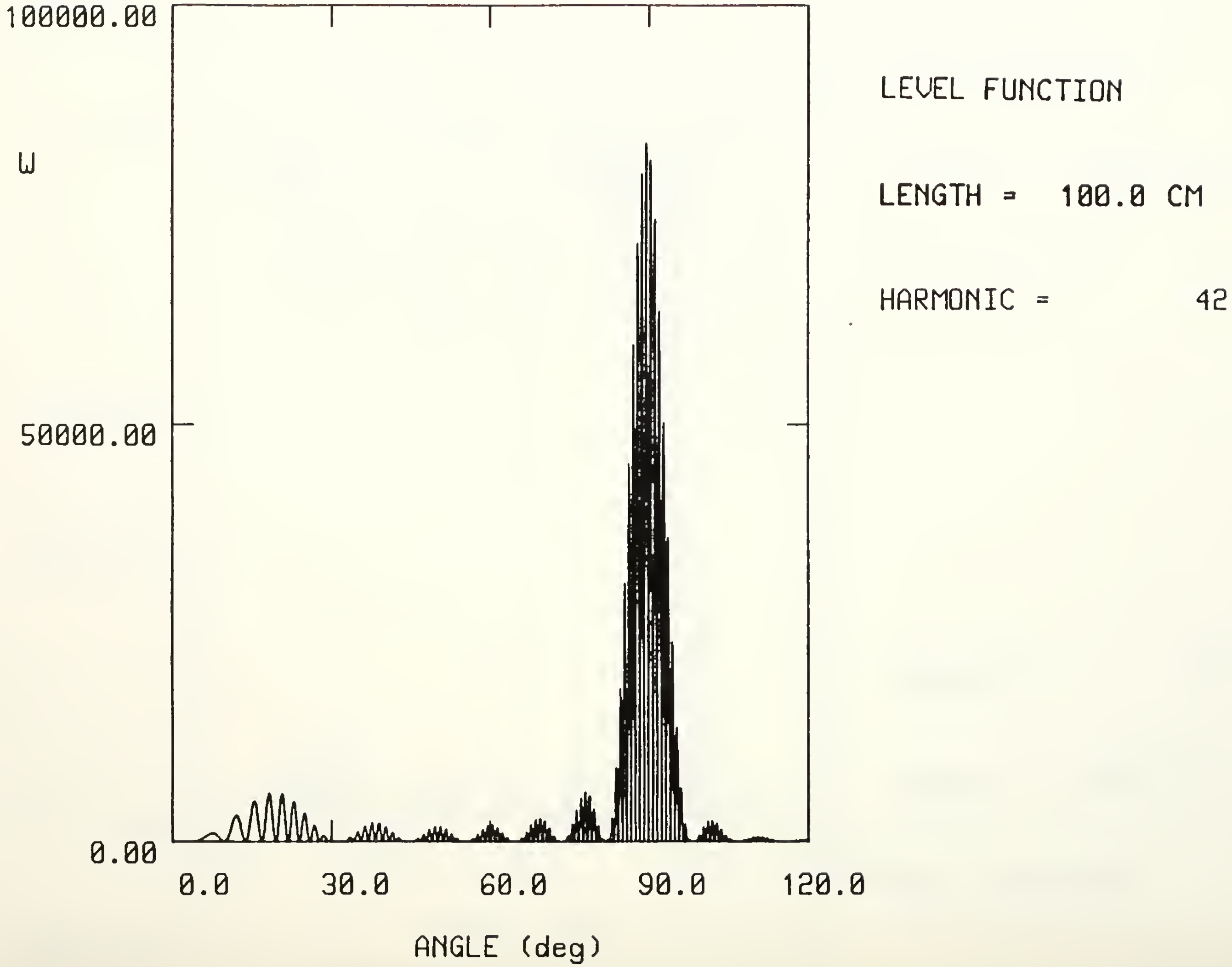
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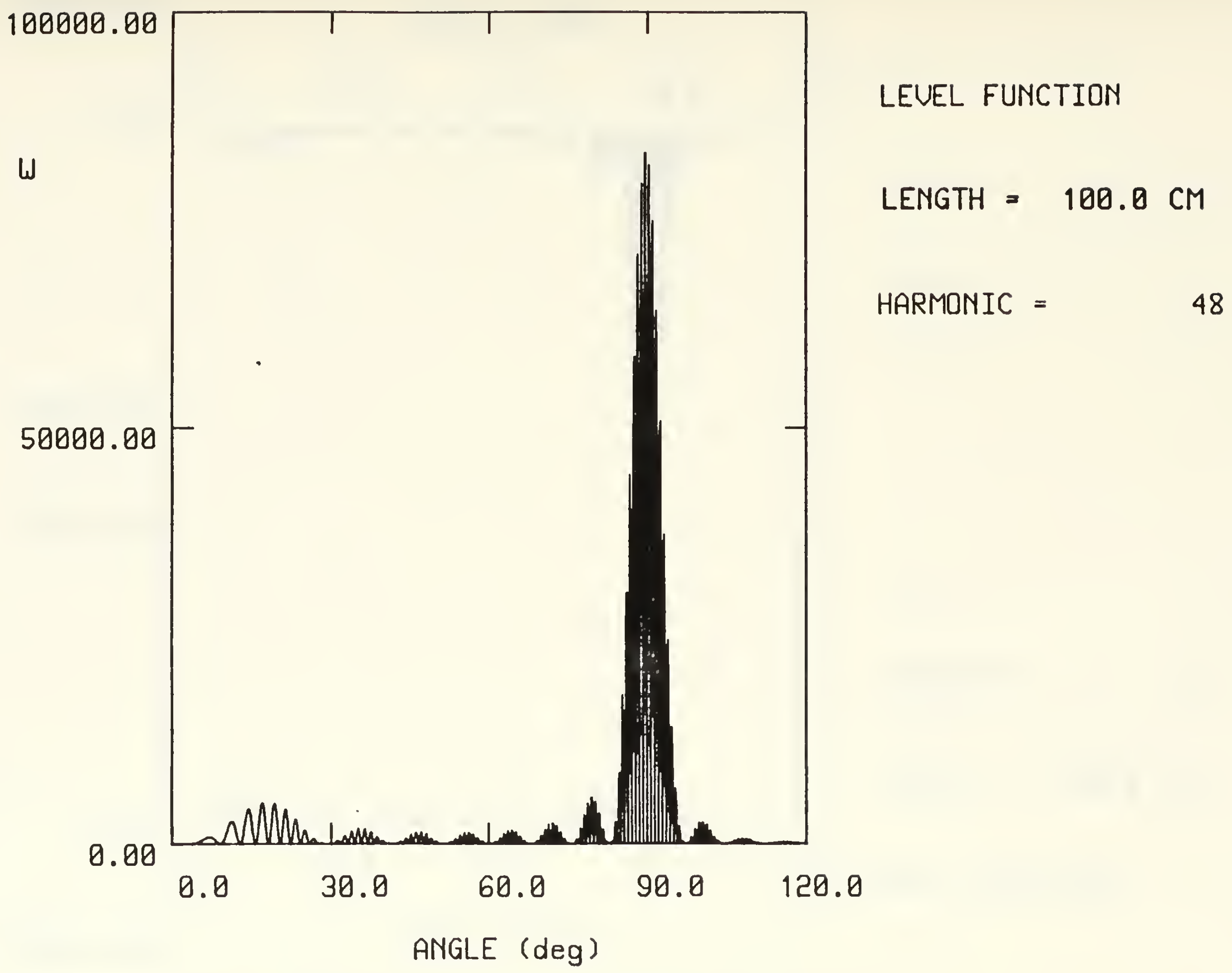
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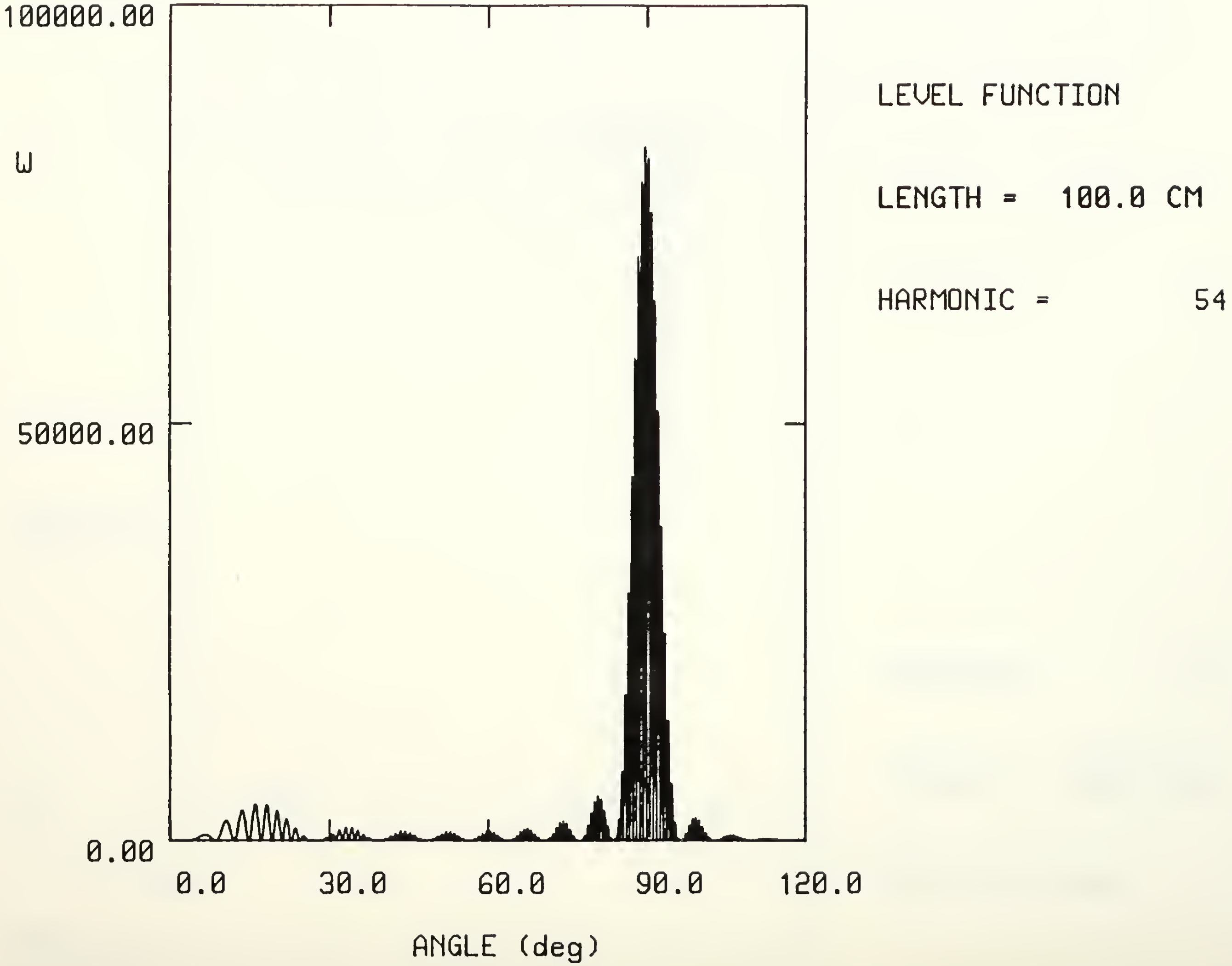
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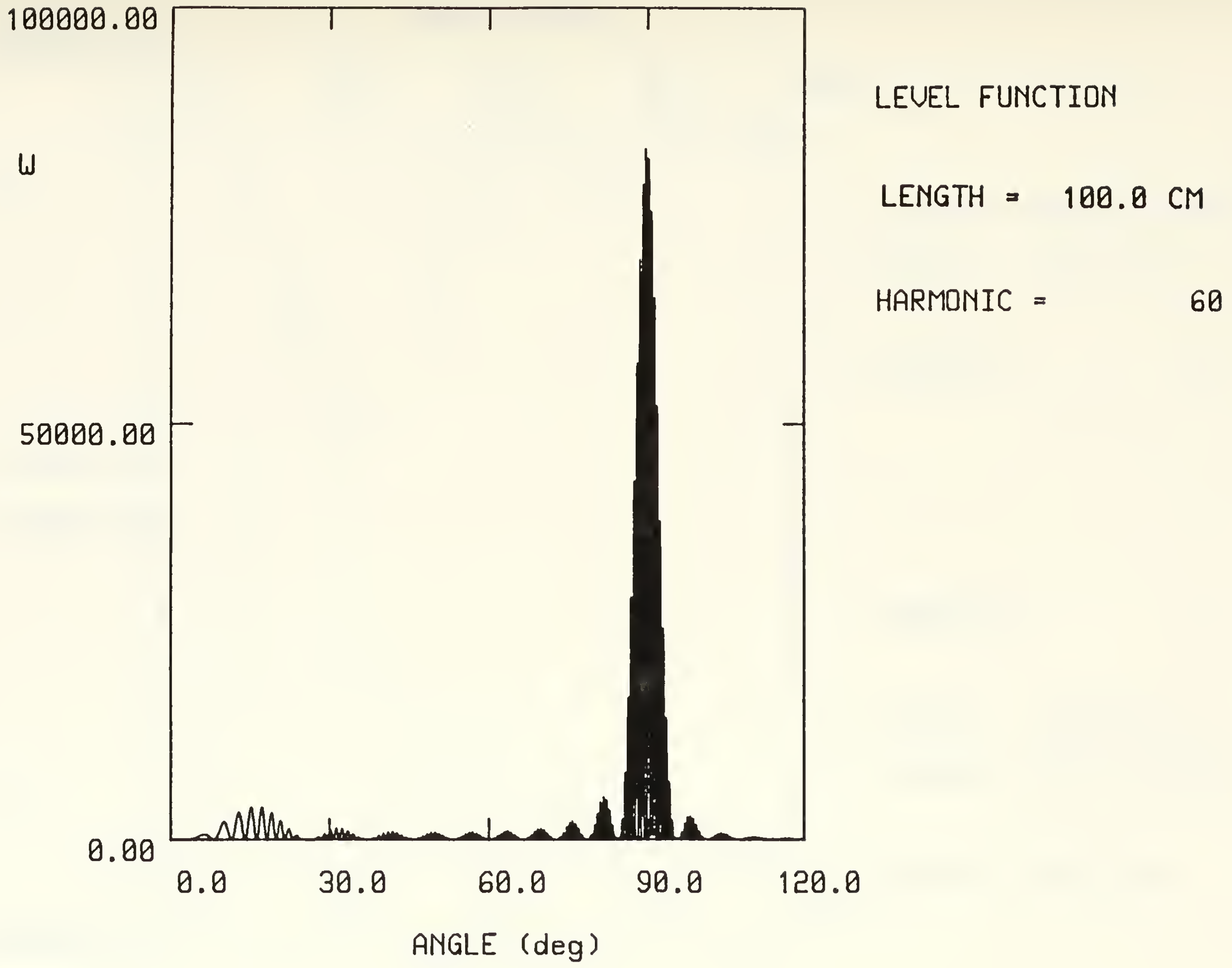
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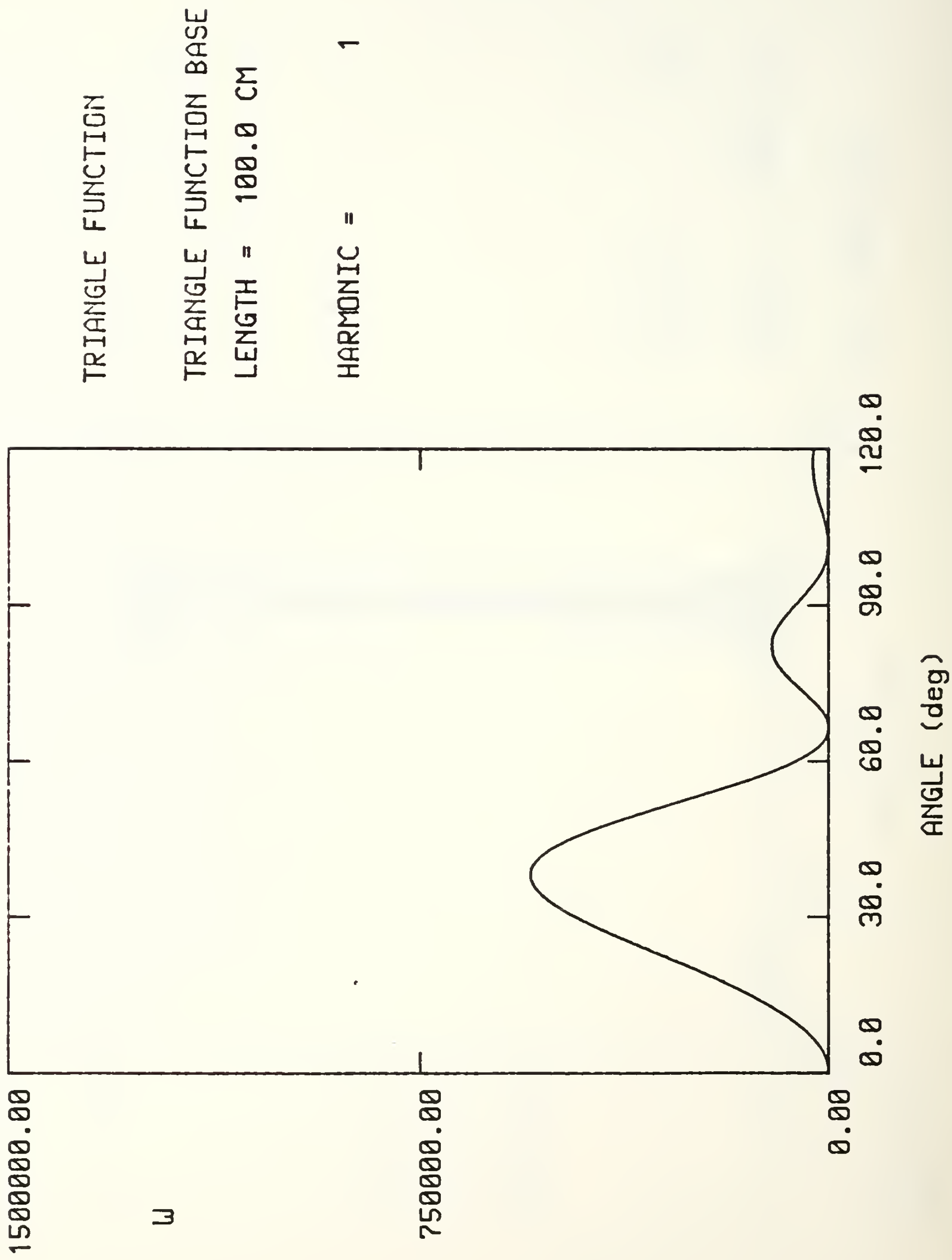
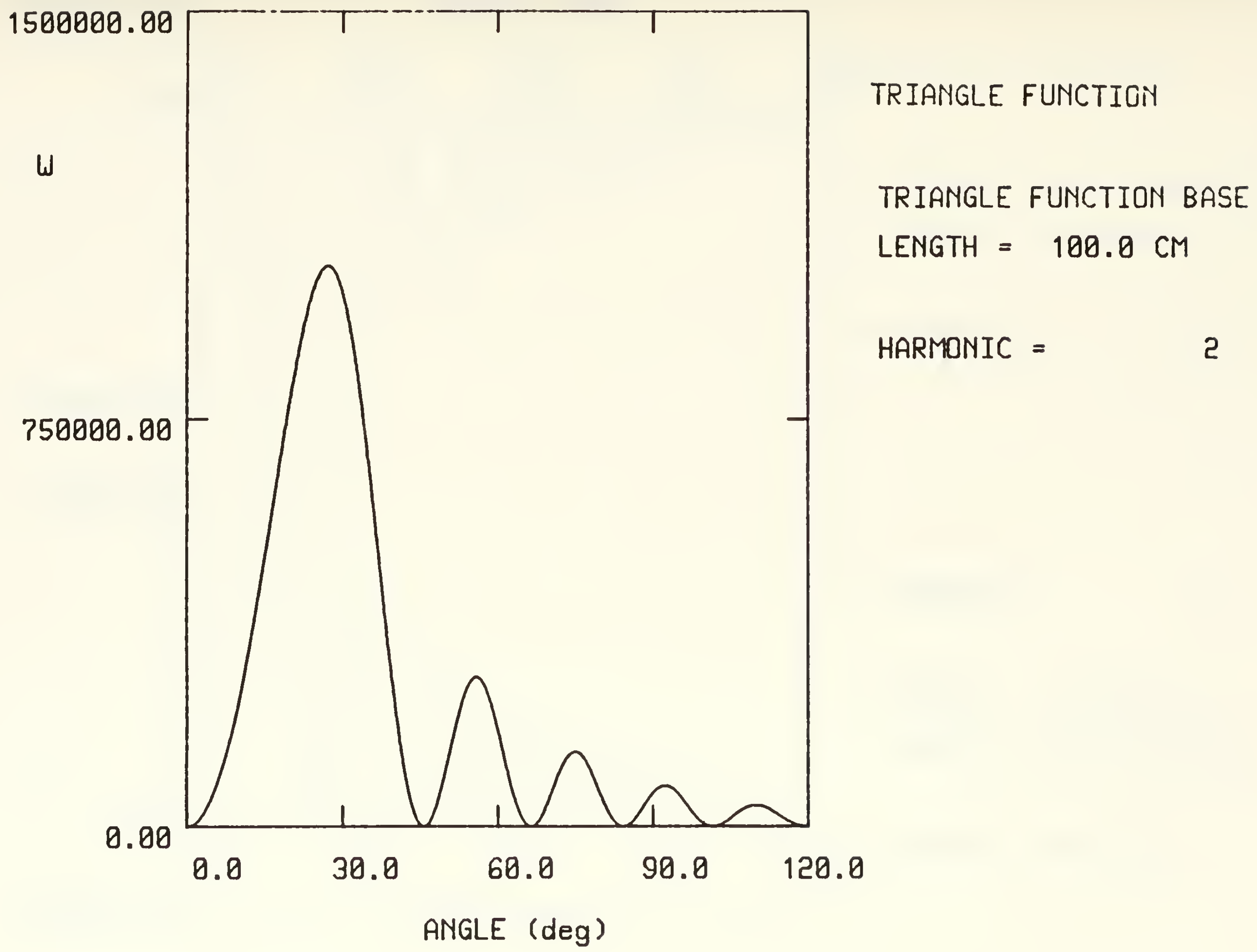
APPENDIX B: TRIANGULAR FUNCTION

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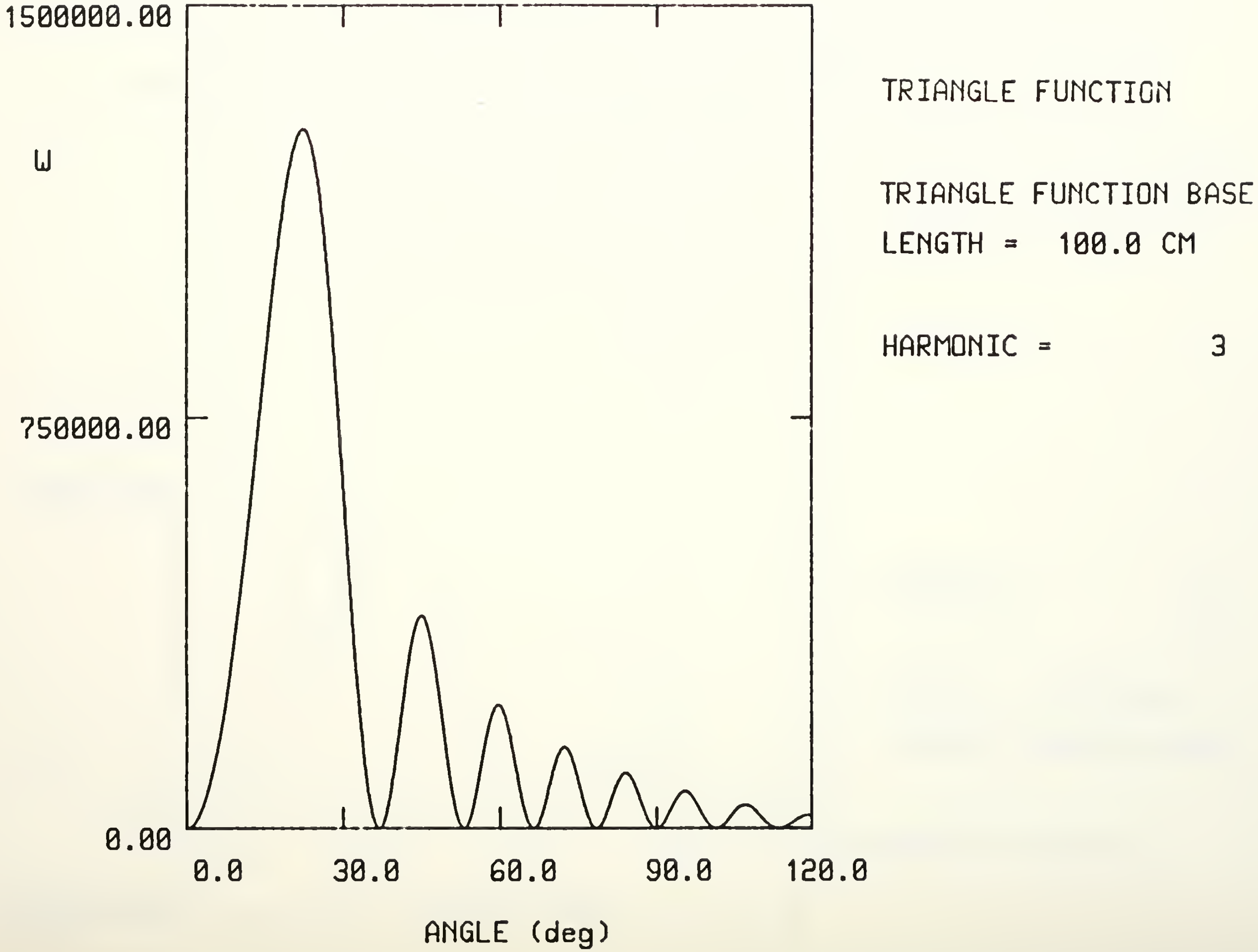
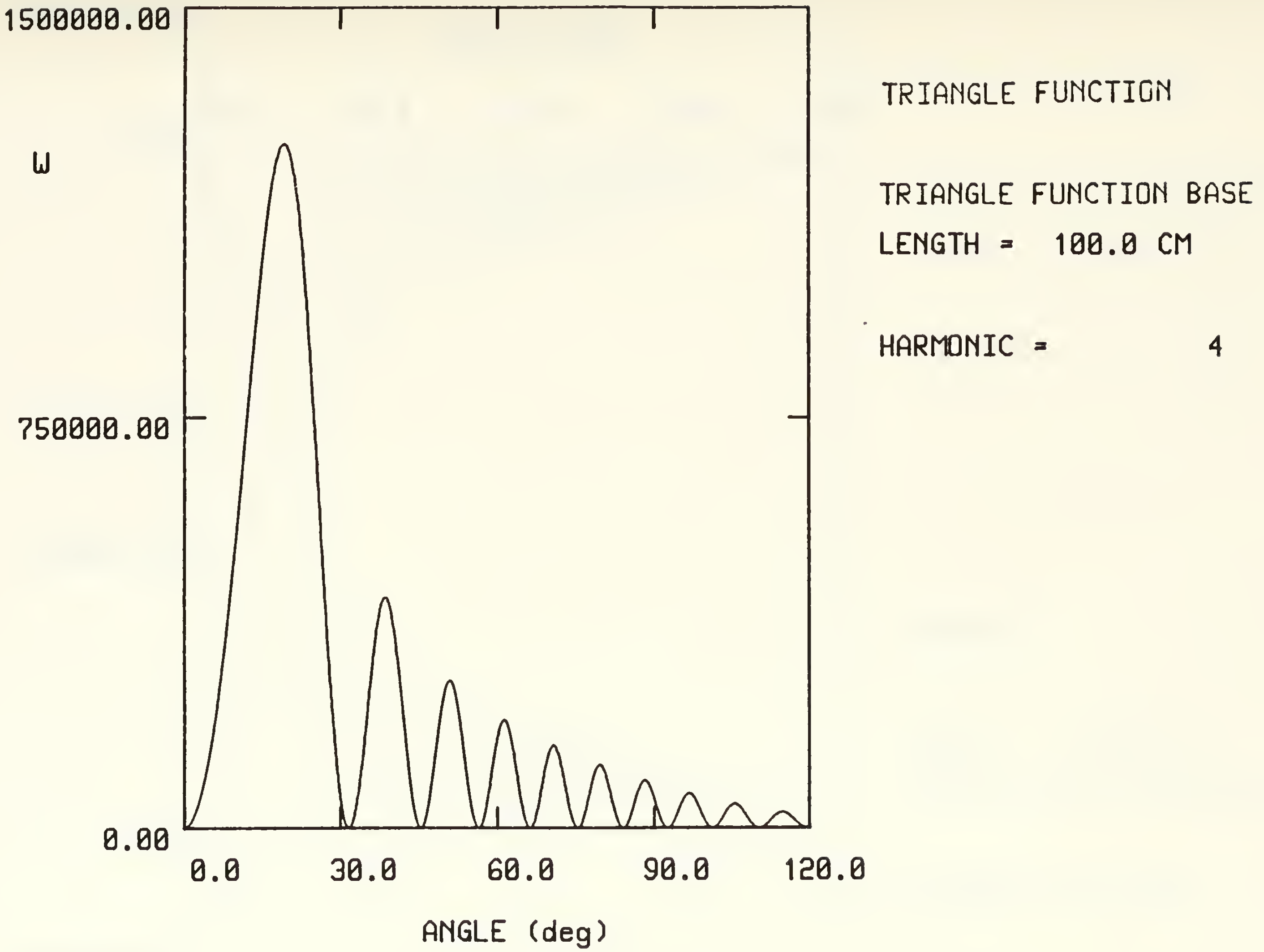
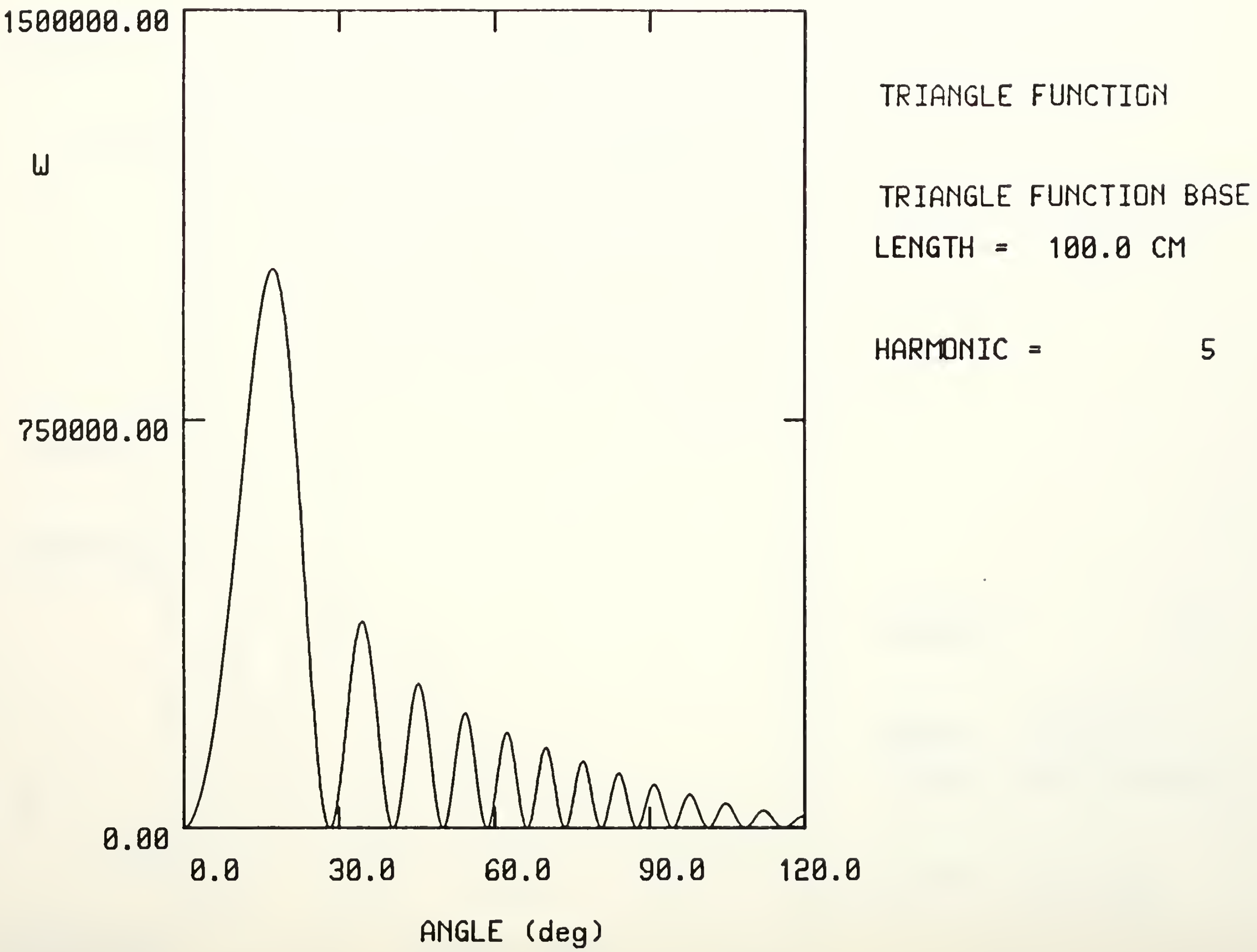
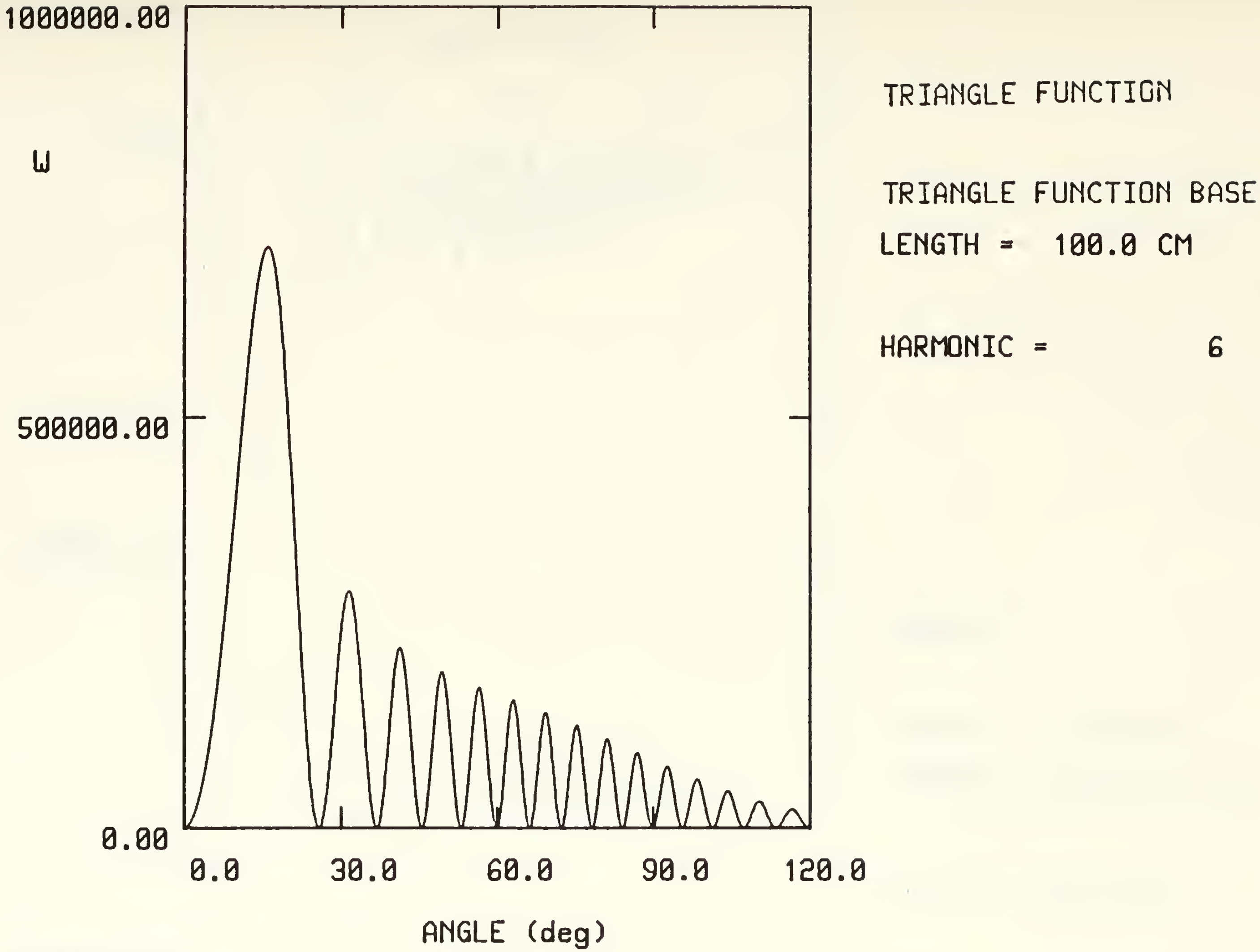

1500000.00
750000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 3


1500000.00
W
750000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 4


1500000.00
750000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 5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0.00
W
0.0
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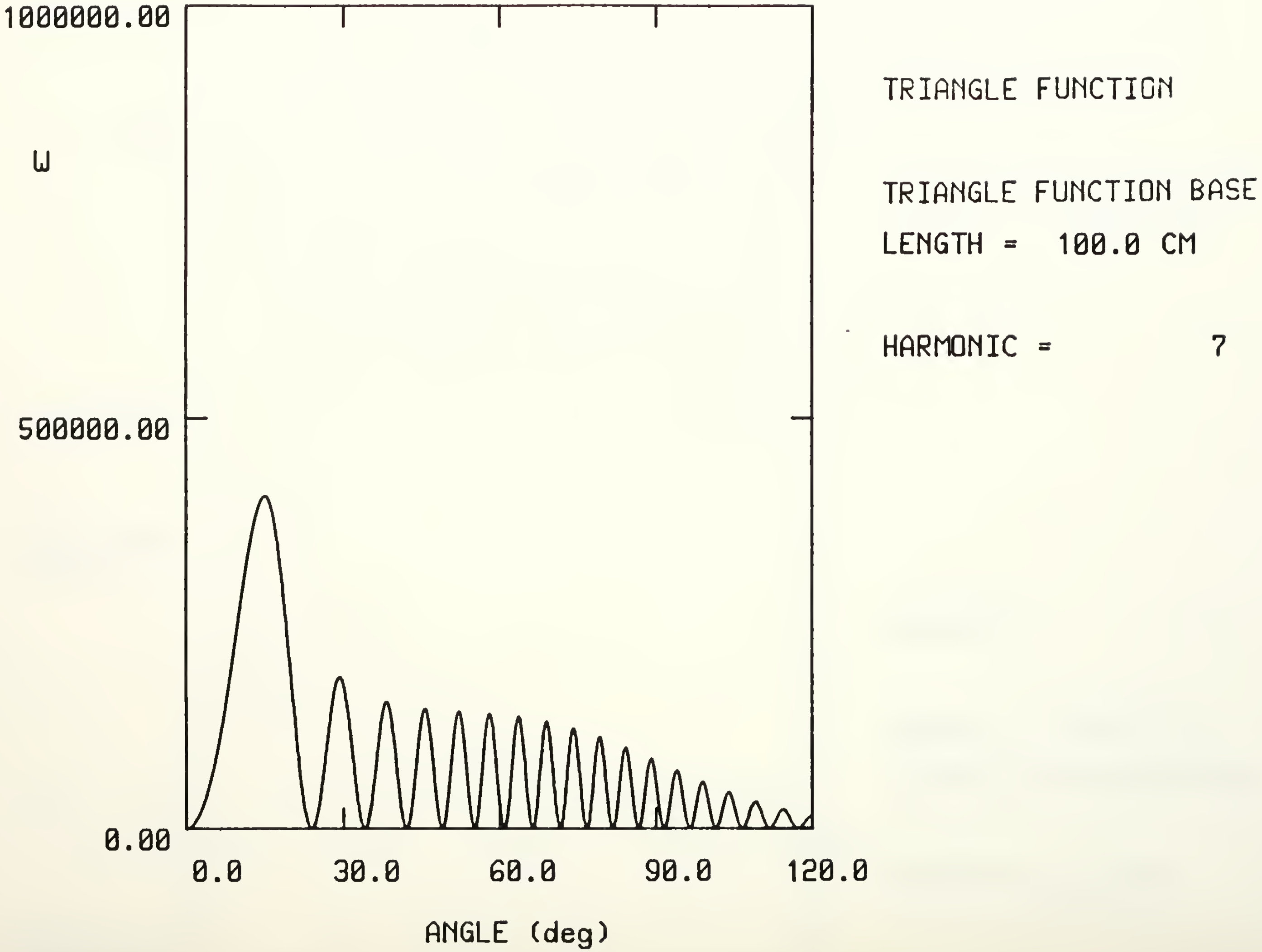
60.0
90.0
120.0
ANGLE (deg)
TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 6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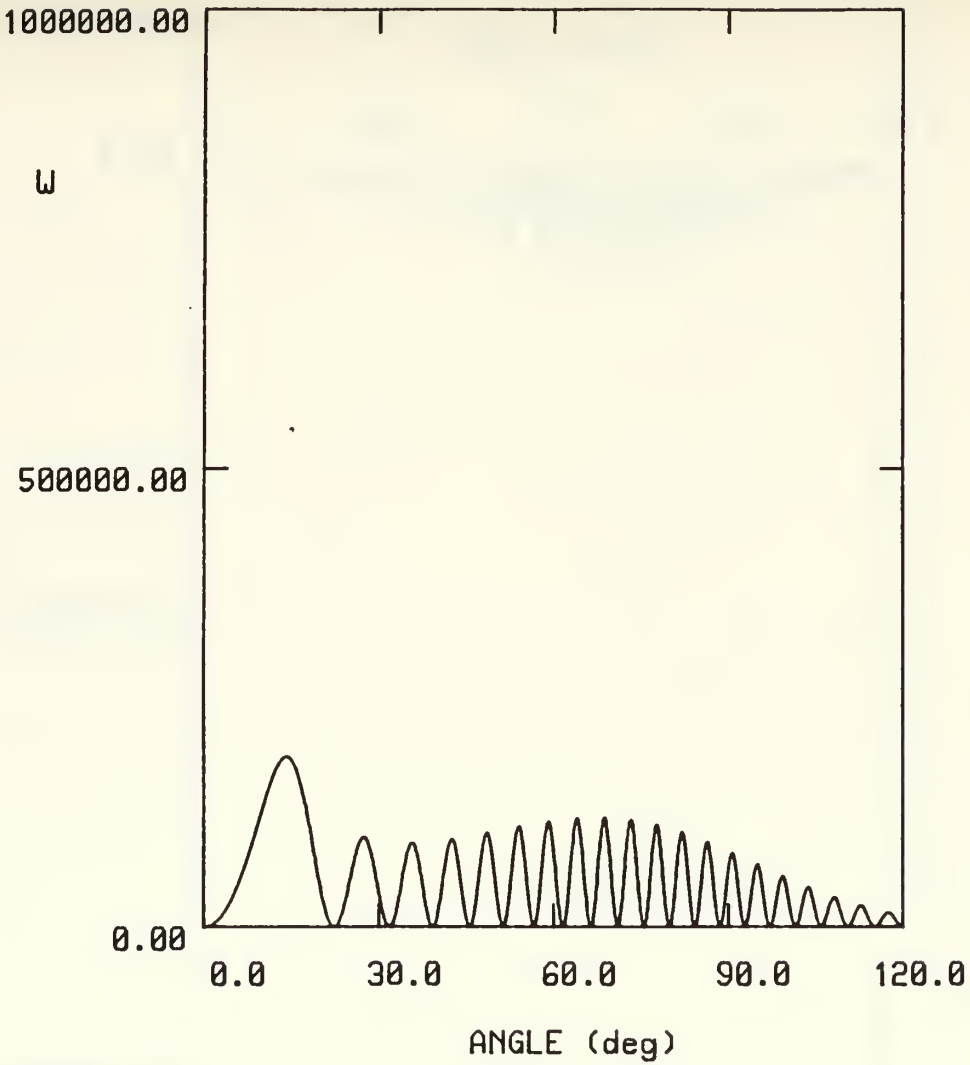
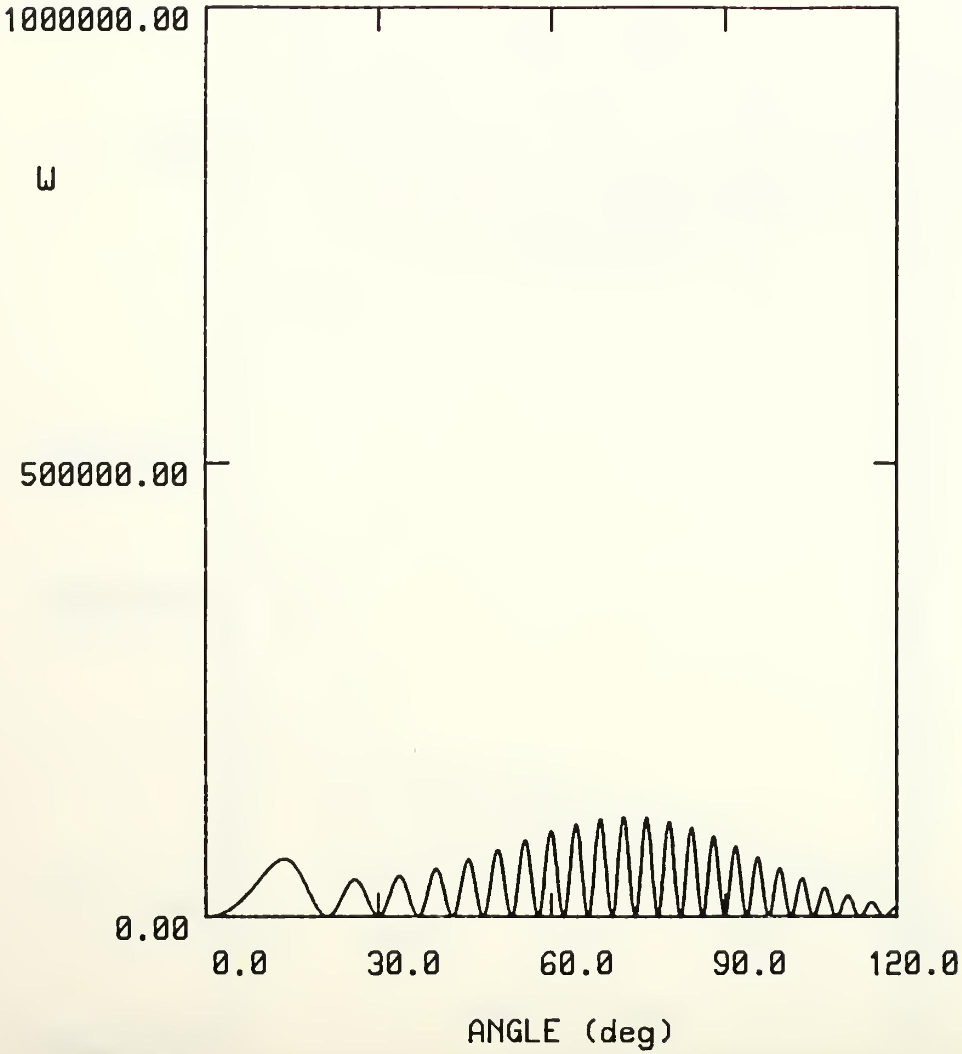
90.0
120.0
ANGLE (deg)
TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 7



TRIANGLE FUNCTION

TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM

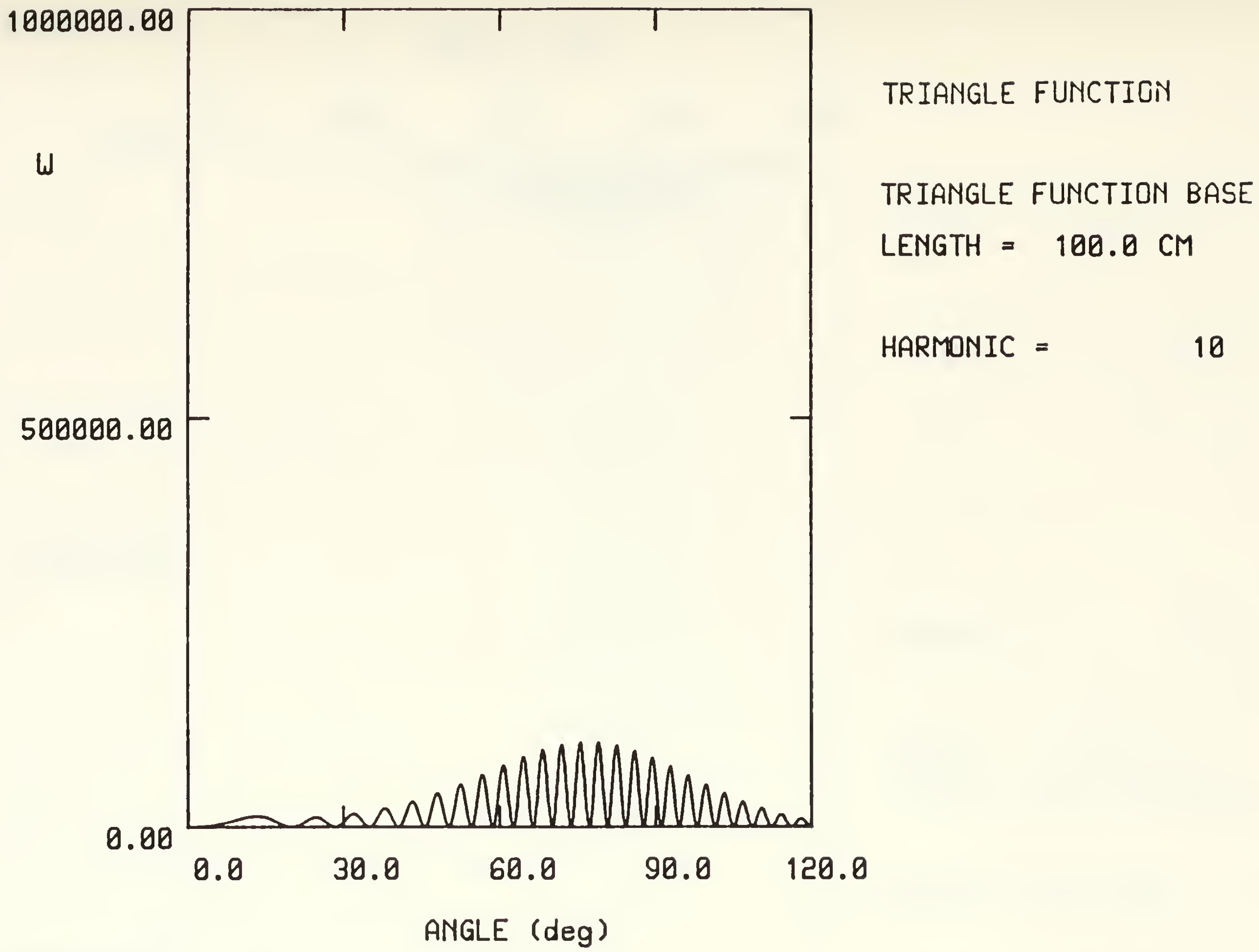
HARMONIC = 8



TRIANGLE FUNCTION

TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM

HARMONIC = 9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W
0.0
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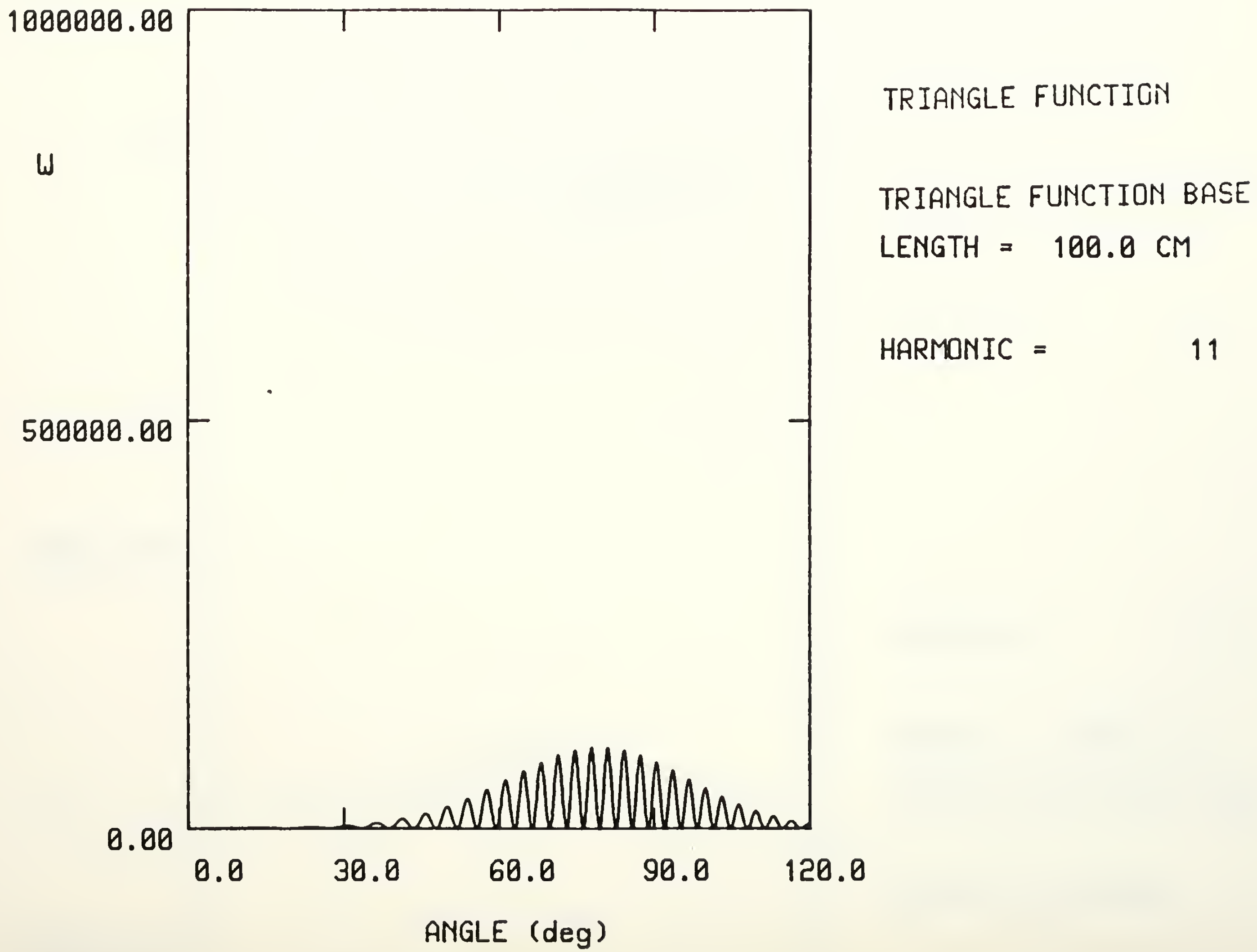
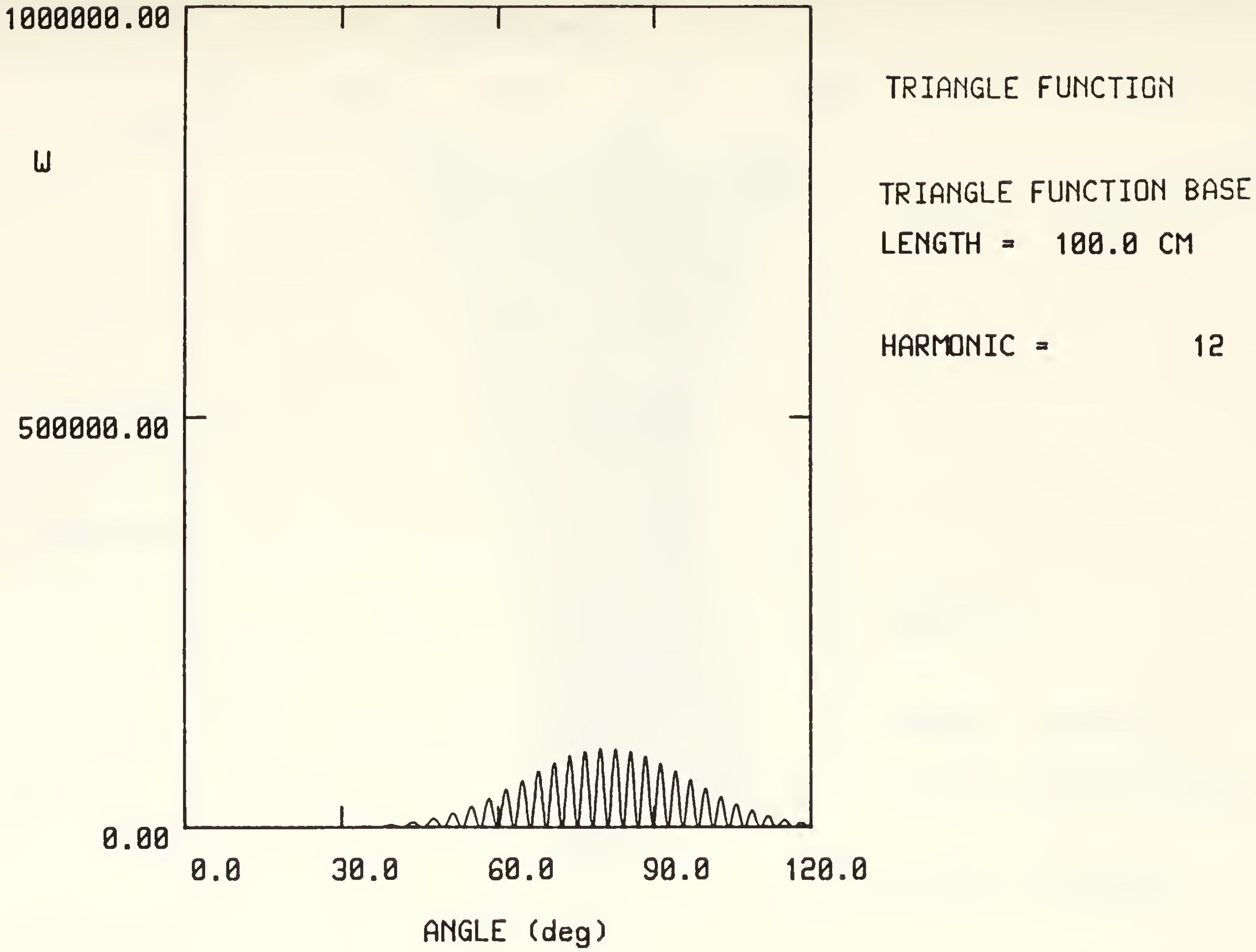
90.0
120.0
ANGLE (deg)
TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 10


1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 11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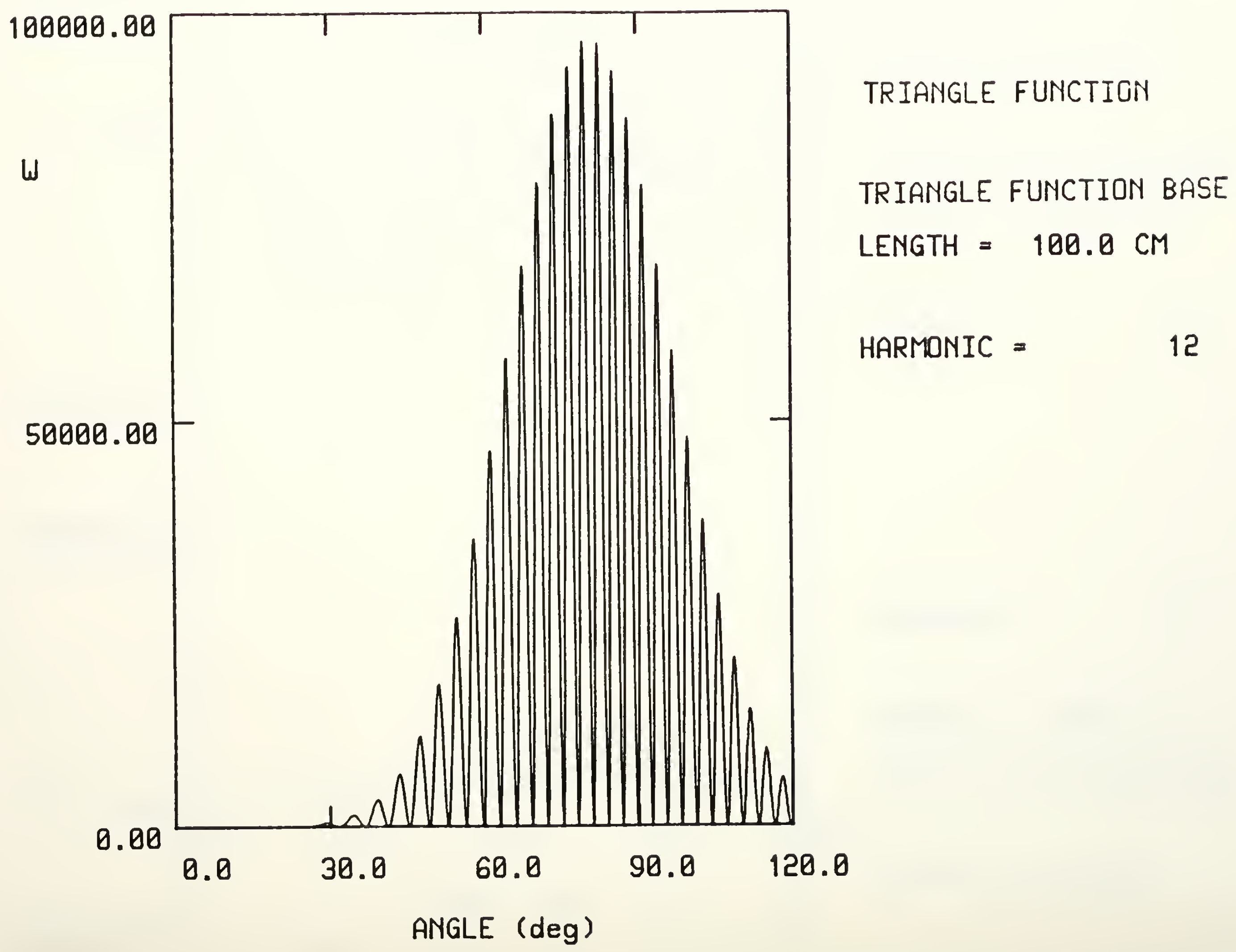
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 12


100000.00
W
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0.00
0.0
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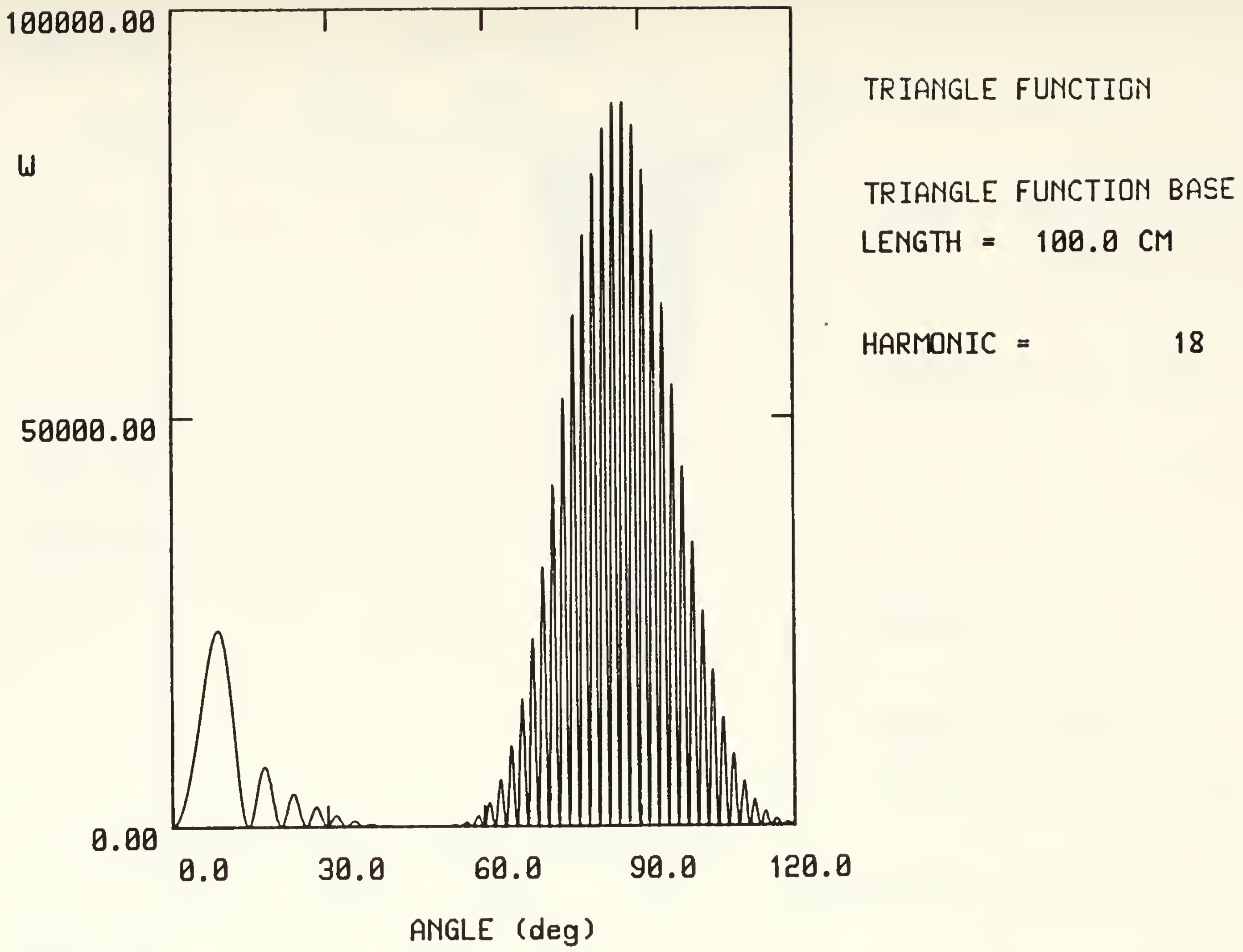
60.0
90.0
120.0
ANGLE (deg)
TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 12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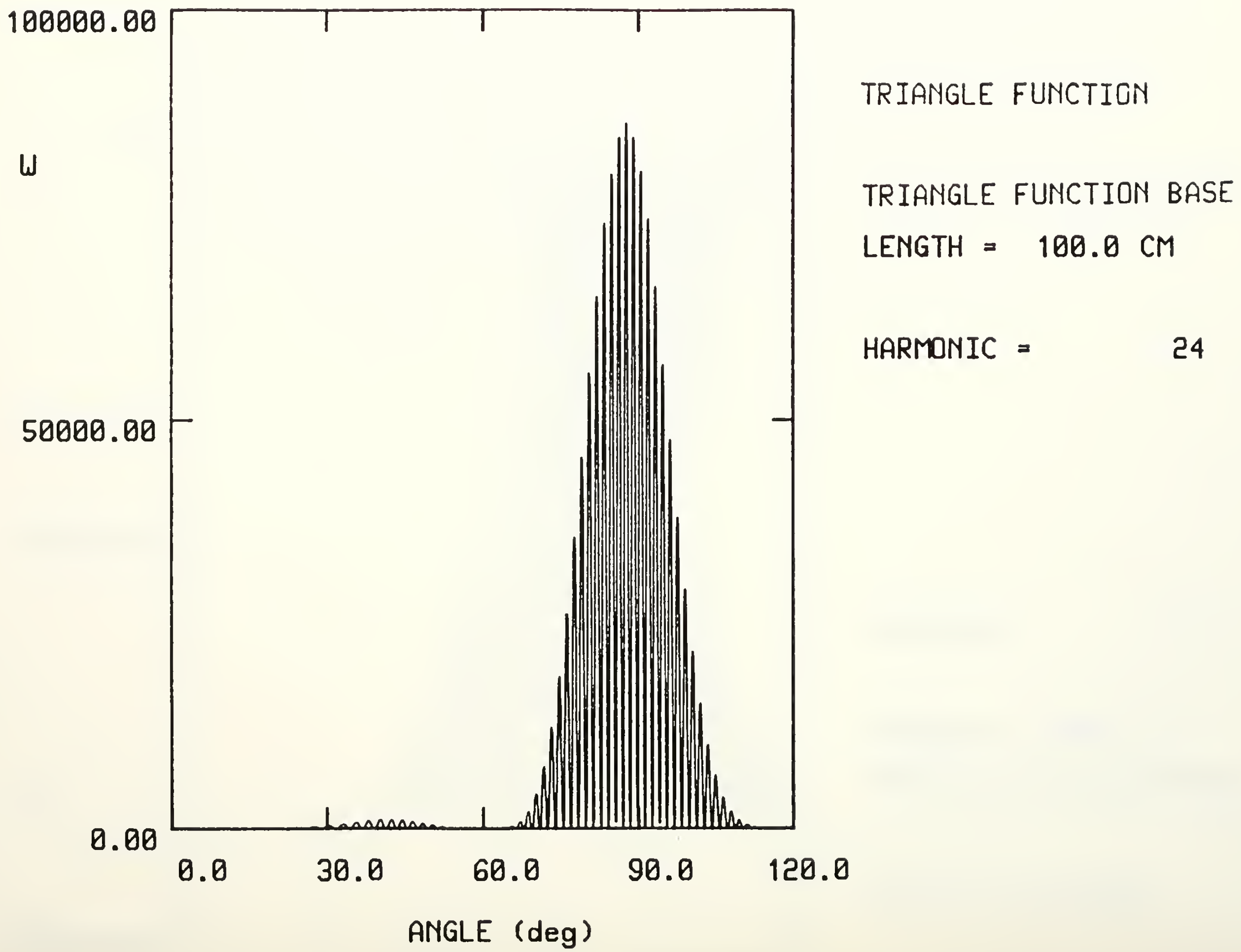
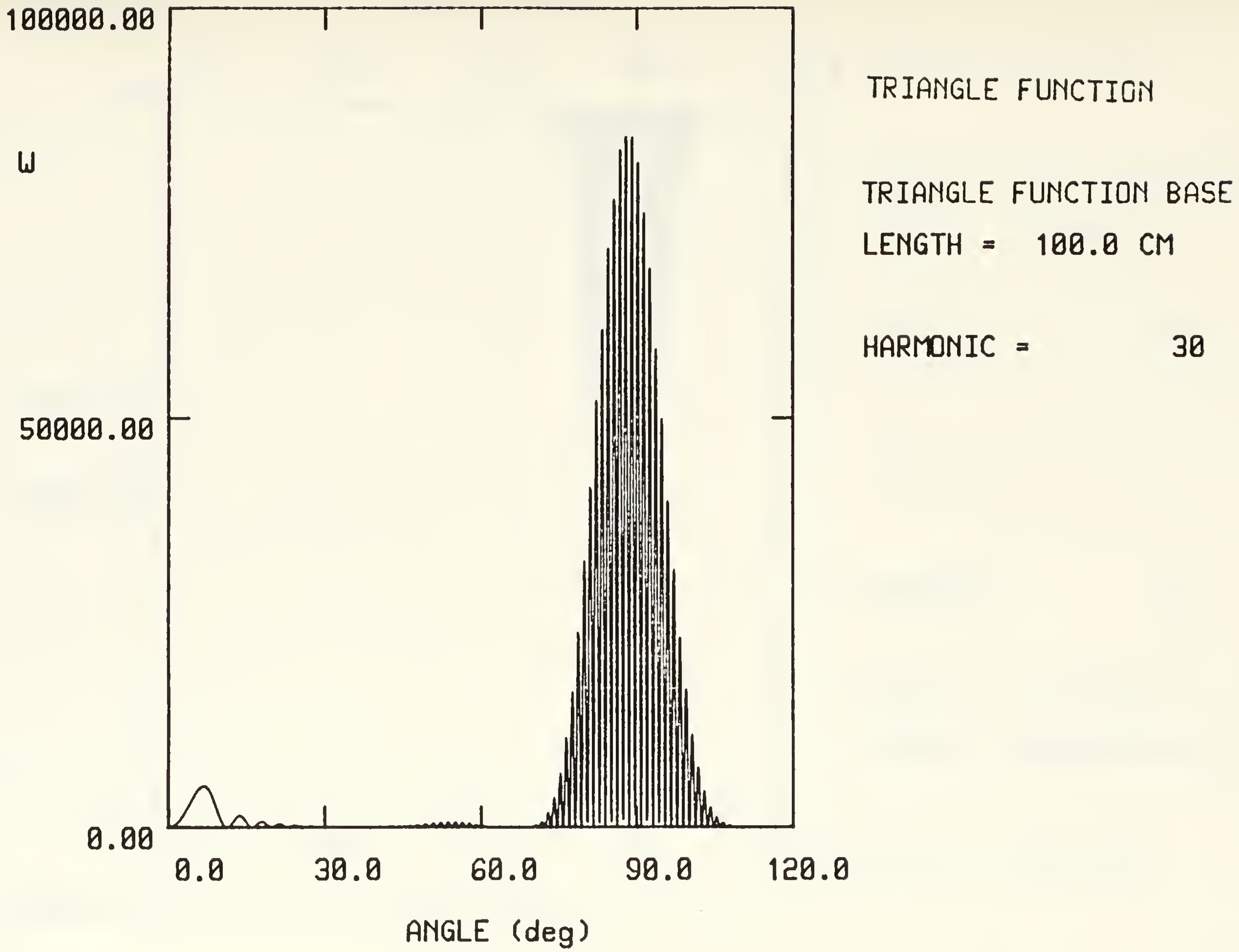
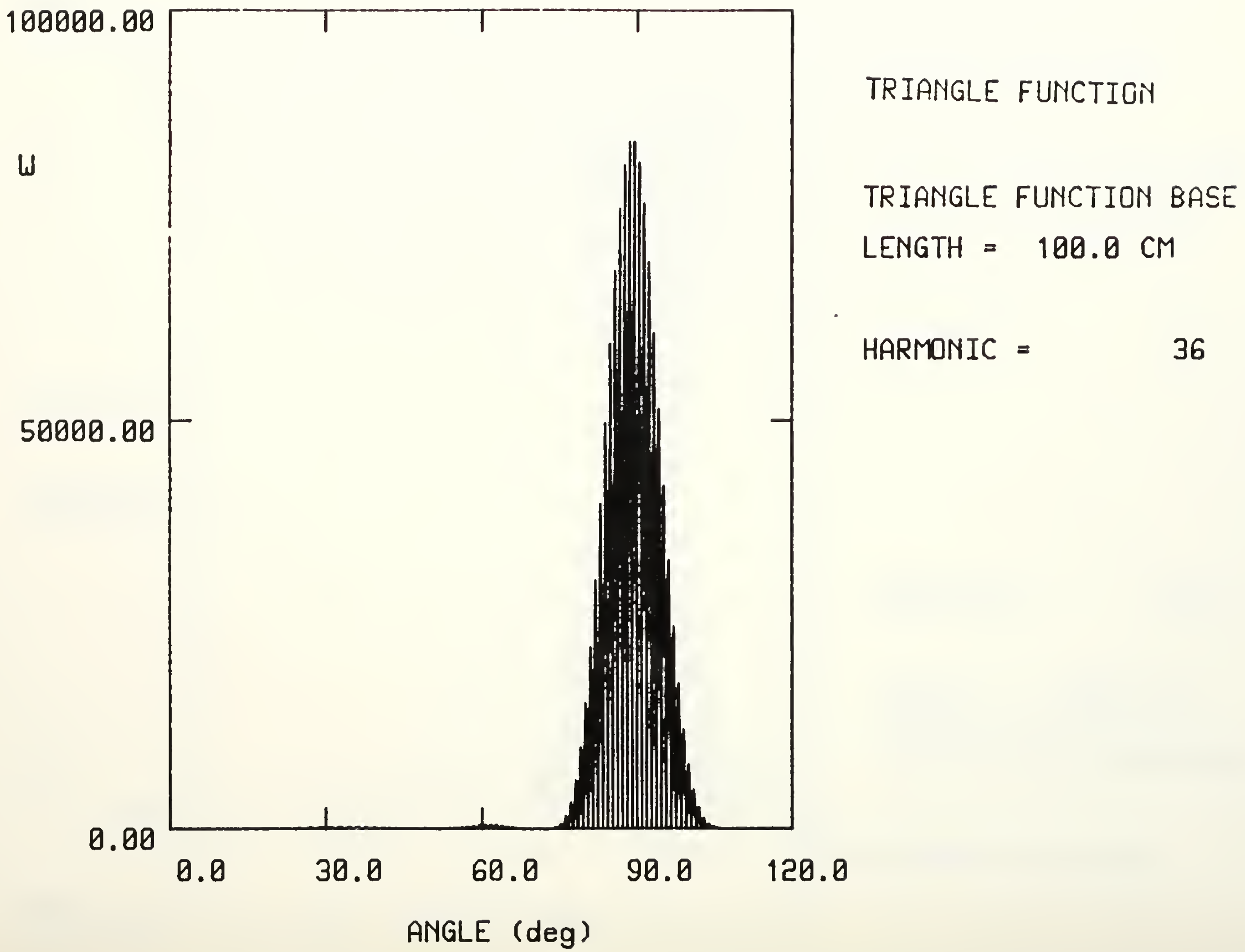
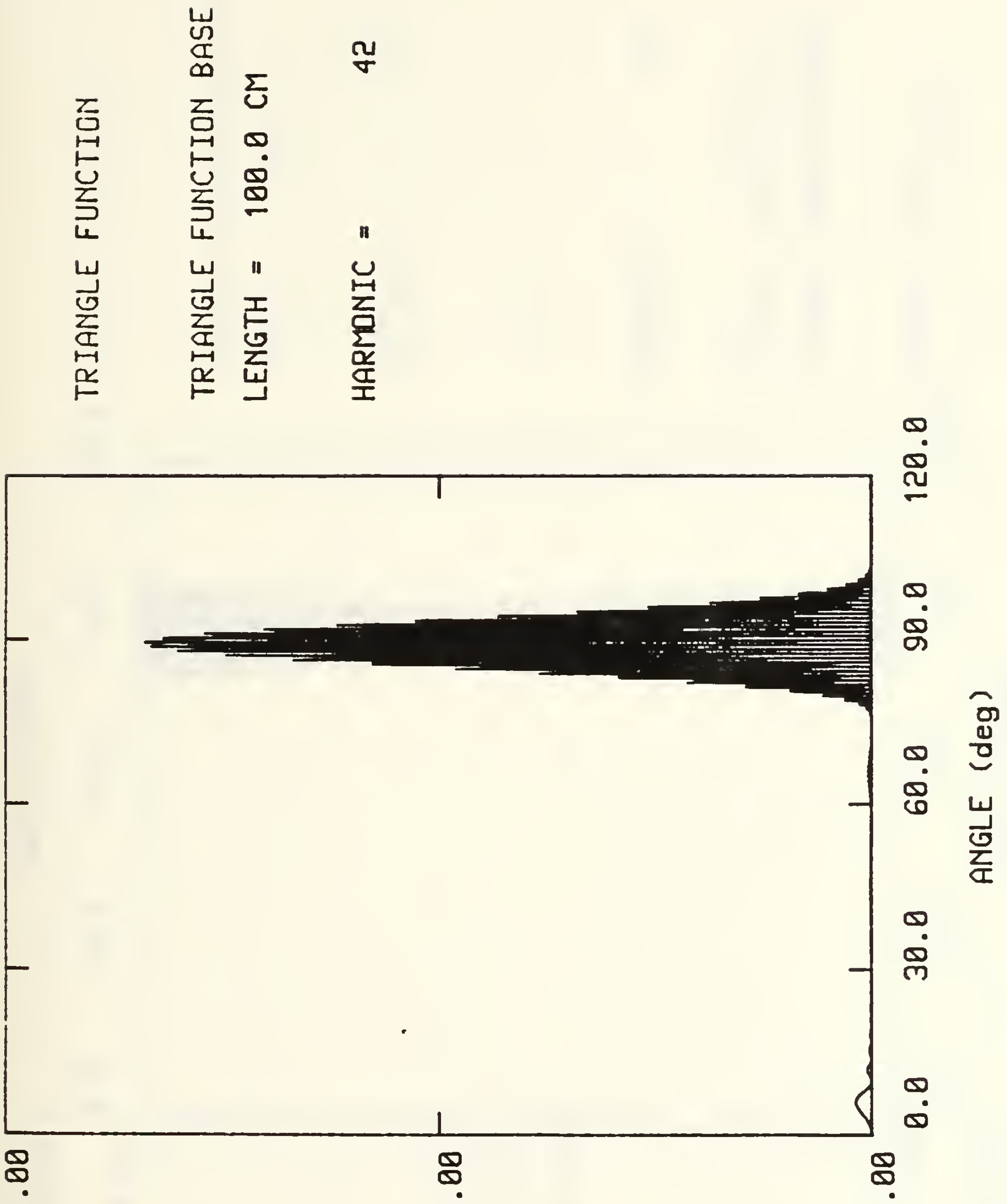
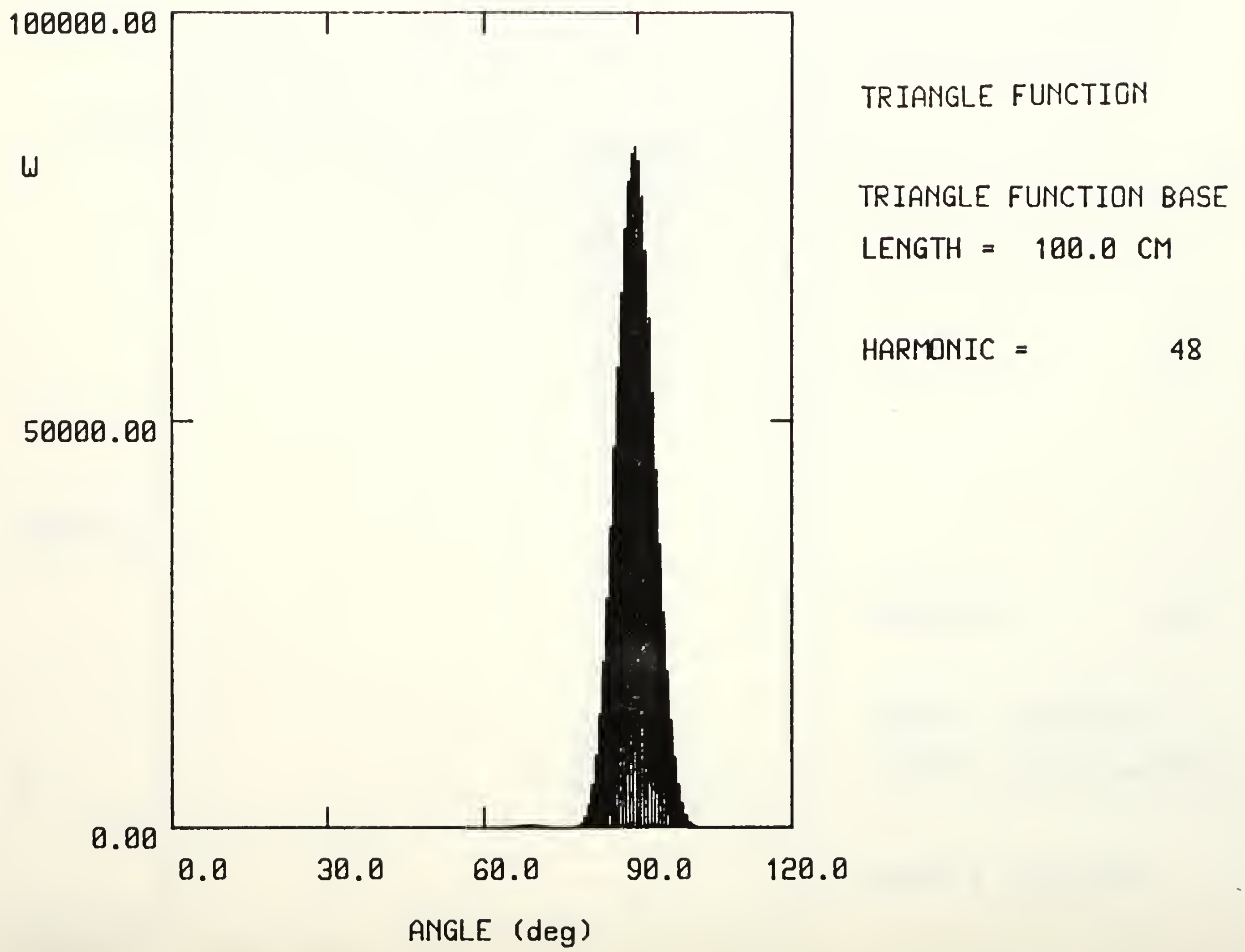
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18

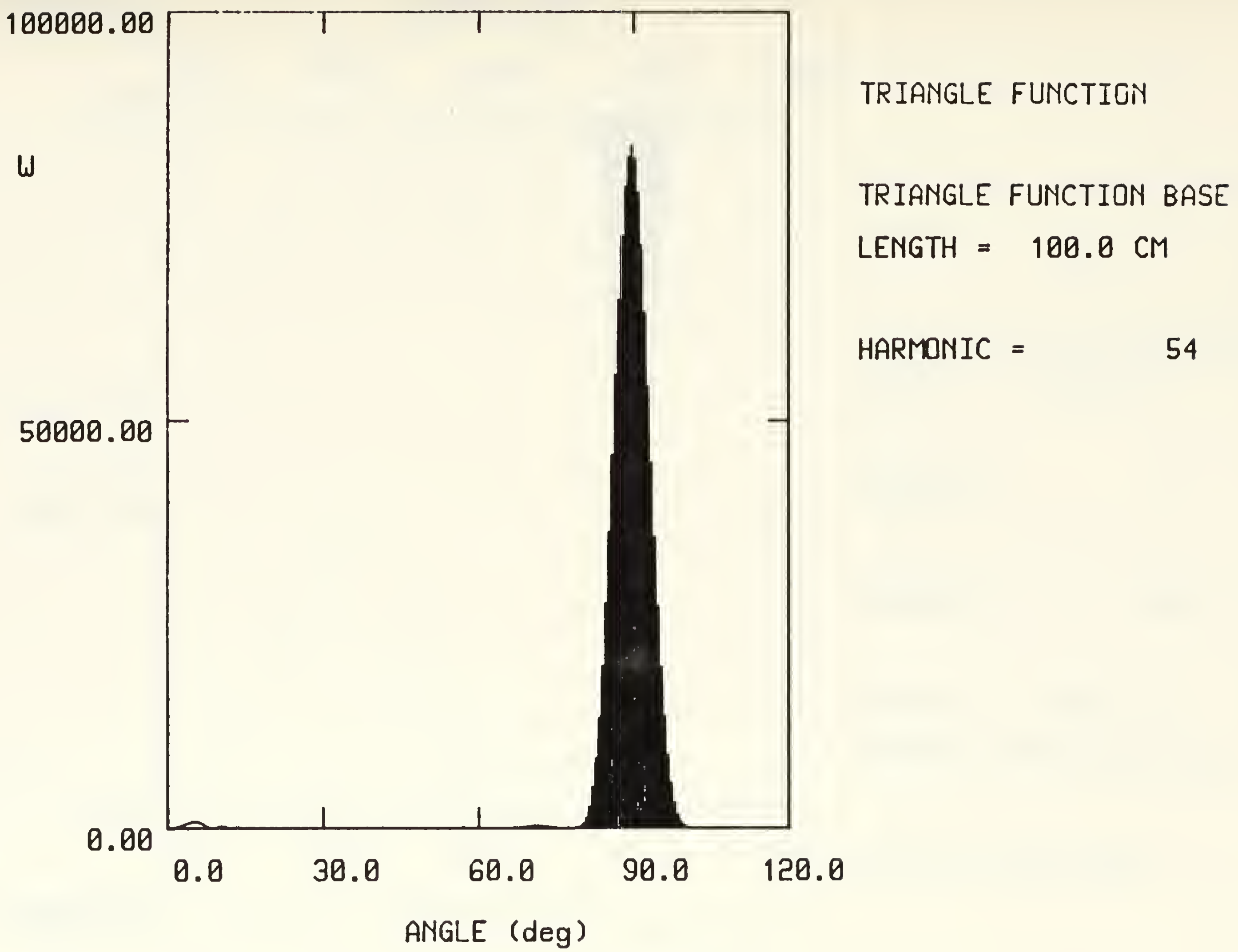

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 24

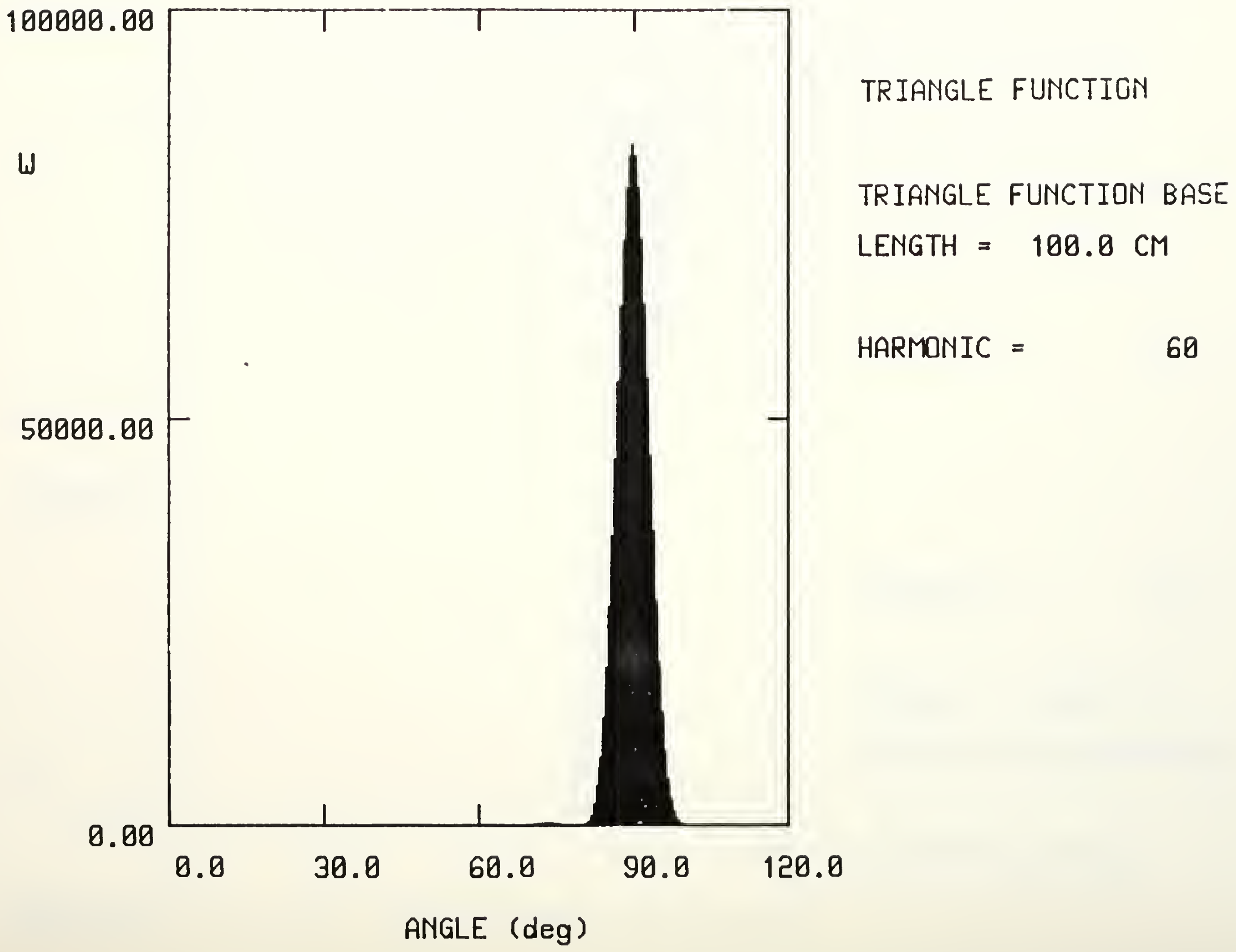

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 30


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 36

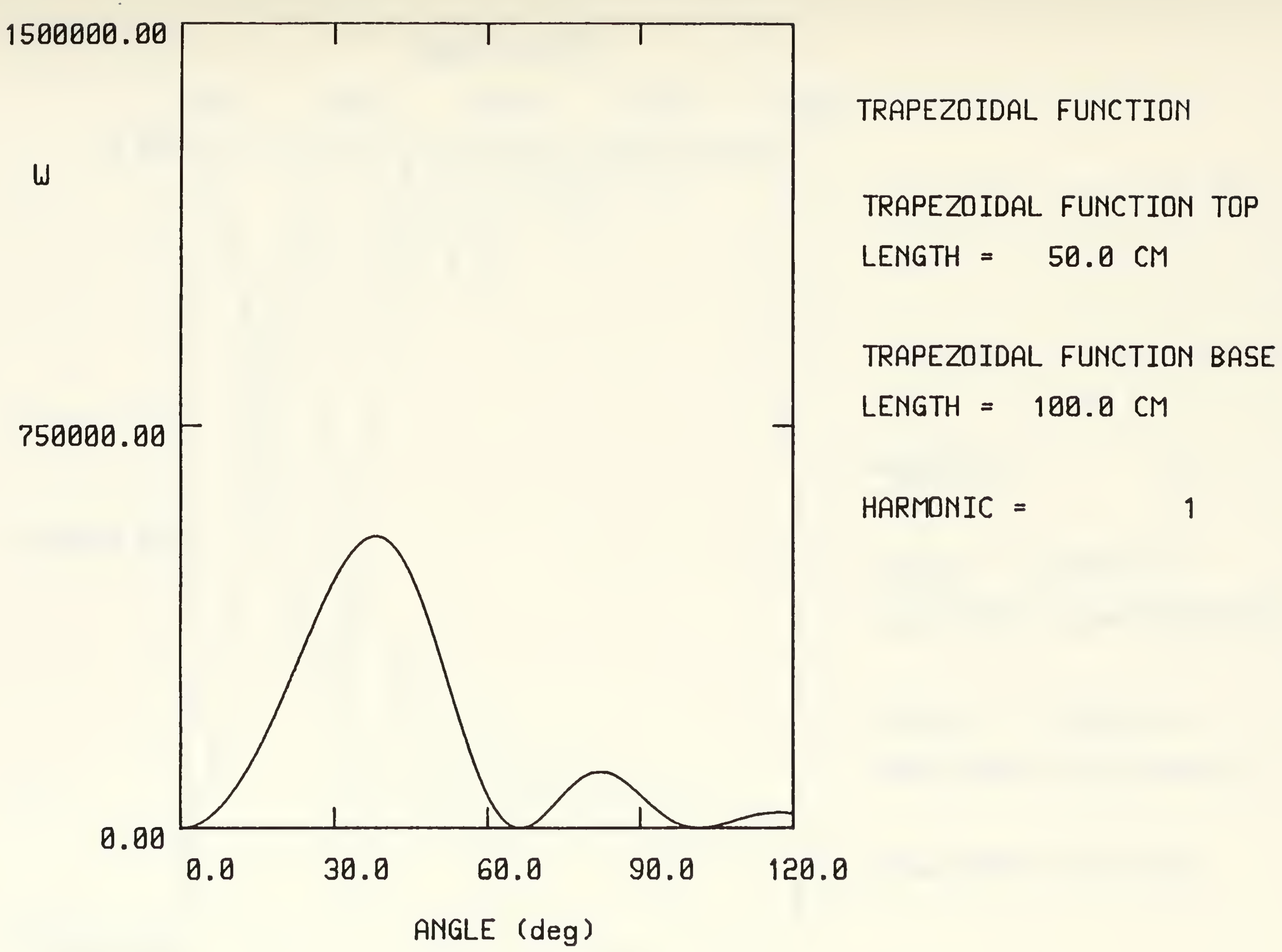

TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 42
100000.00
50000.00
0.00
E
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)

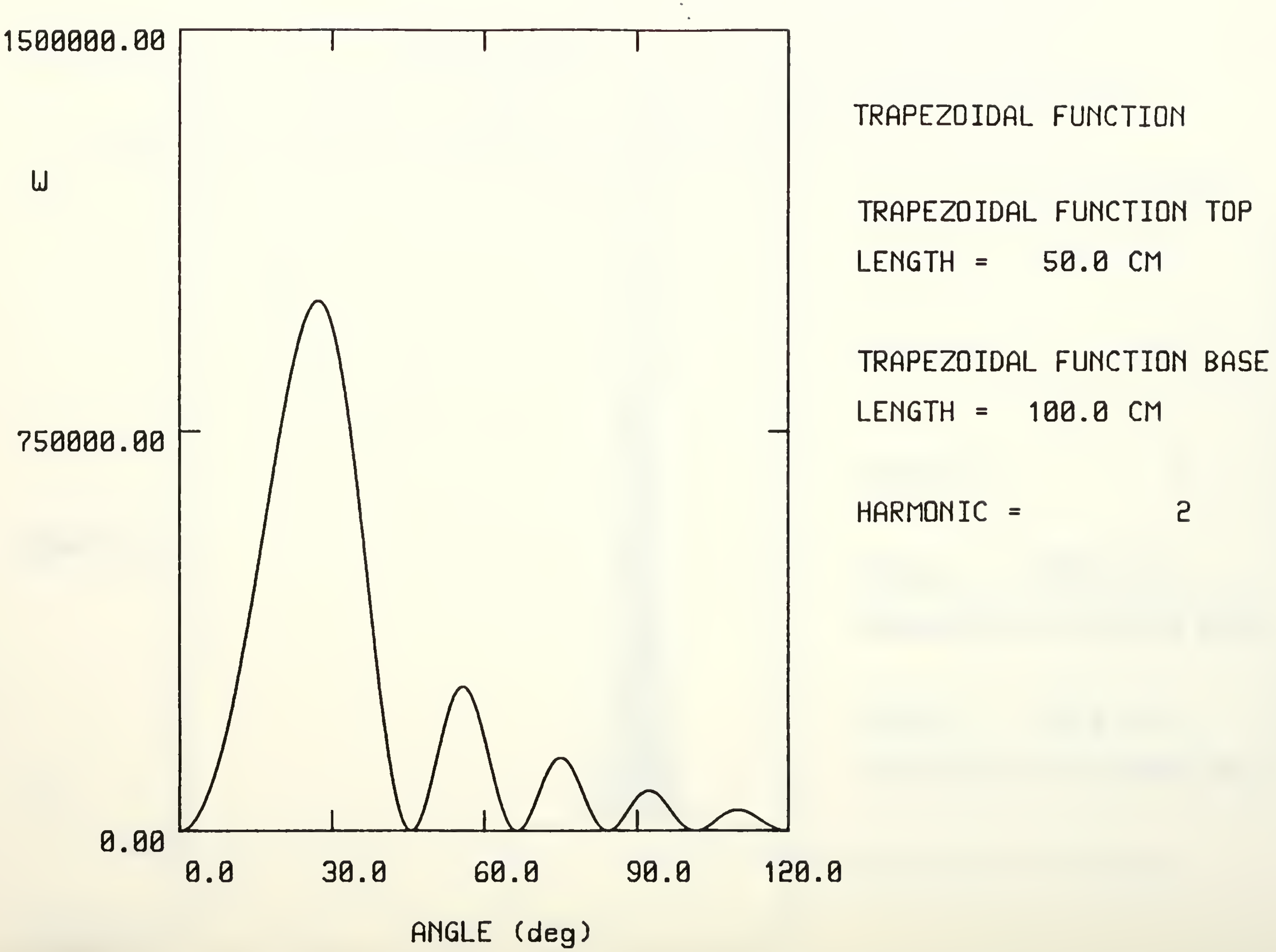

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 48

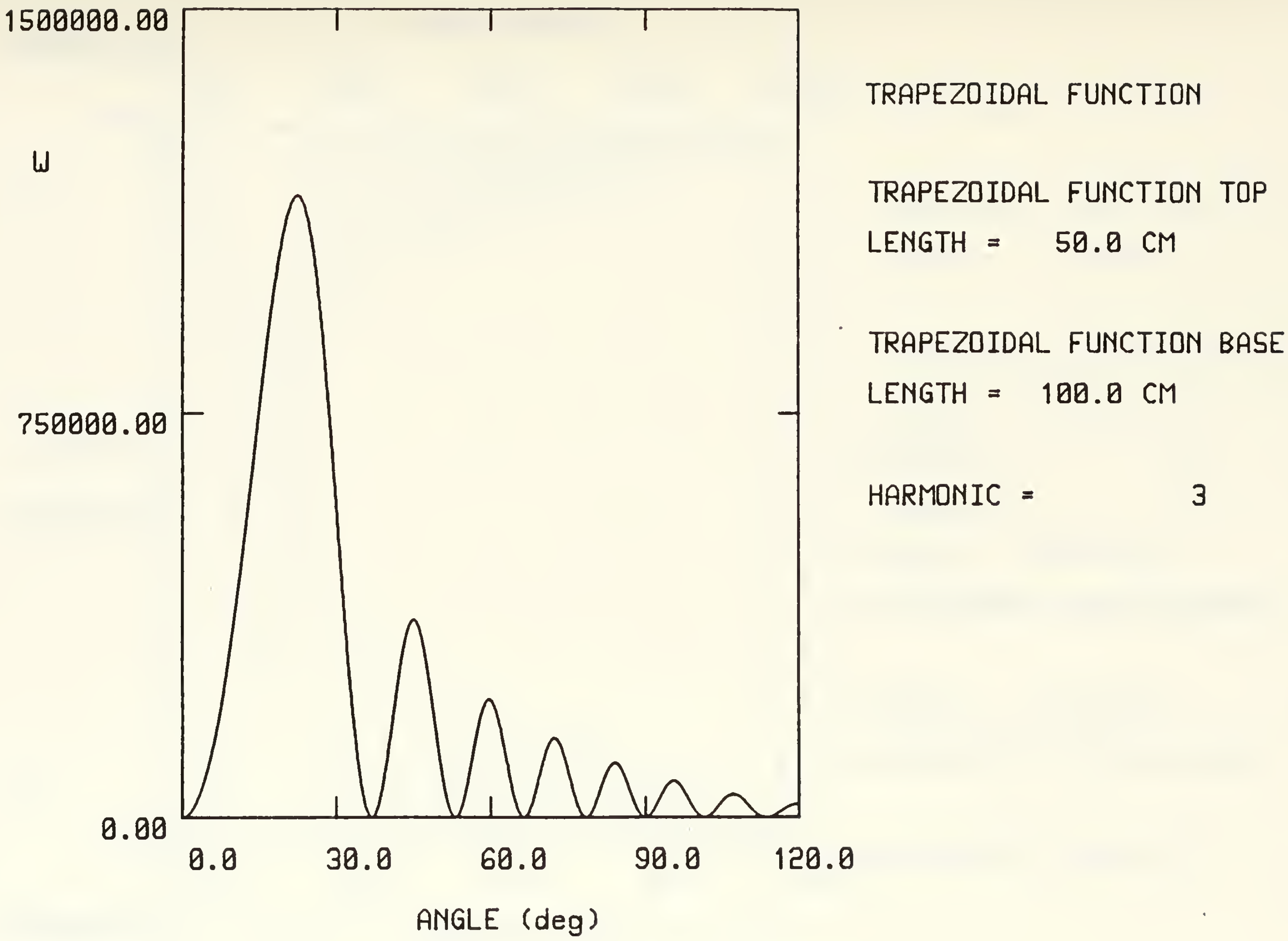

100000.00
50000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 54

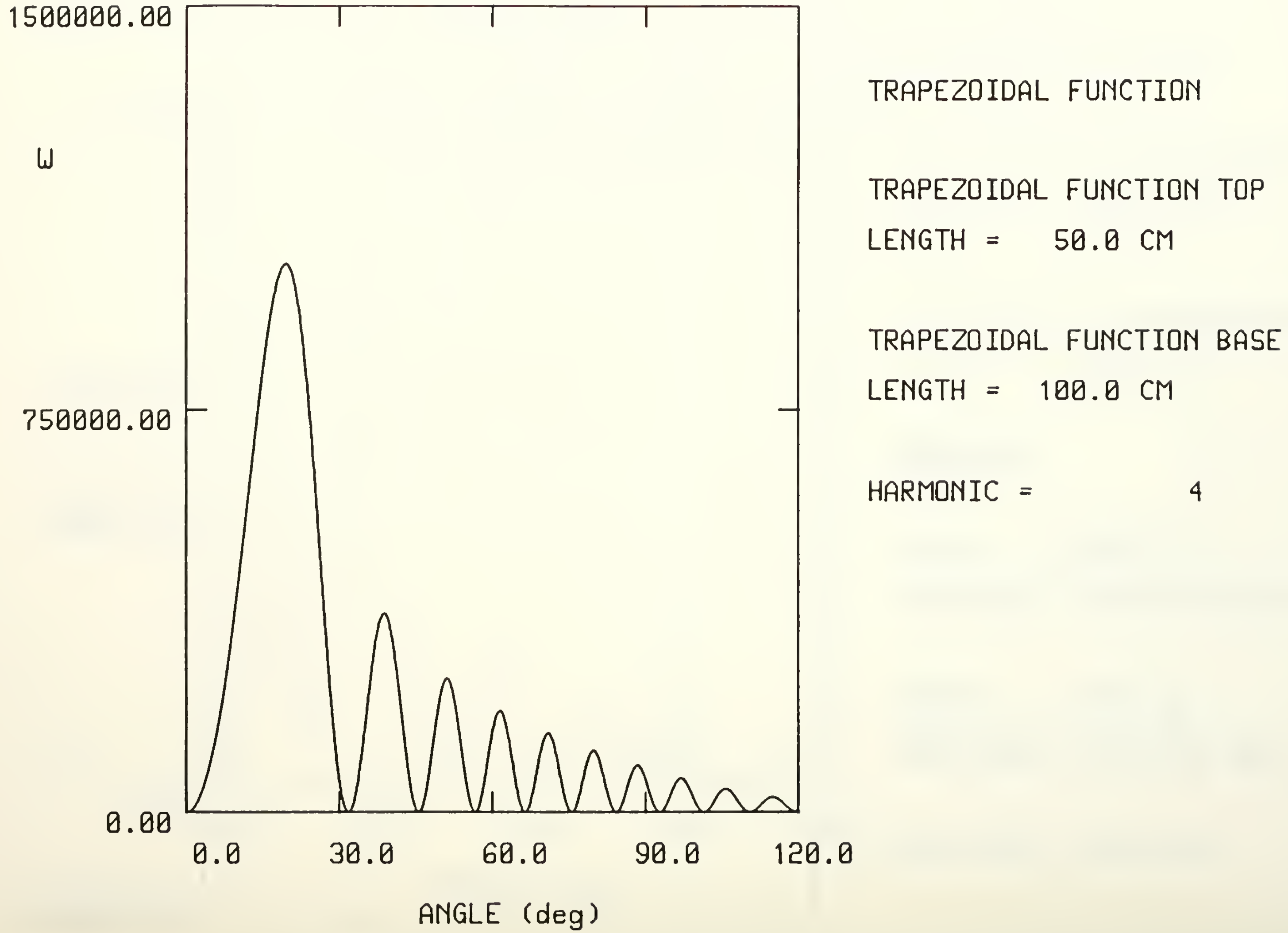

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRIANGLE FUNCTION
TRIANGLE FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 60

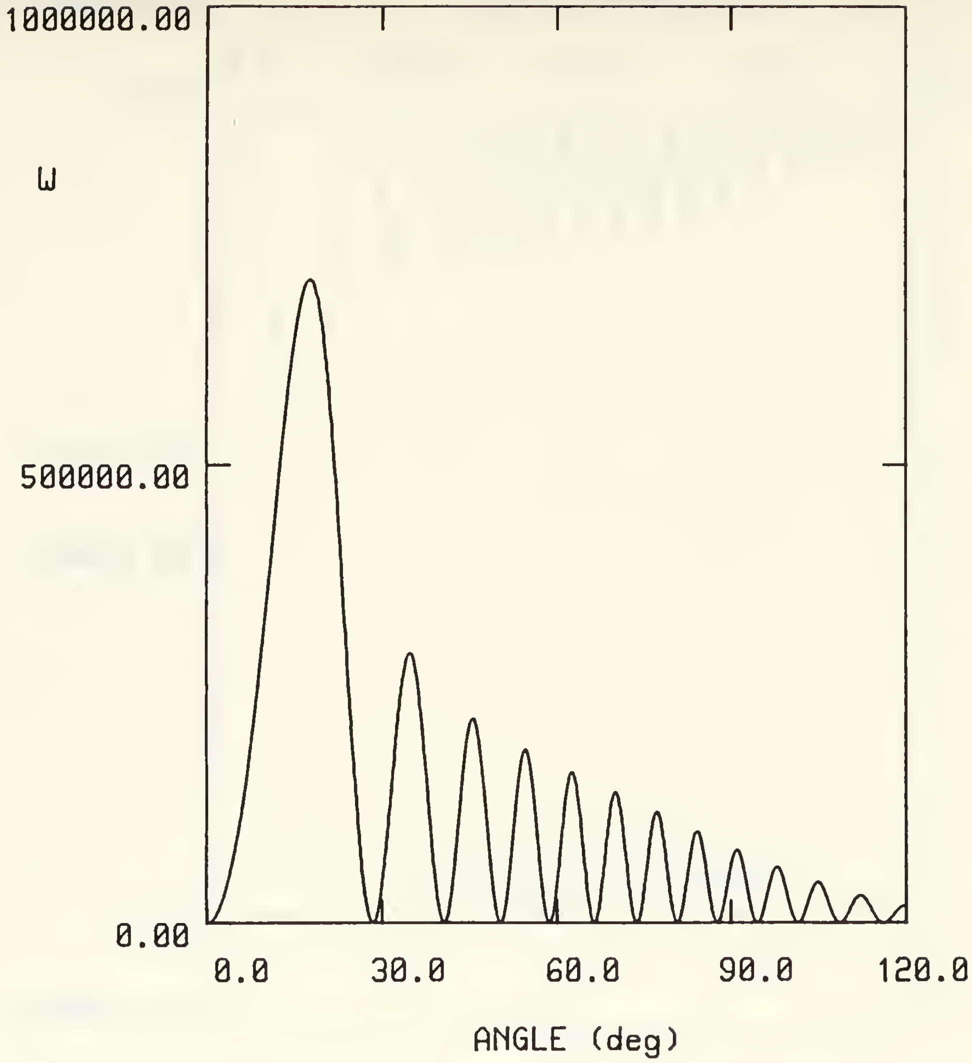
APPENDIX C: TRAPEZOIDAL FUNCTION




1500000.00
750000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 2


1500000.00
W
750000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 3


1500000.00
750000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 4

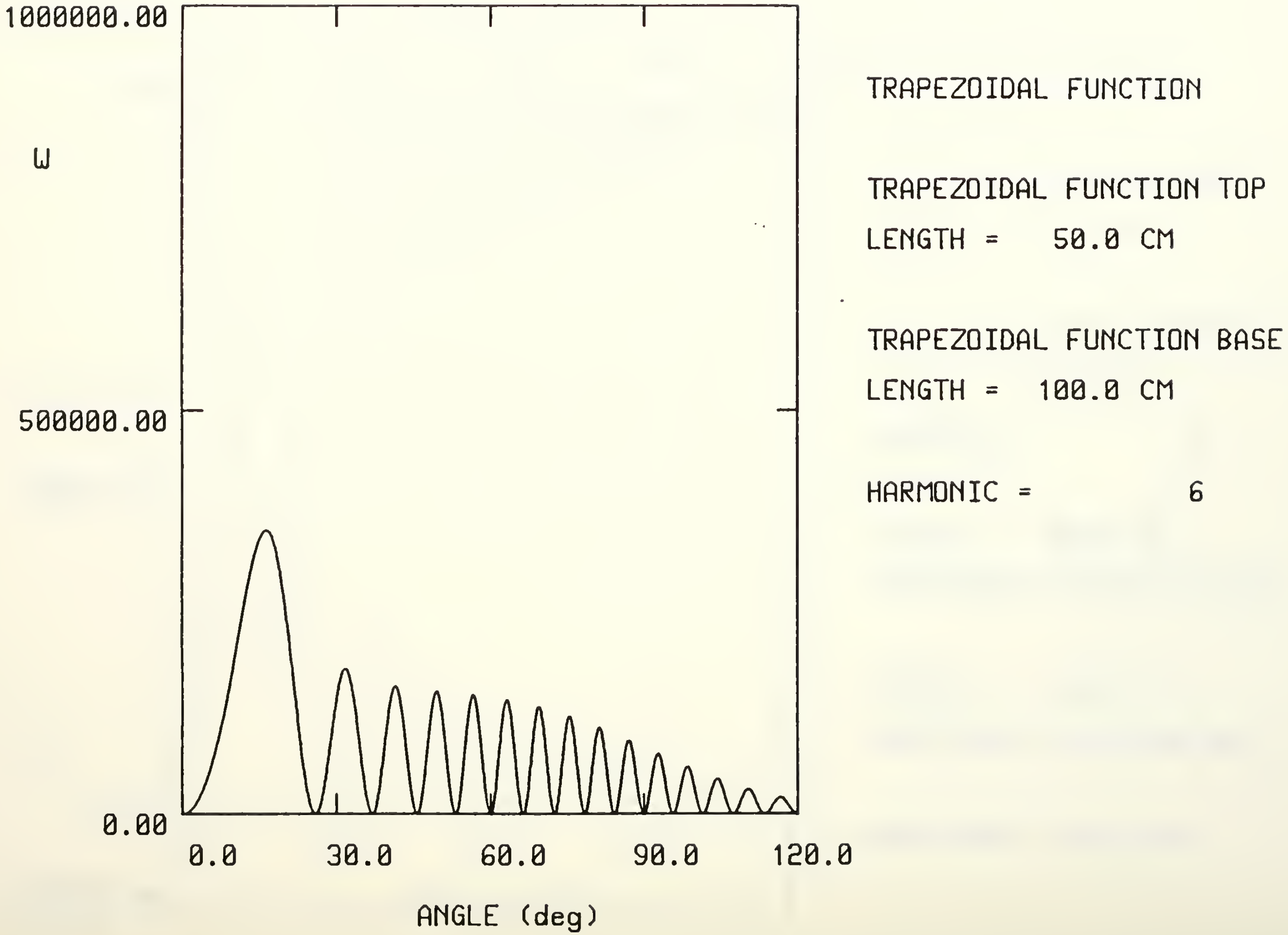


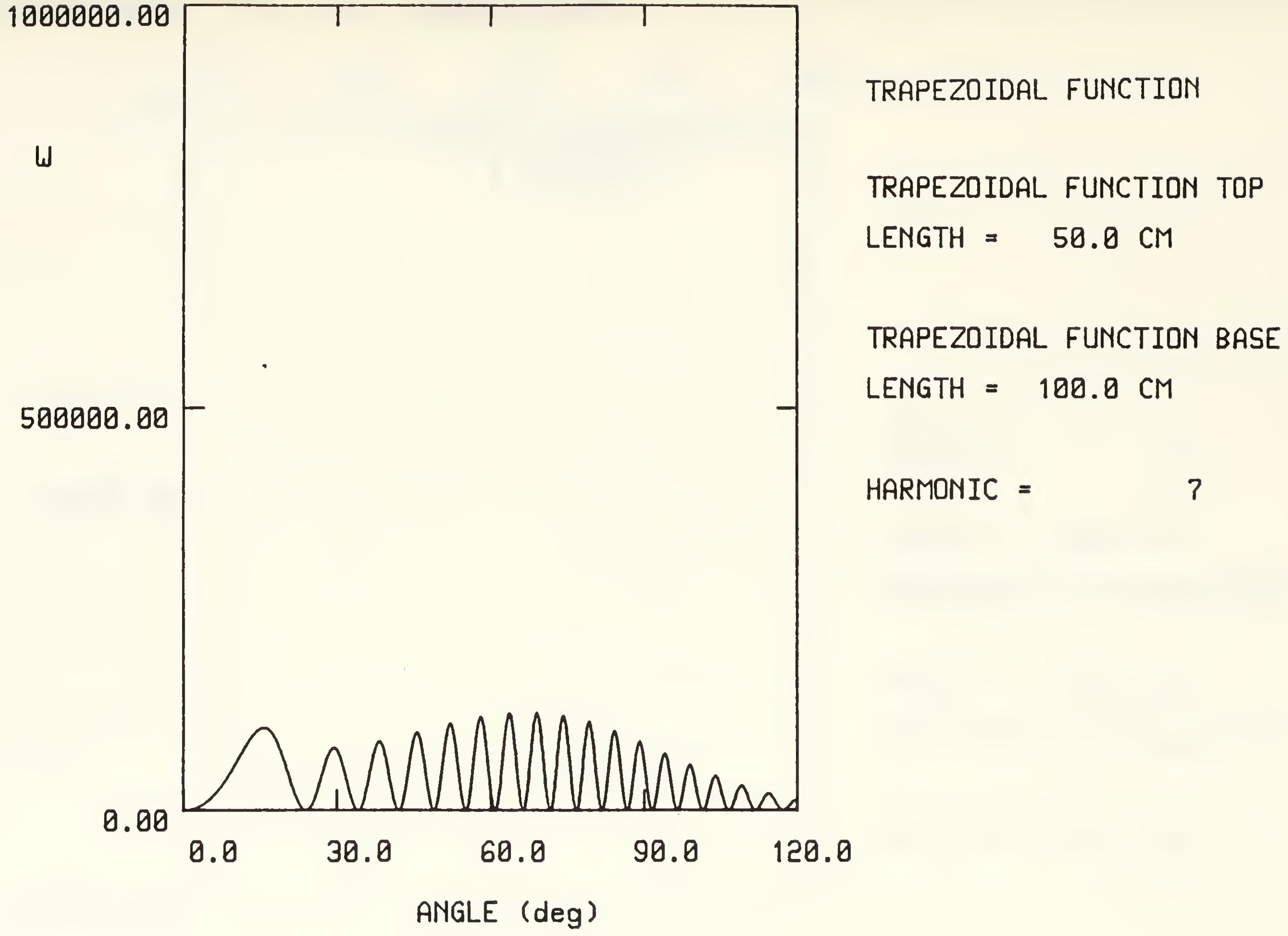
TRAPEZOIDAL FUNCTION

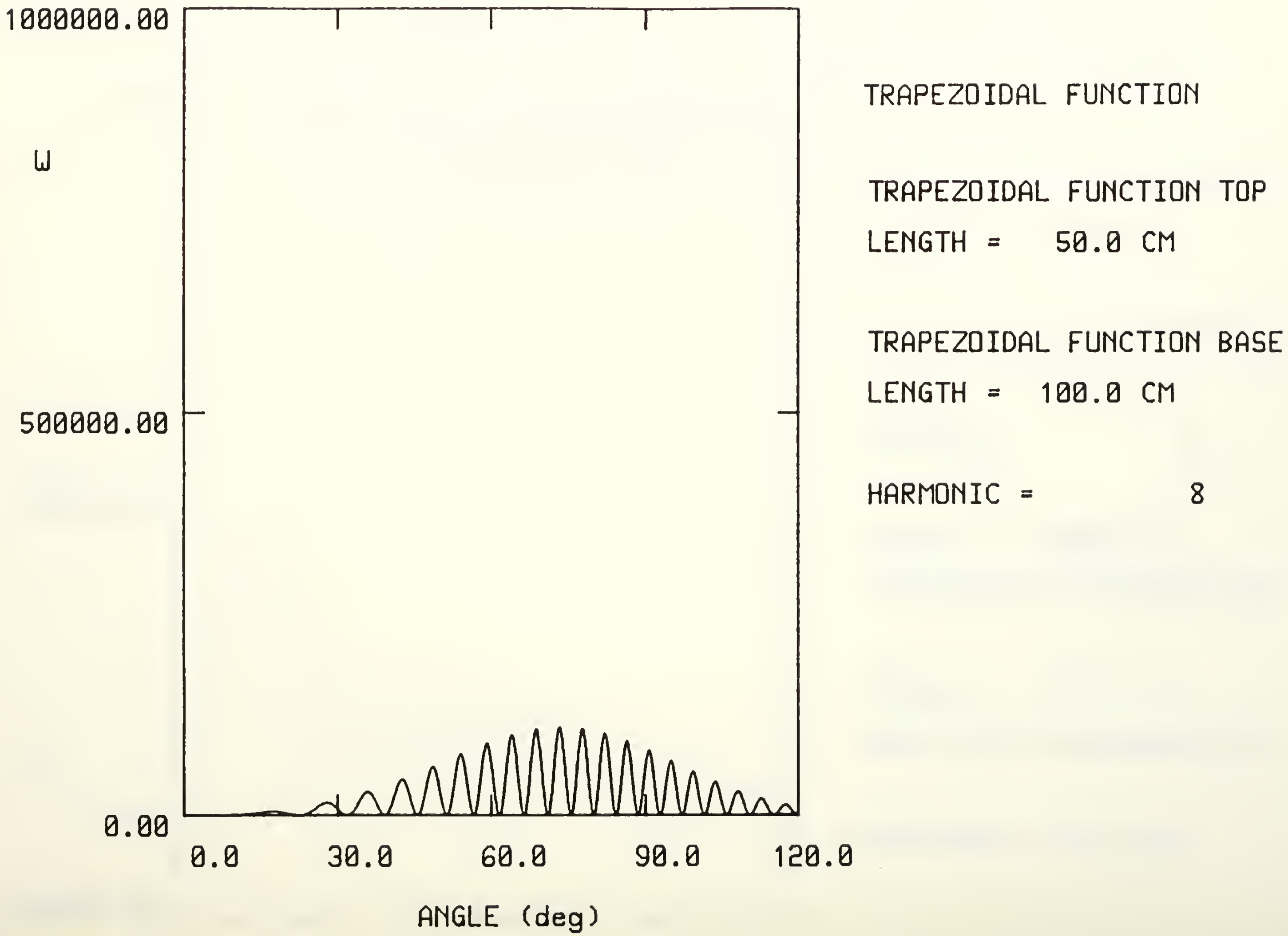
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM

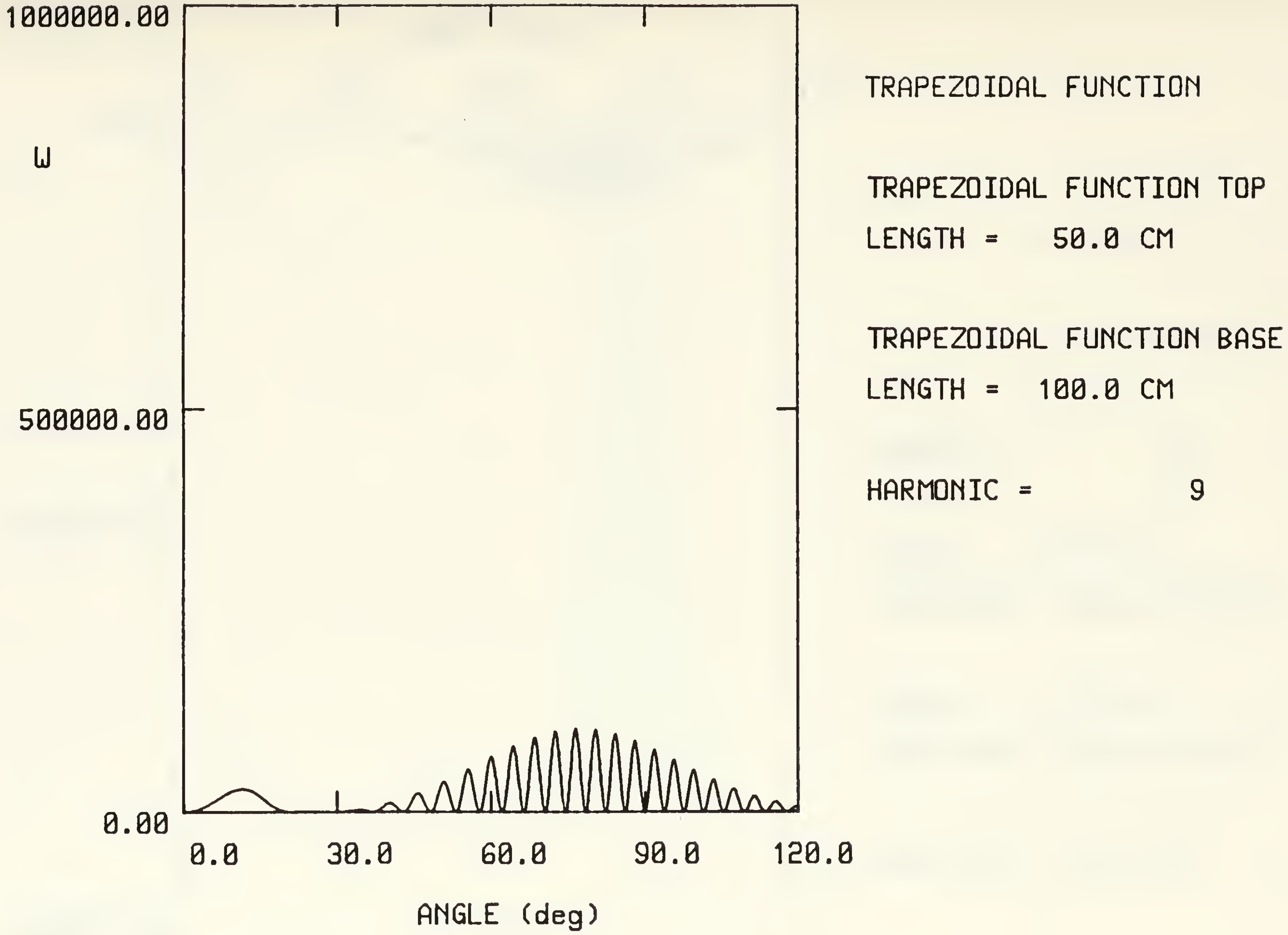
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM

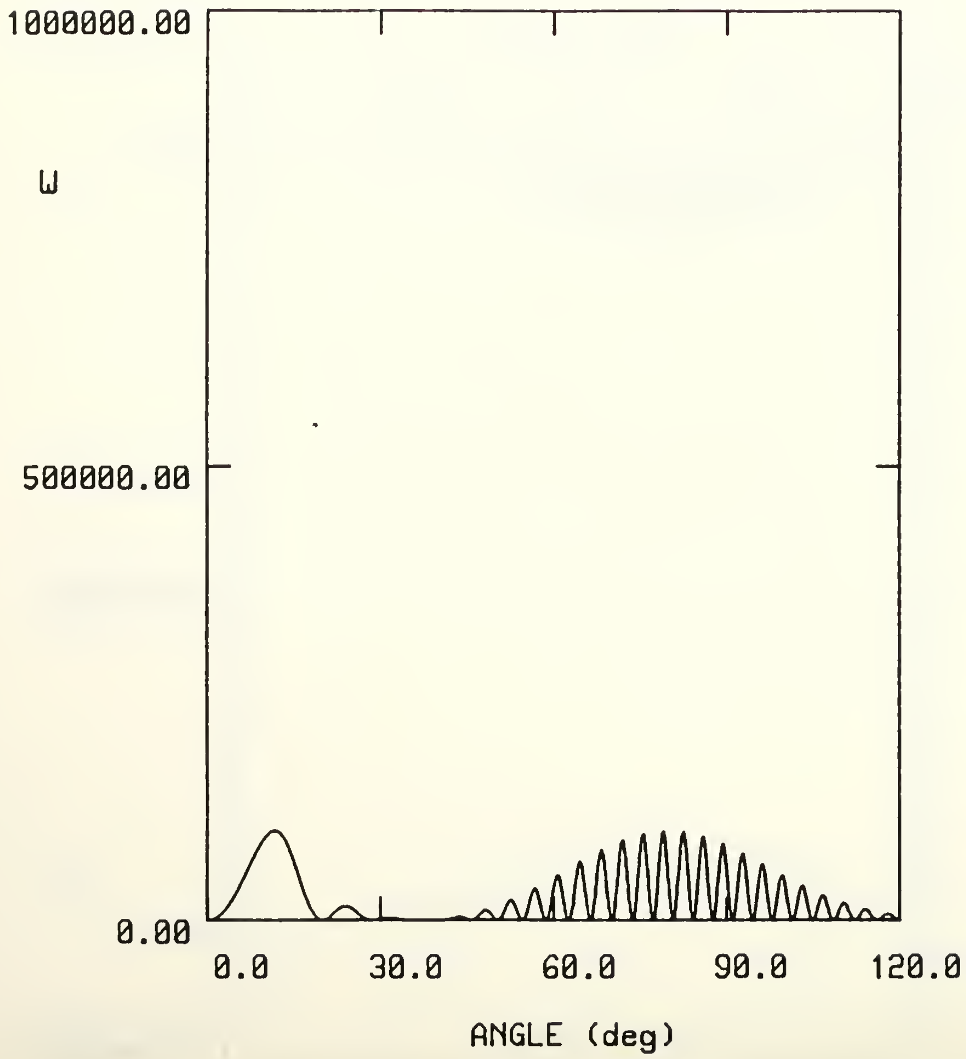
HARMONIC = 5


1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 6


1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 7


1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 8


1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 9

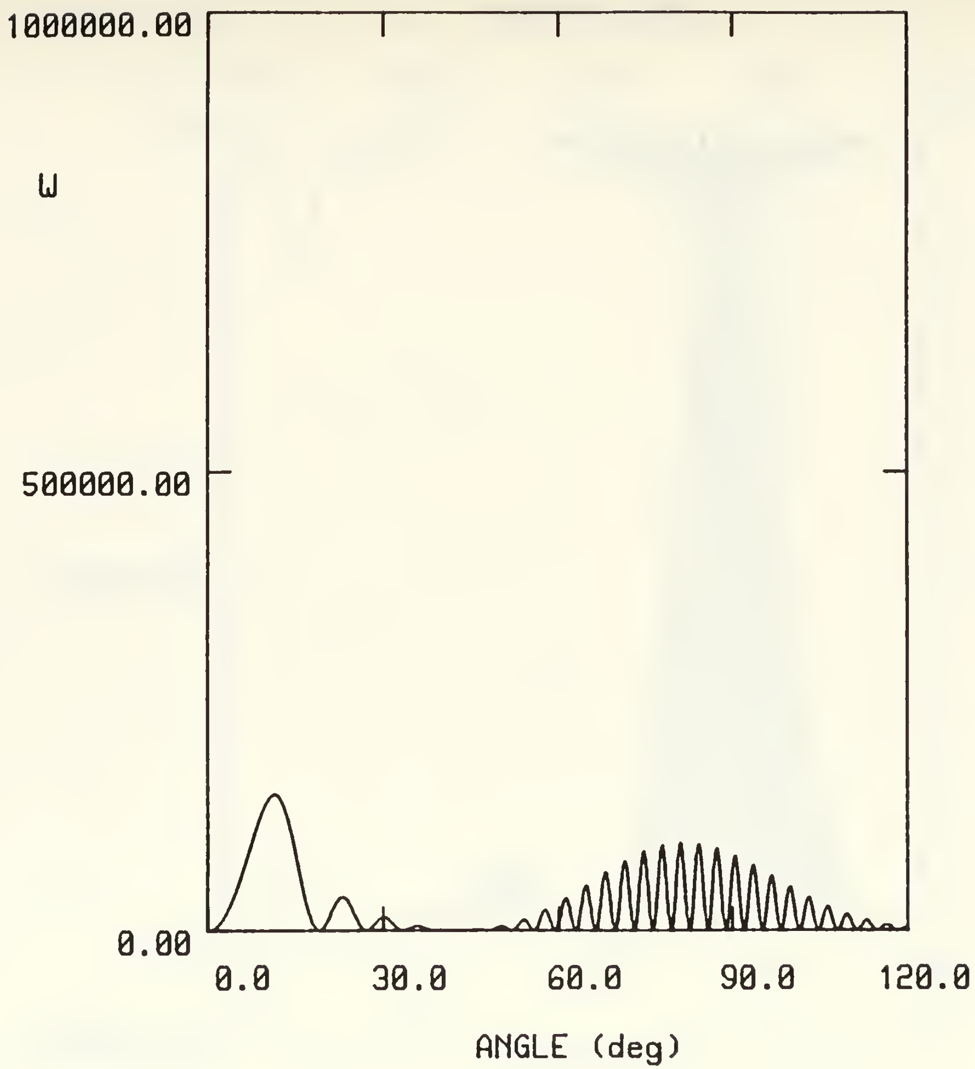


TRAPEZOIDAL FUNCTION

TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM

TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM

HARMONIC = 10

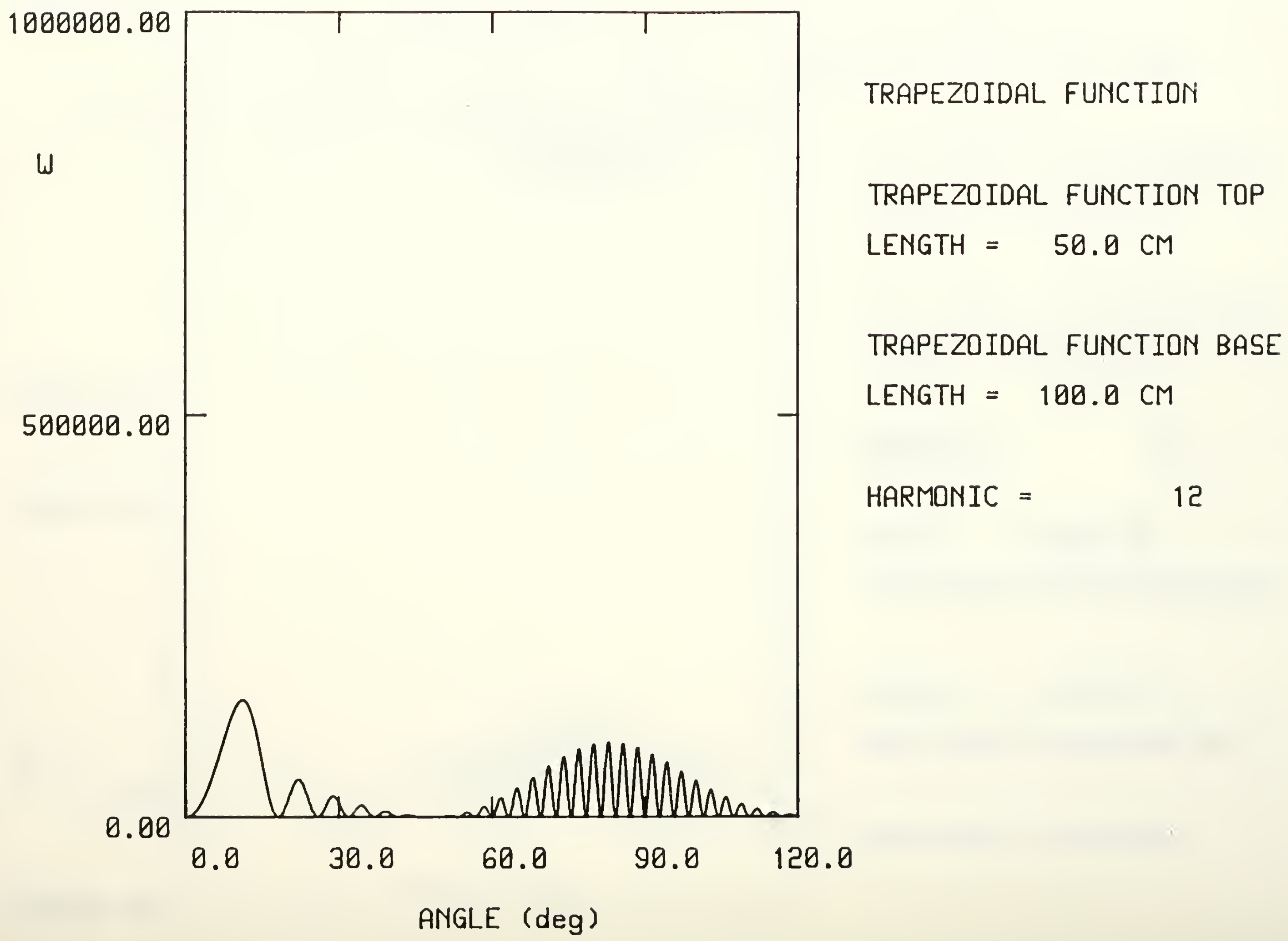


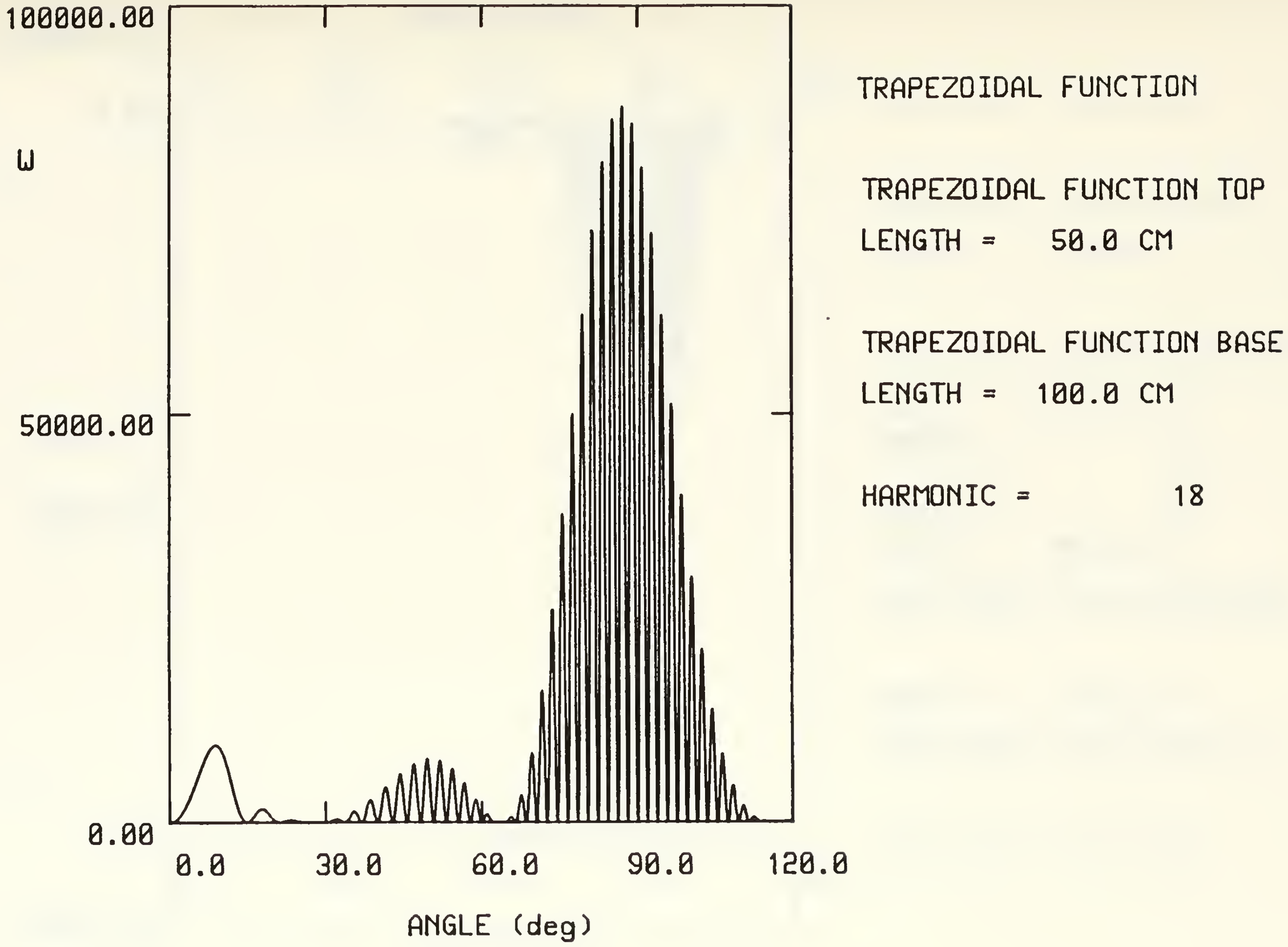
TRAPEZOIDAL FUNCTION

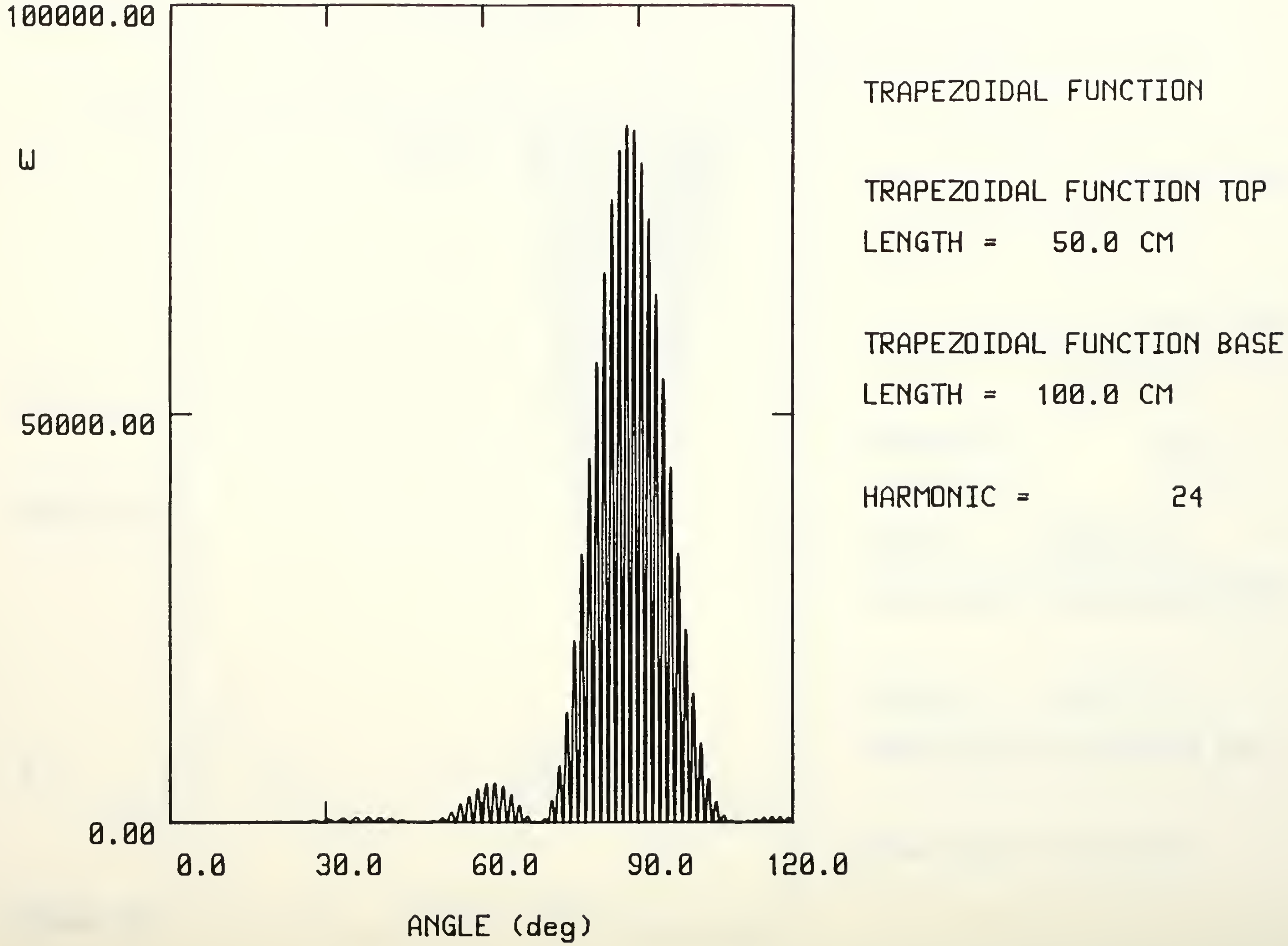
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM

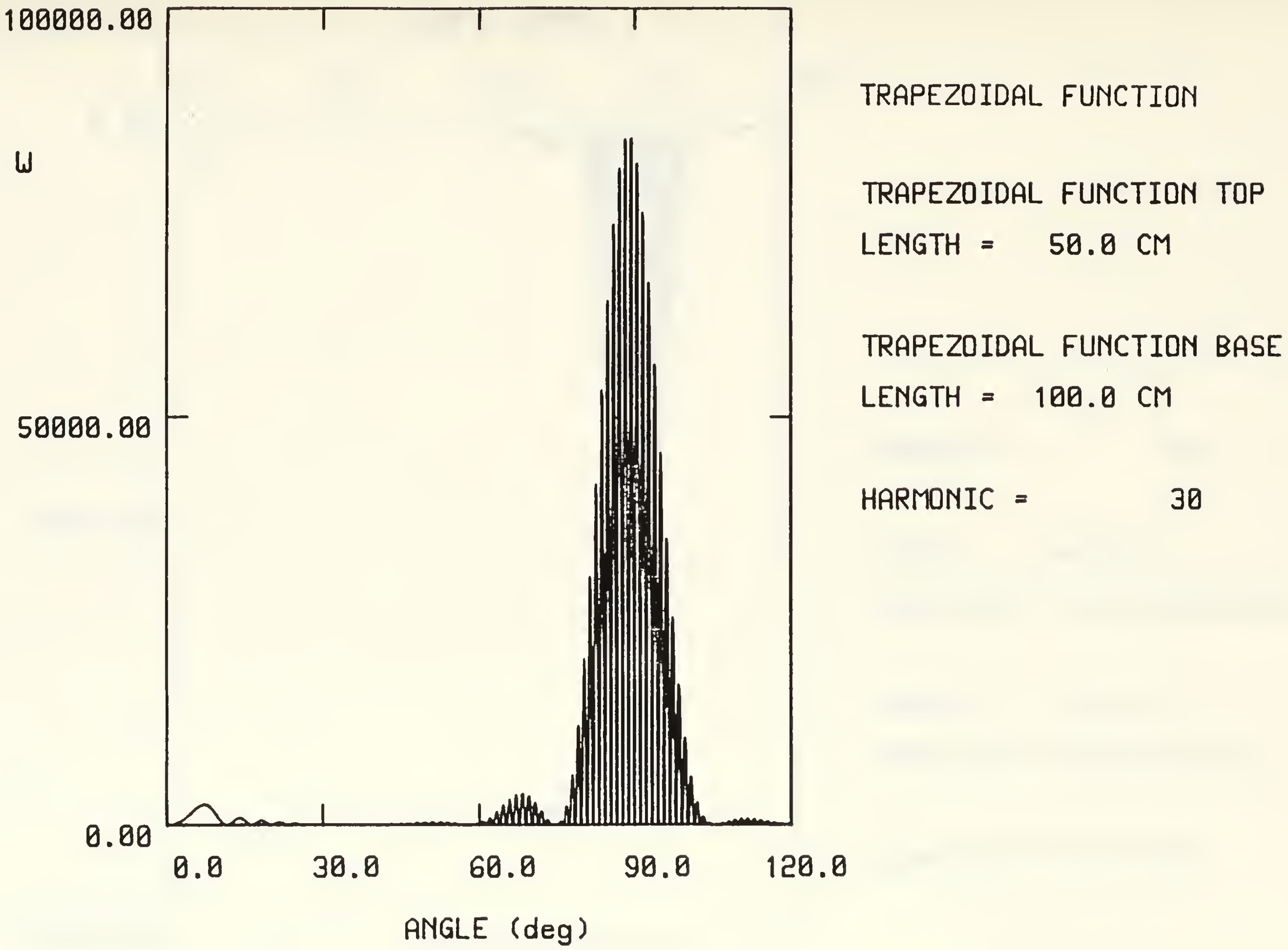
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM

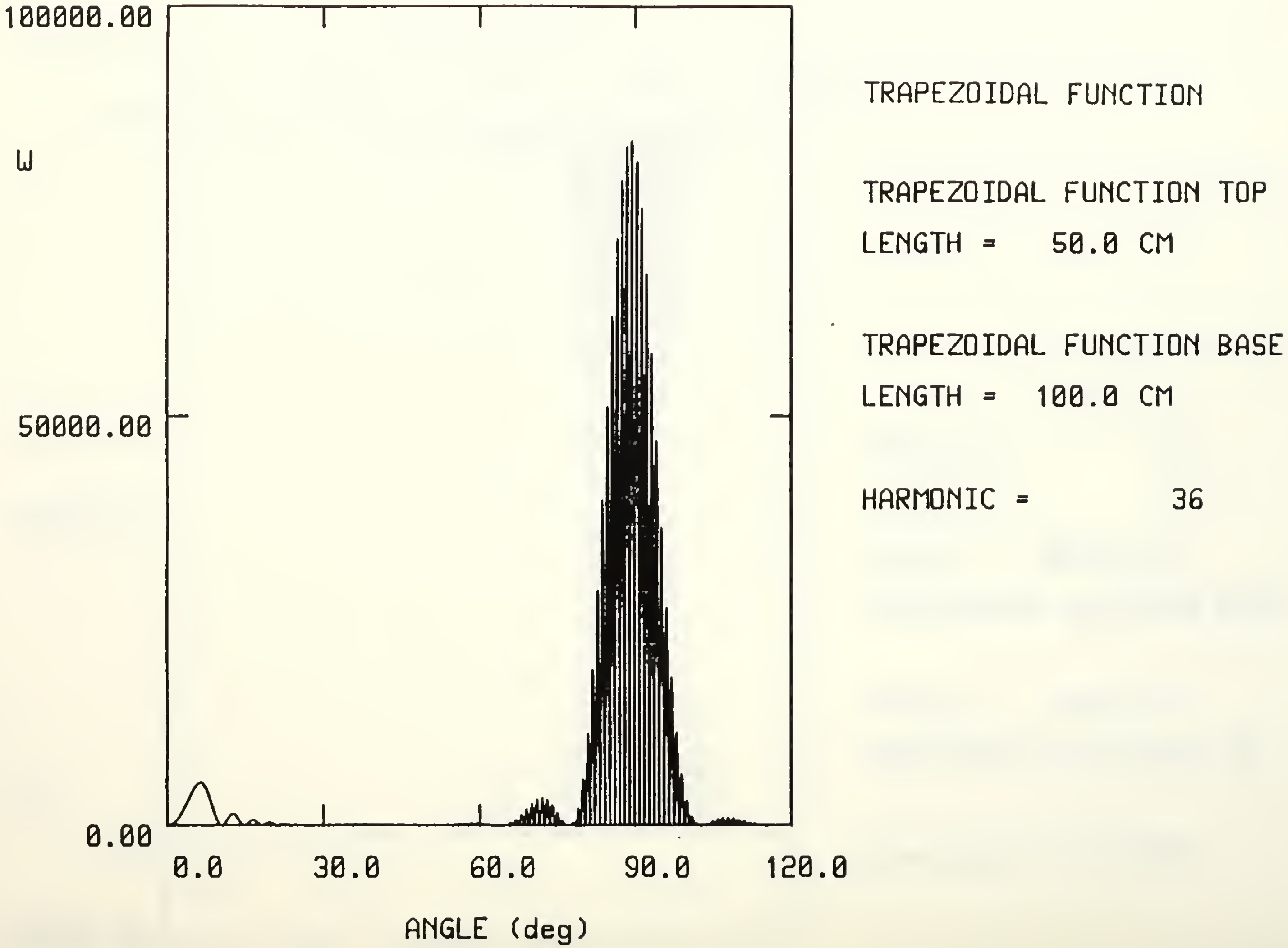
HARMONIC = 11

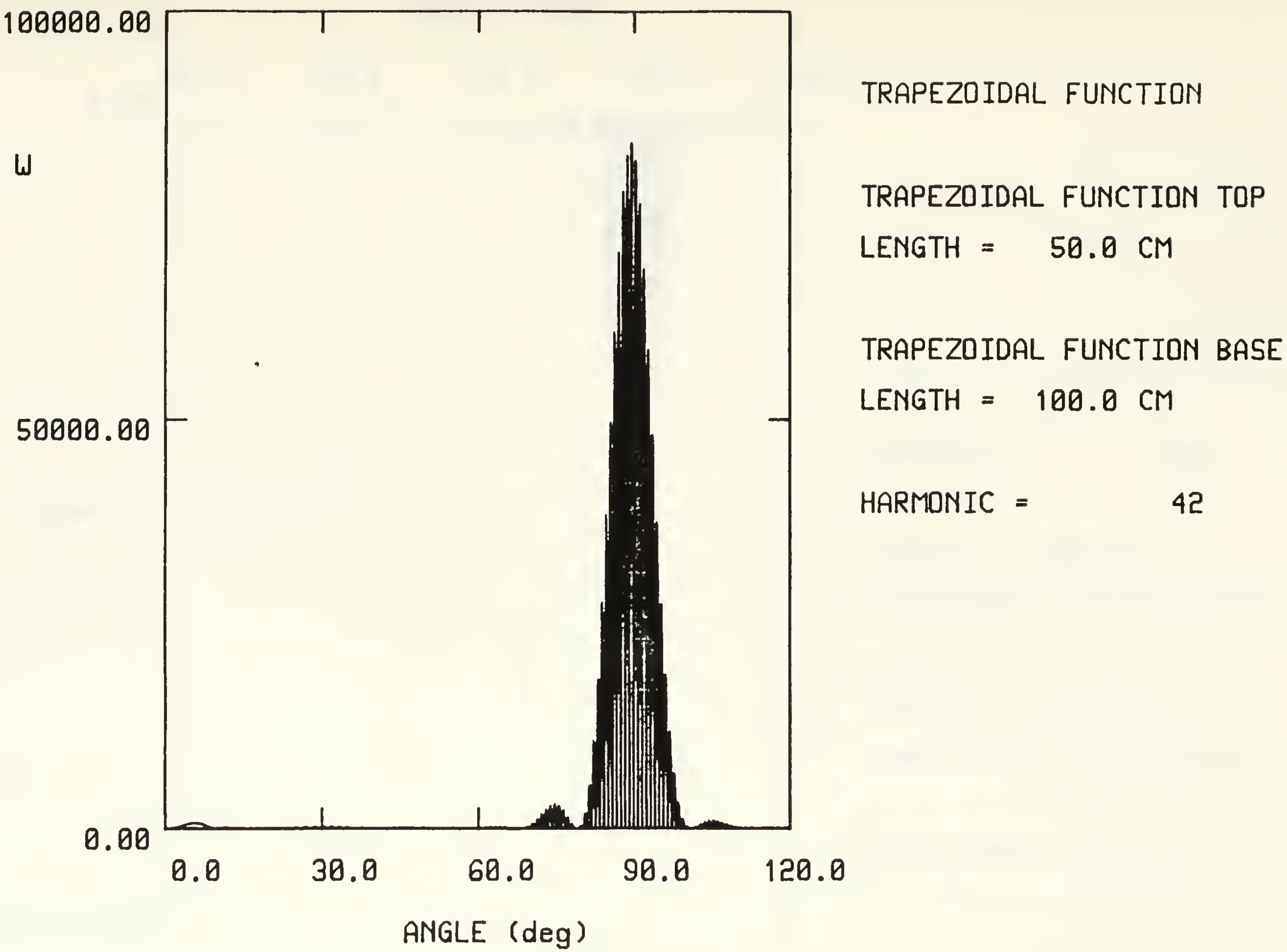

1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 12


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 24


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 30


100000.00
50000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 36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W
50000.00
0.00
0.0
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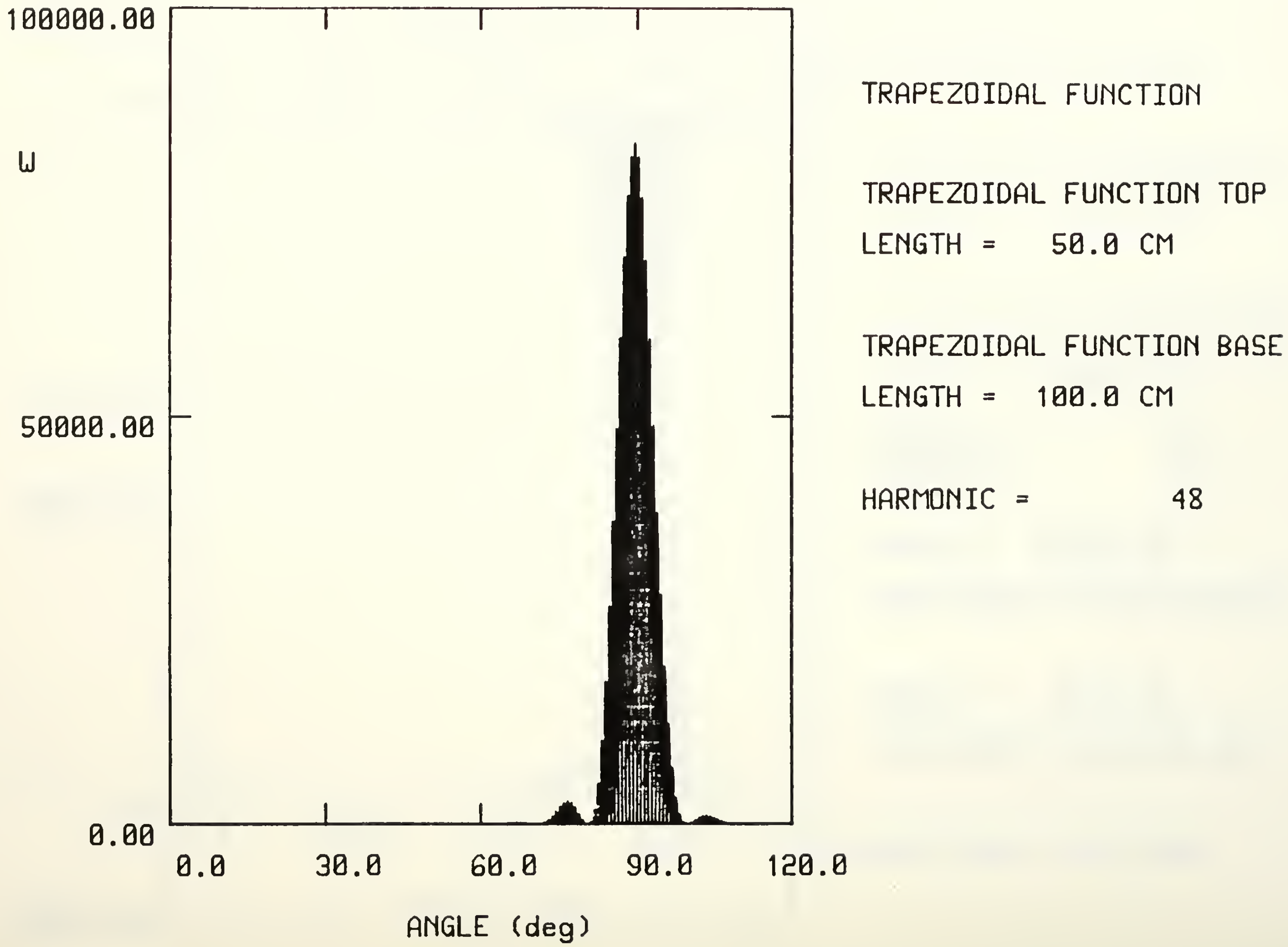
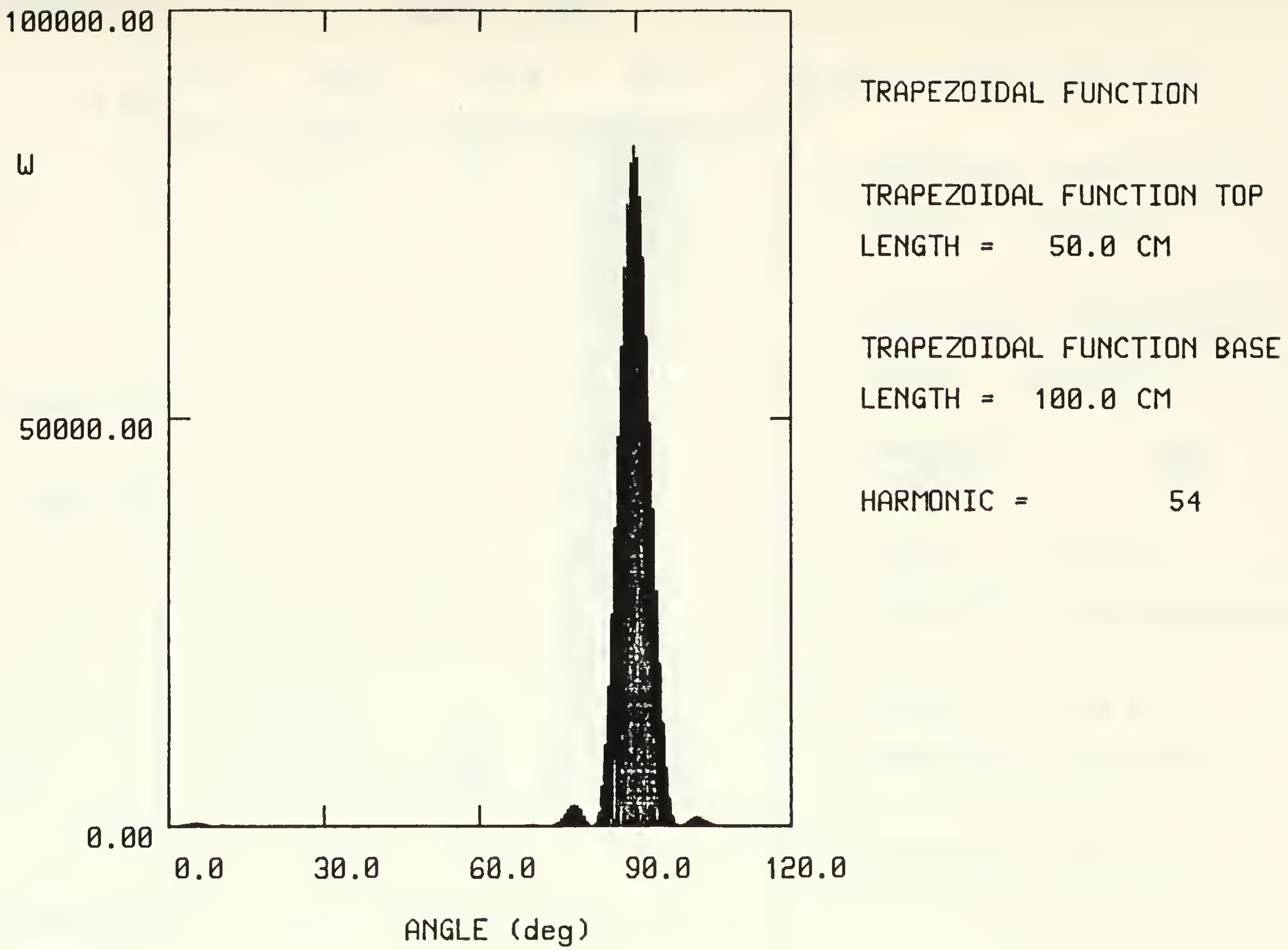
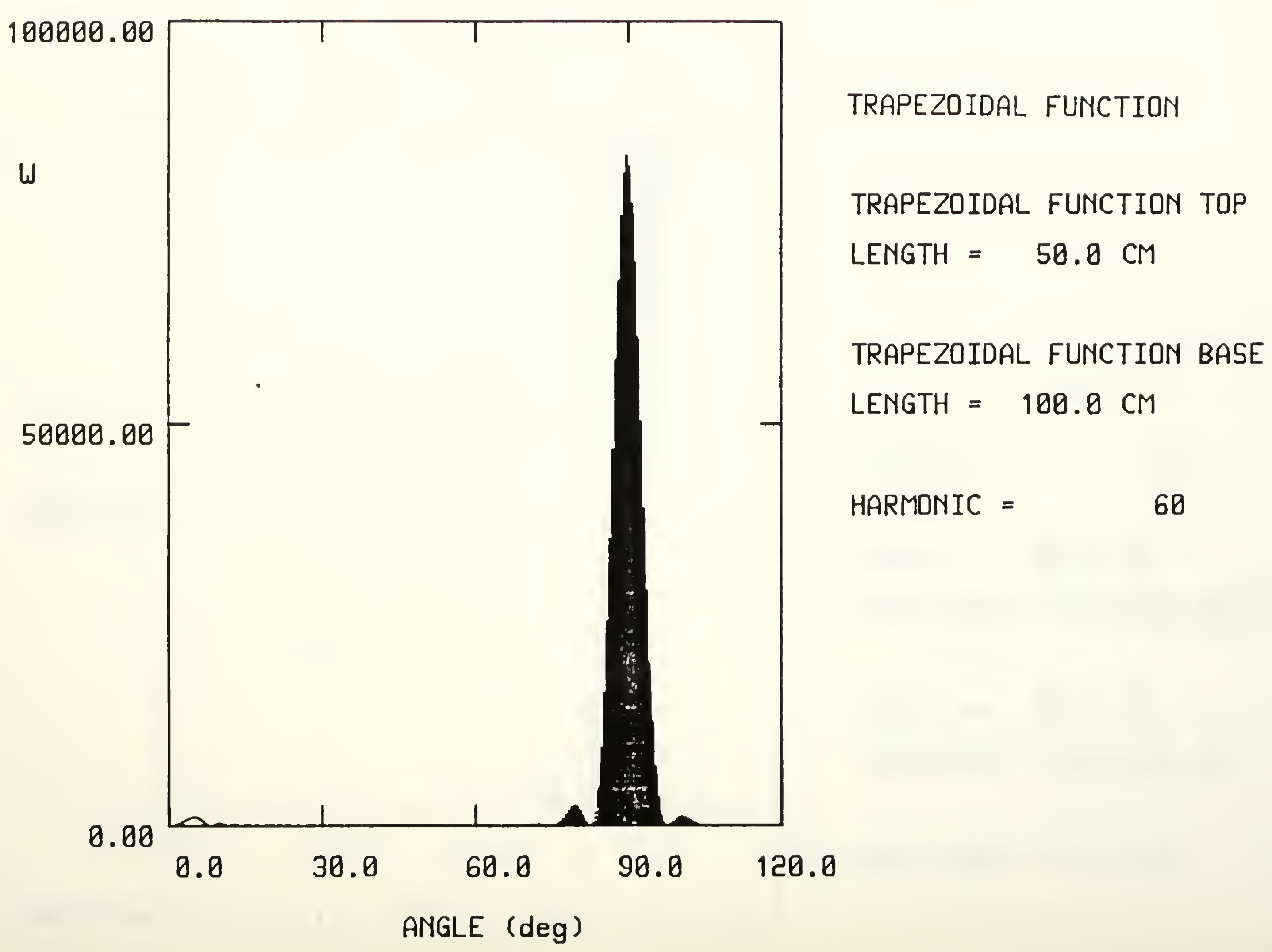
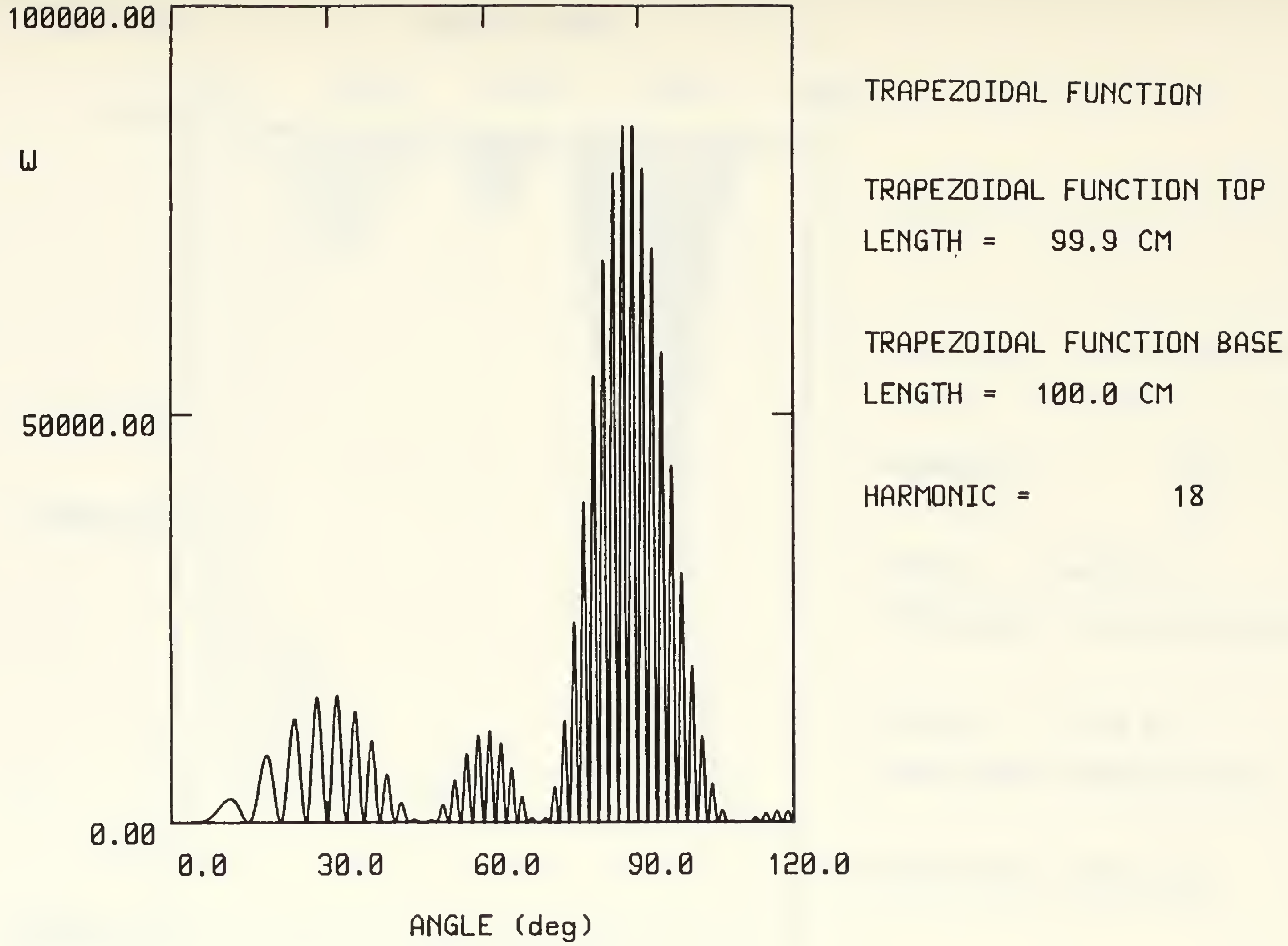
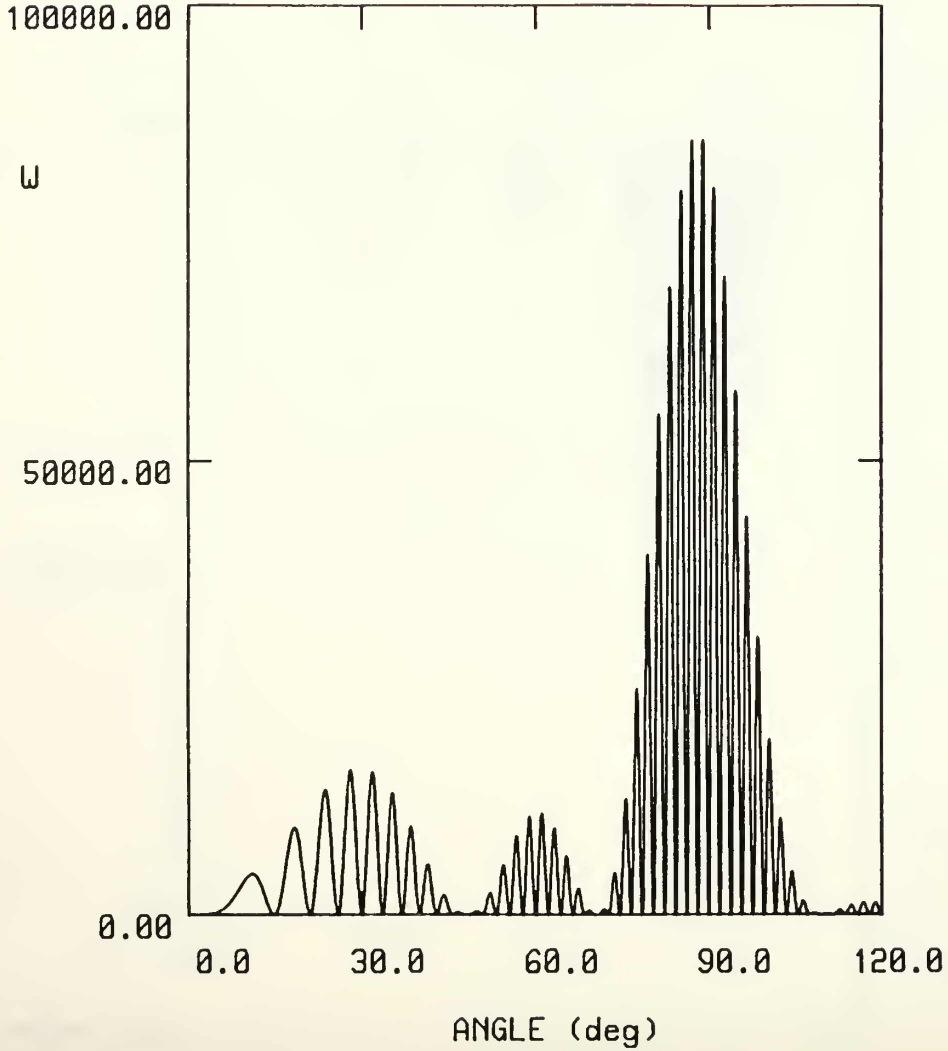
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 42


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 48


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 54


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 60


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 99.9 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18

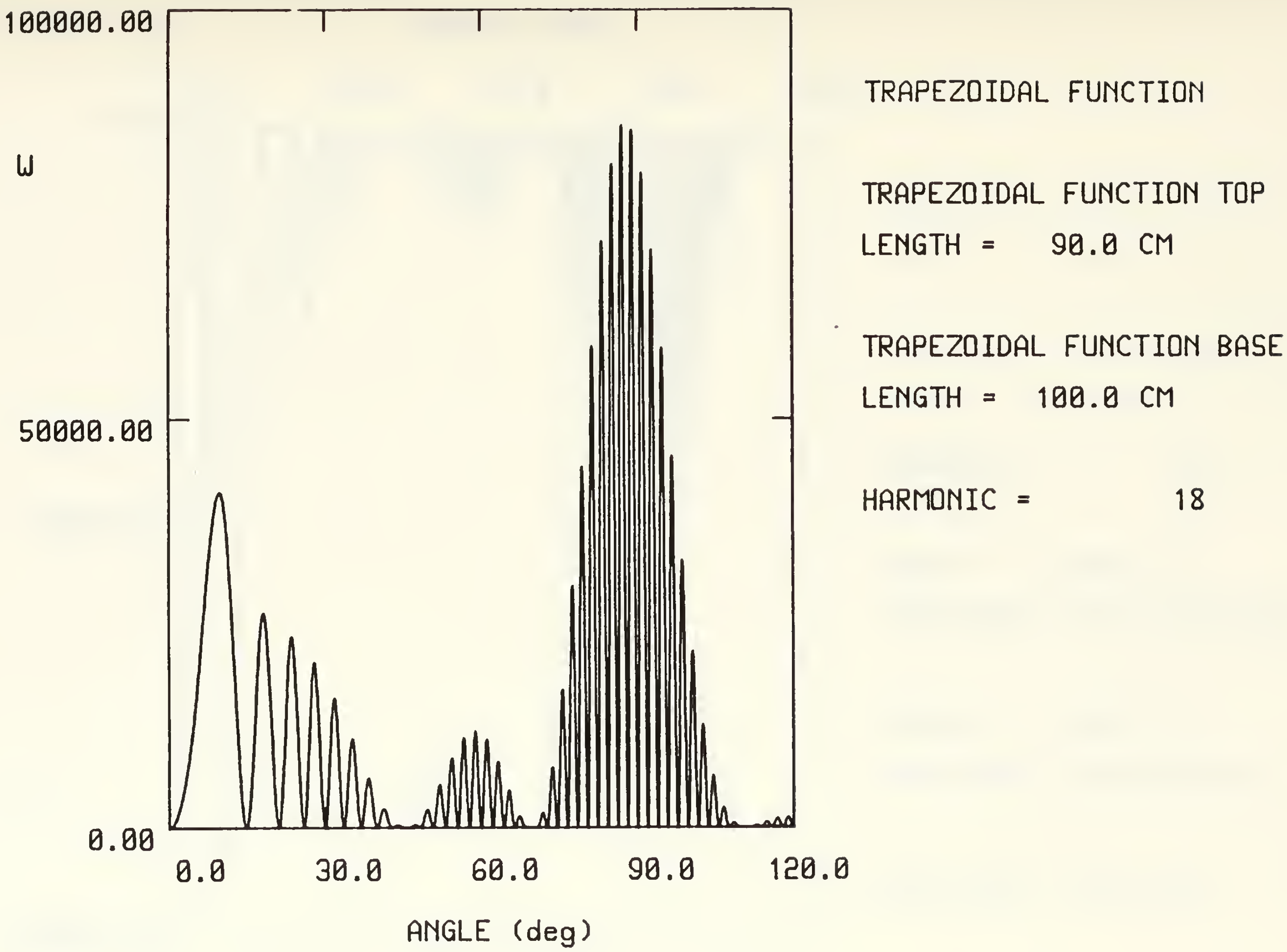


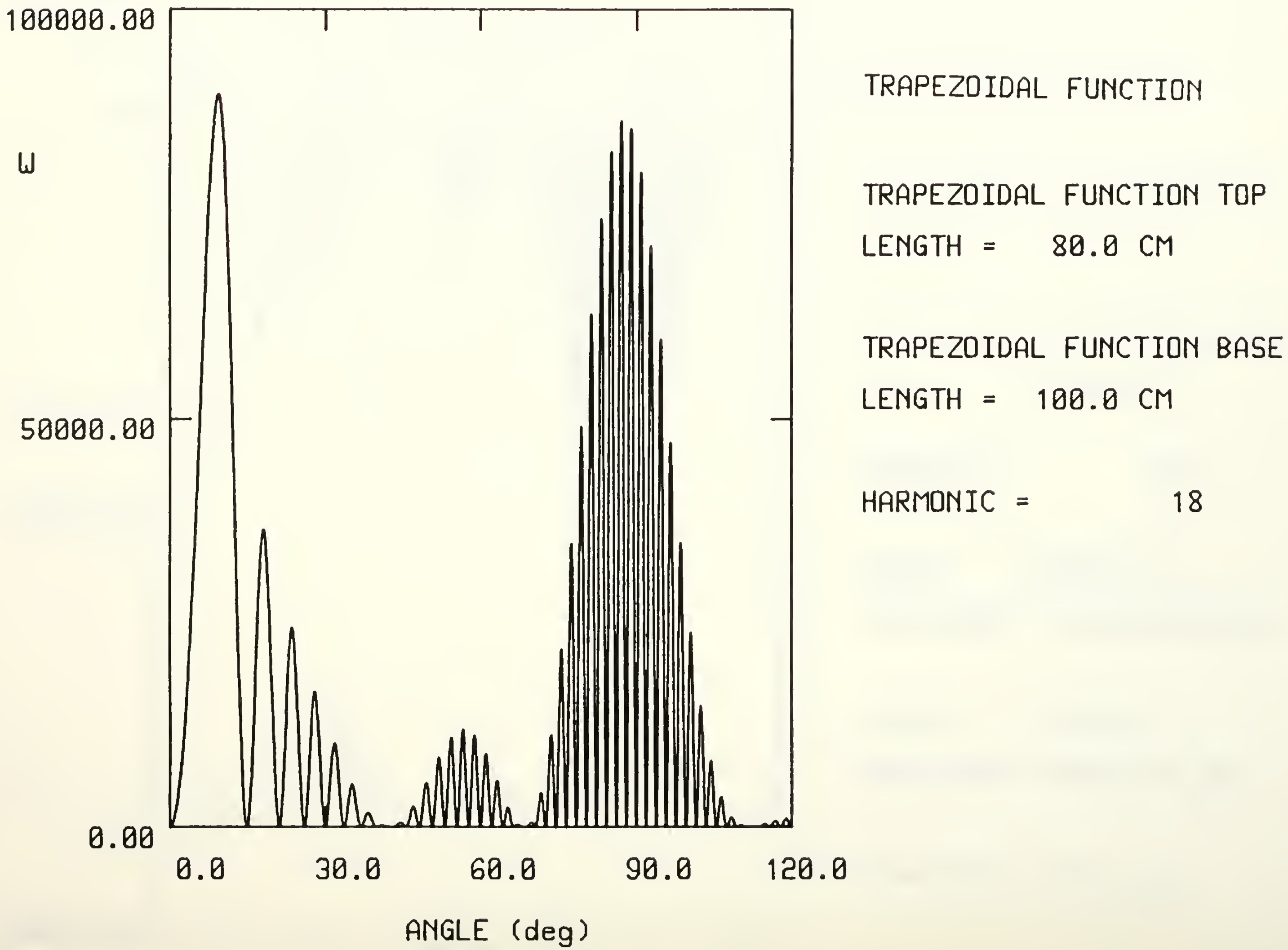
TRAPEZOIDAL FUNCTION

TRAPEZOIDAL FUNCTION TOP
LENGTH = 99.0 CM

TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM

HARMONIC = 18


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 90.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18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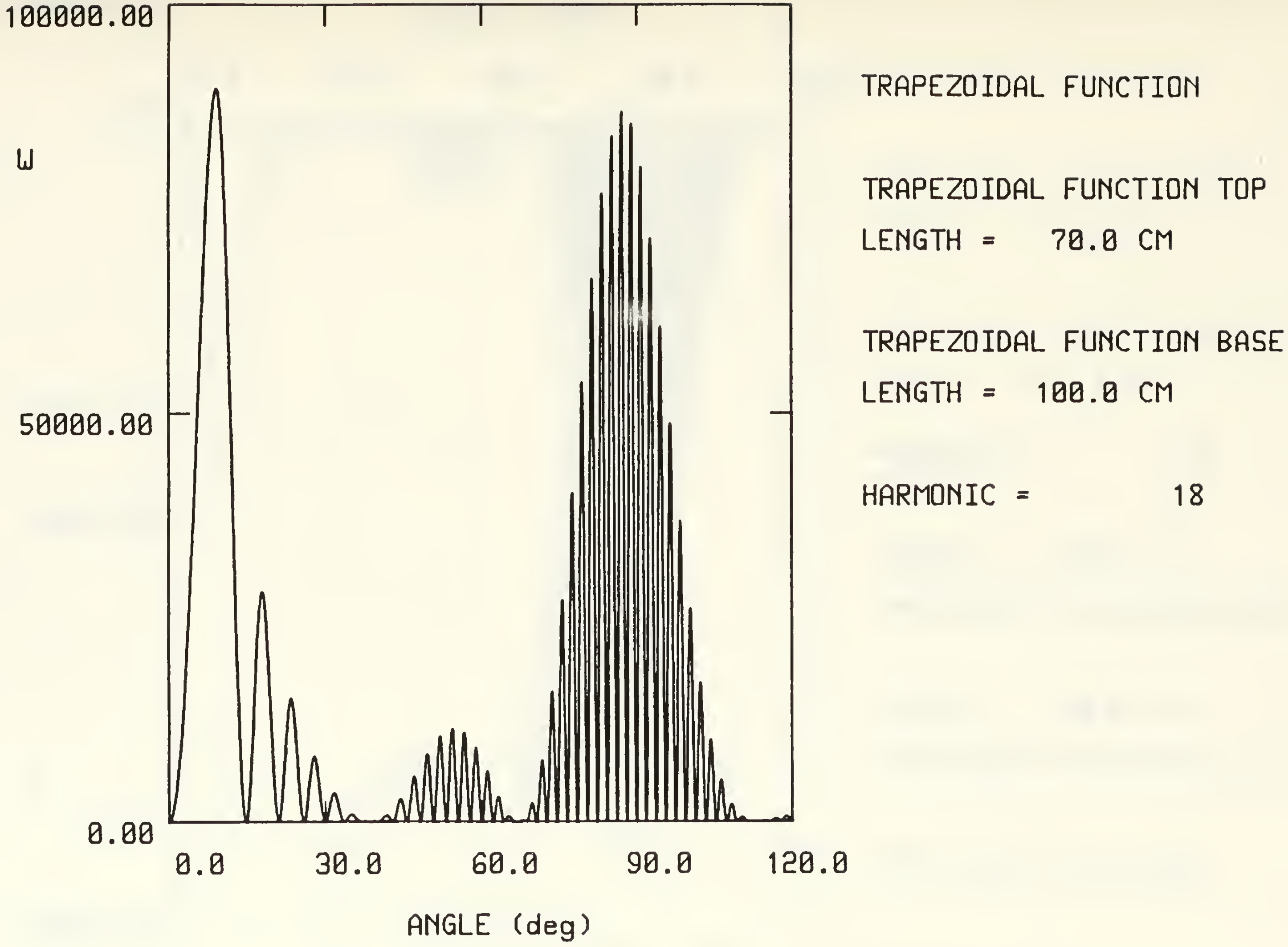
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 80.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18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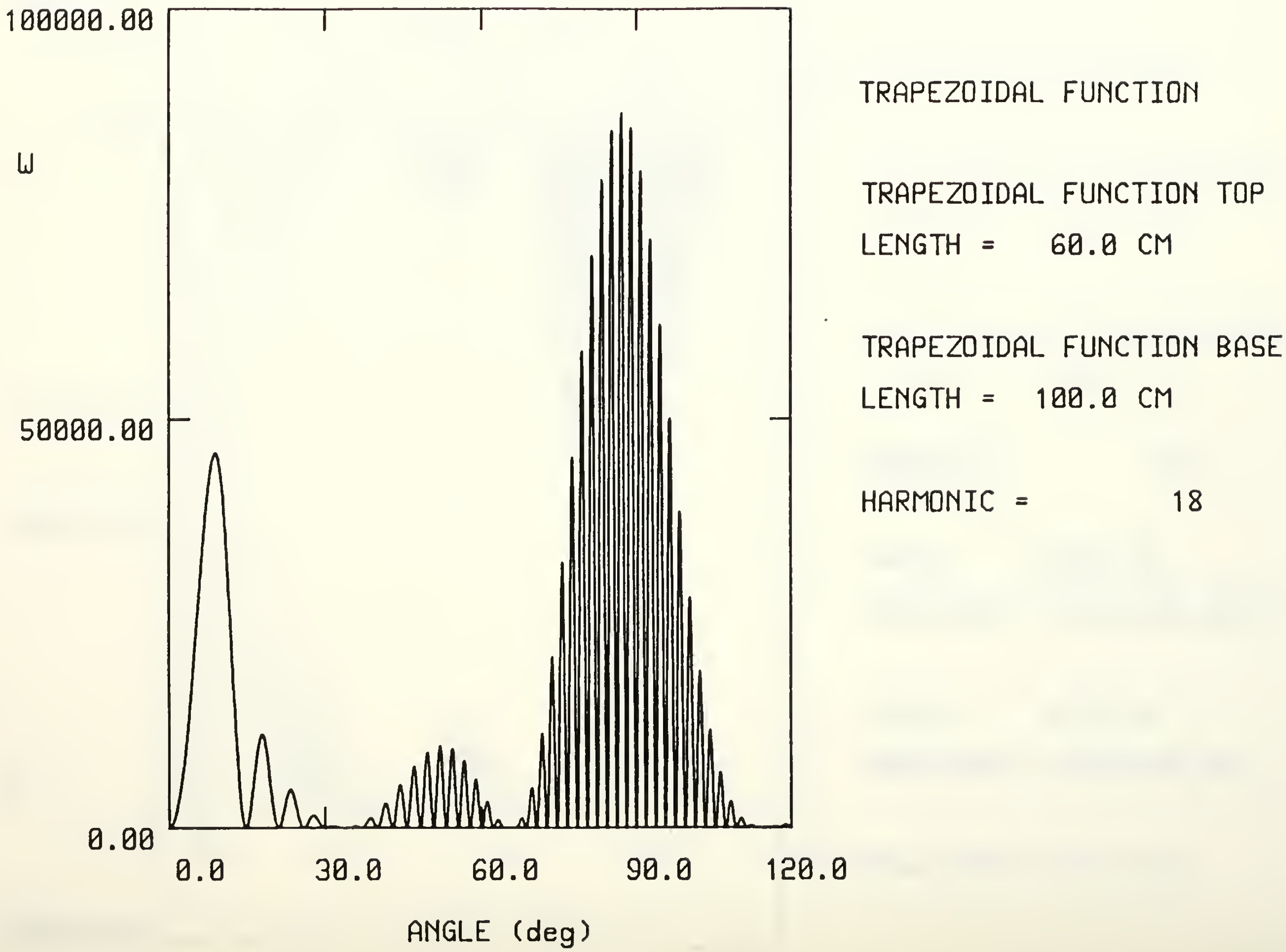
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 70.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18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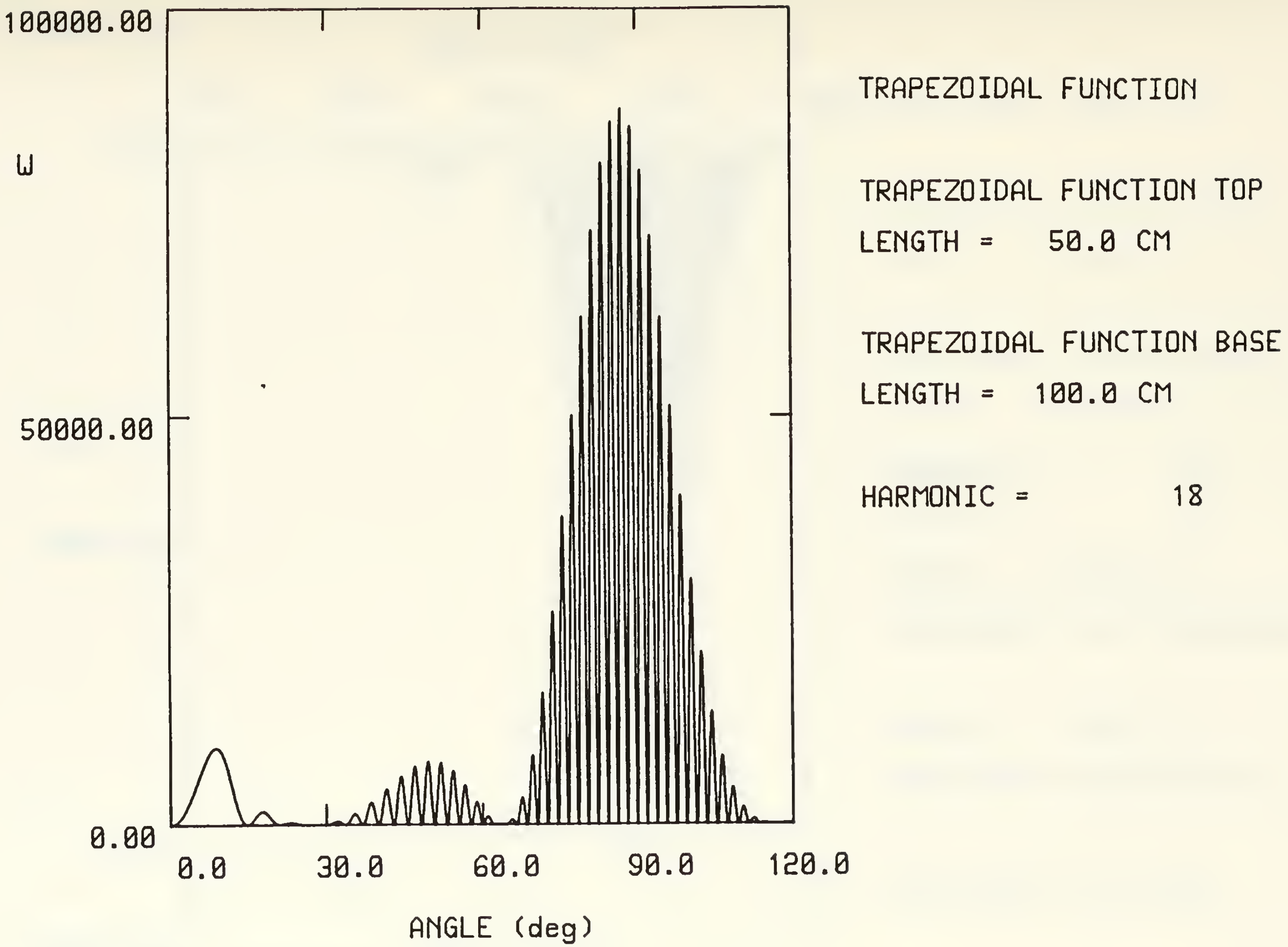
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 60.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18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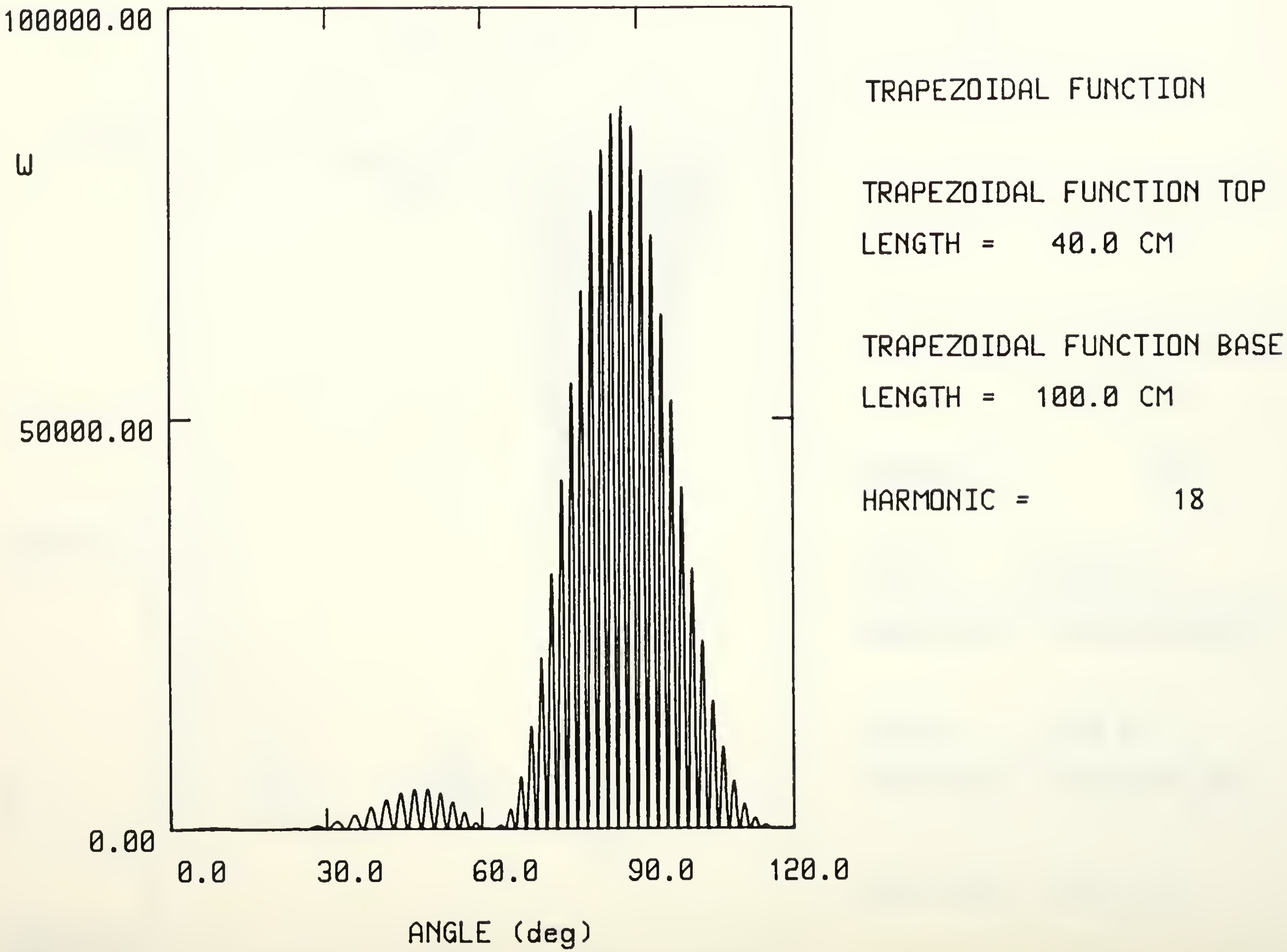
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 50.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18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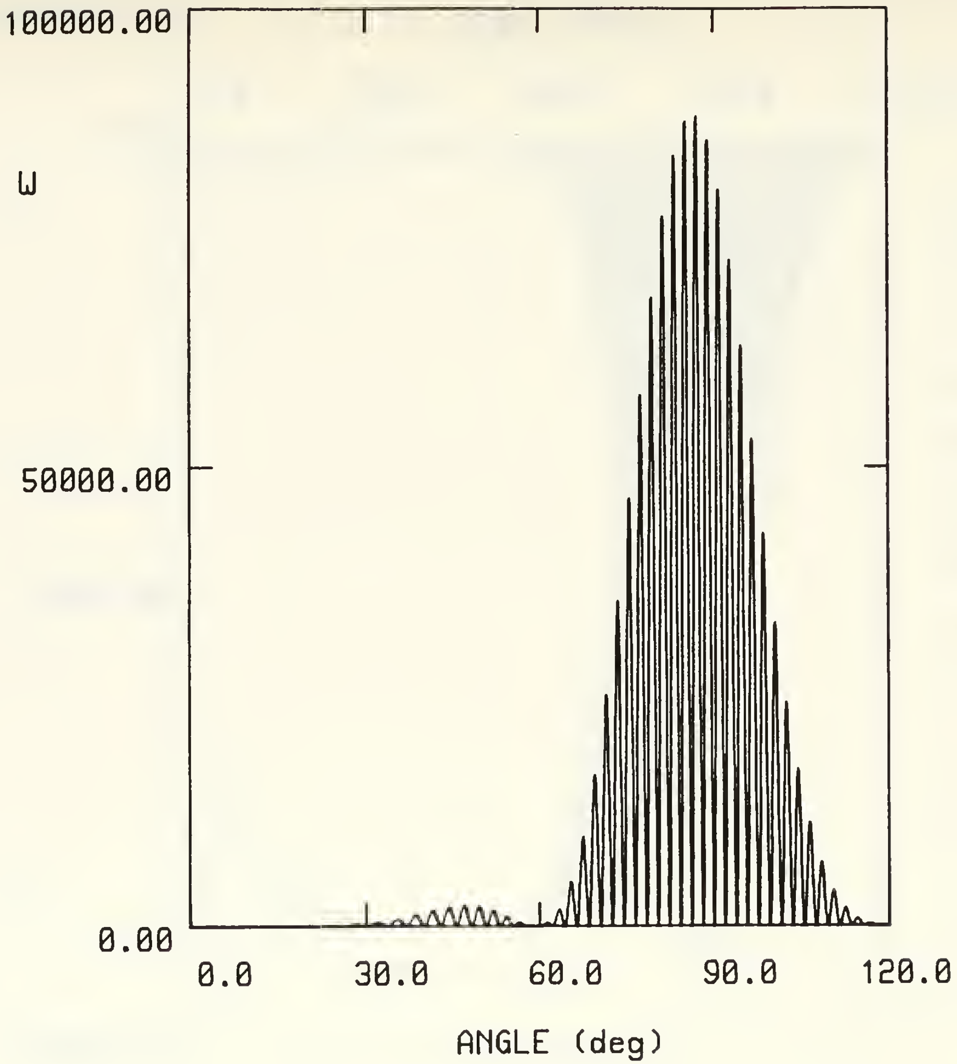
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 40.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18

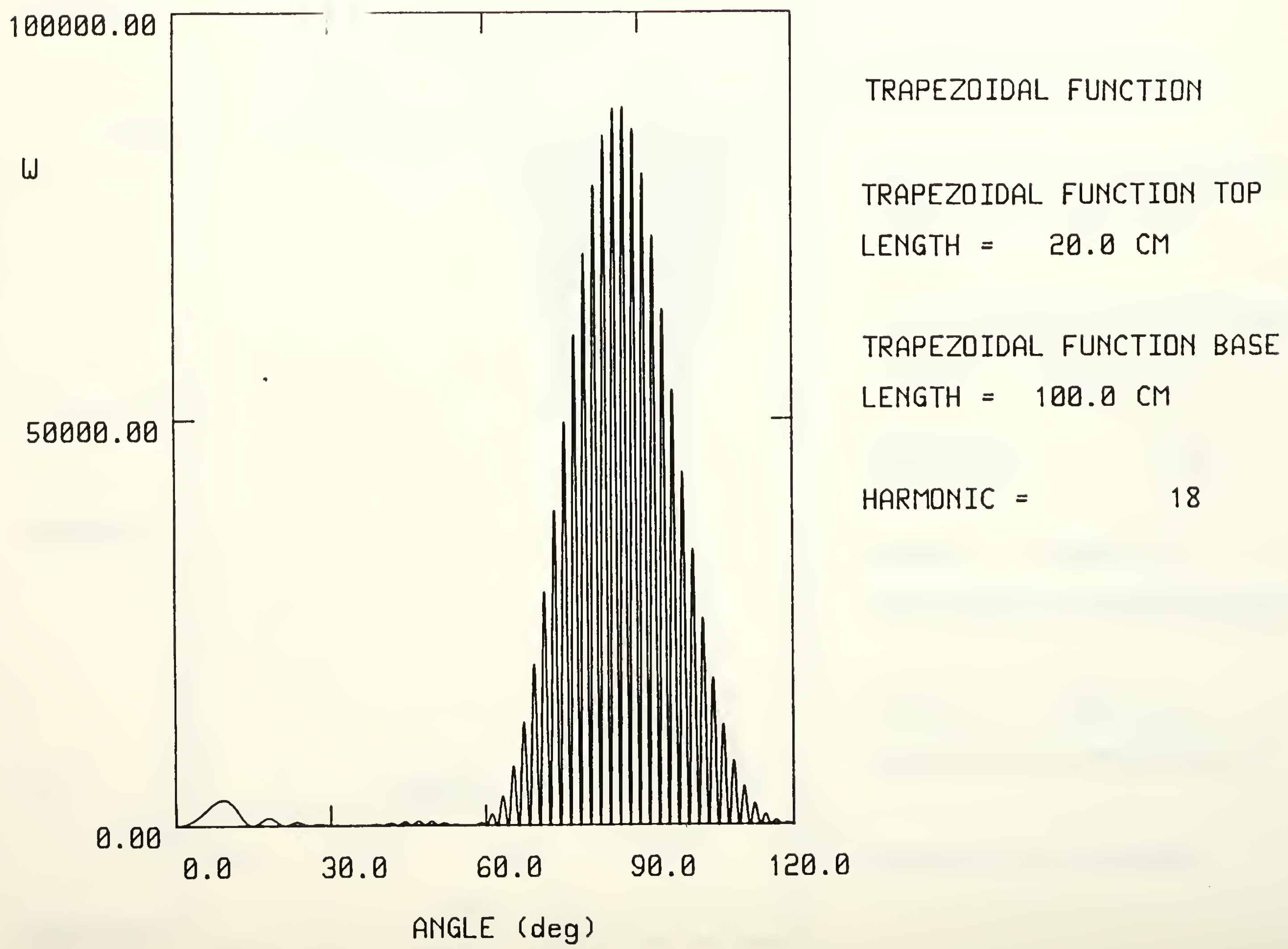


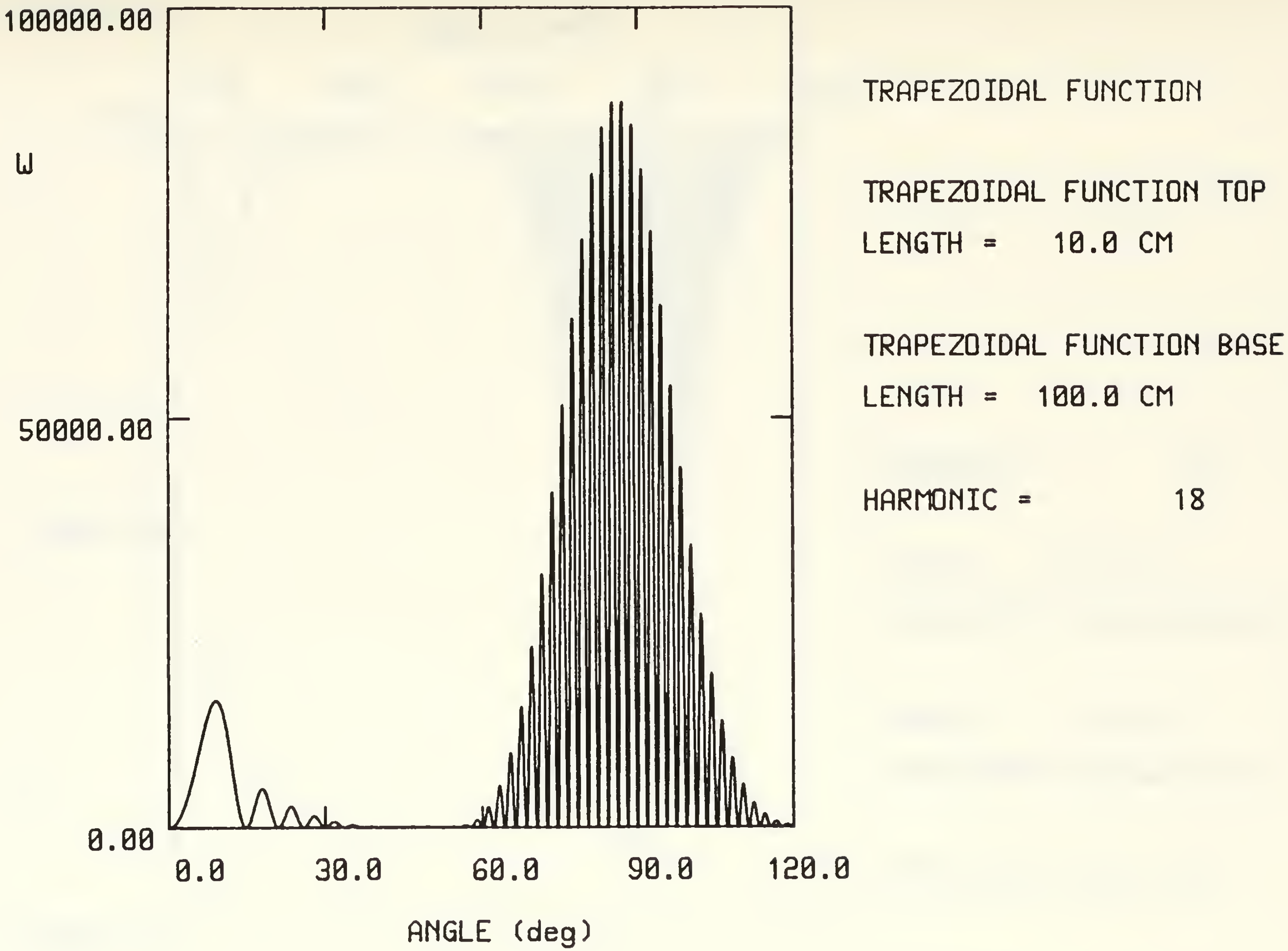
TRAPEZOIDAL FUNCTION

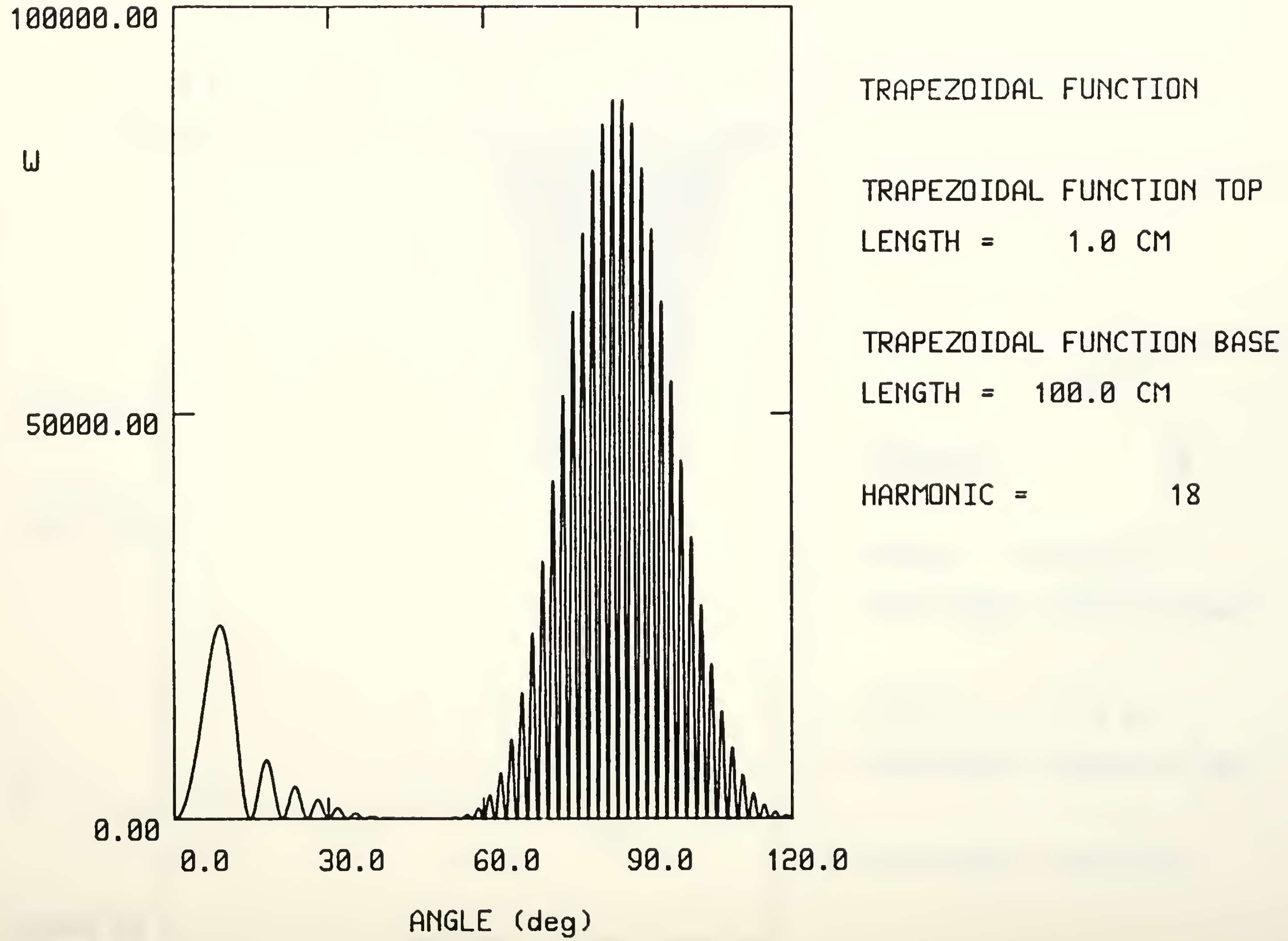
TRAPEZOIDAL FUNCTION TOP
LENGTH = 30.0 CM

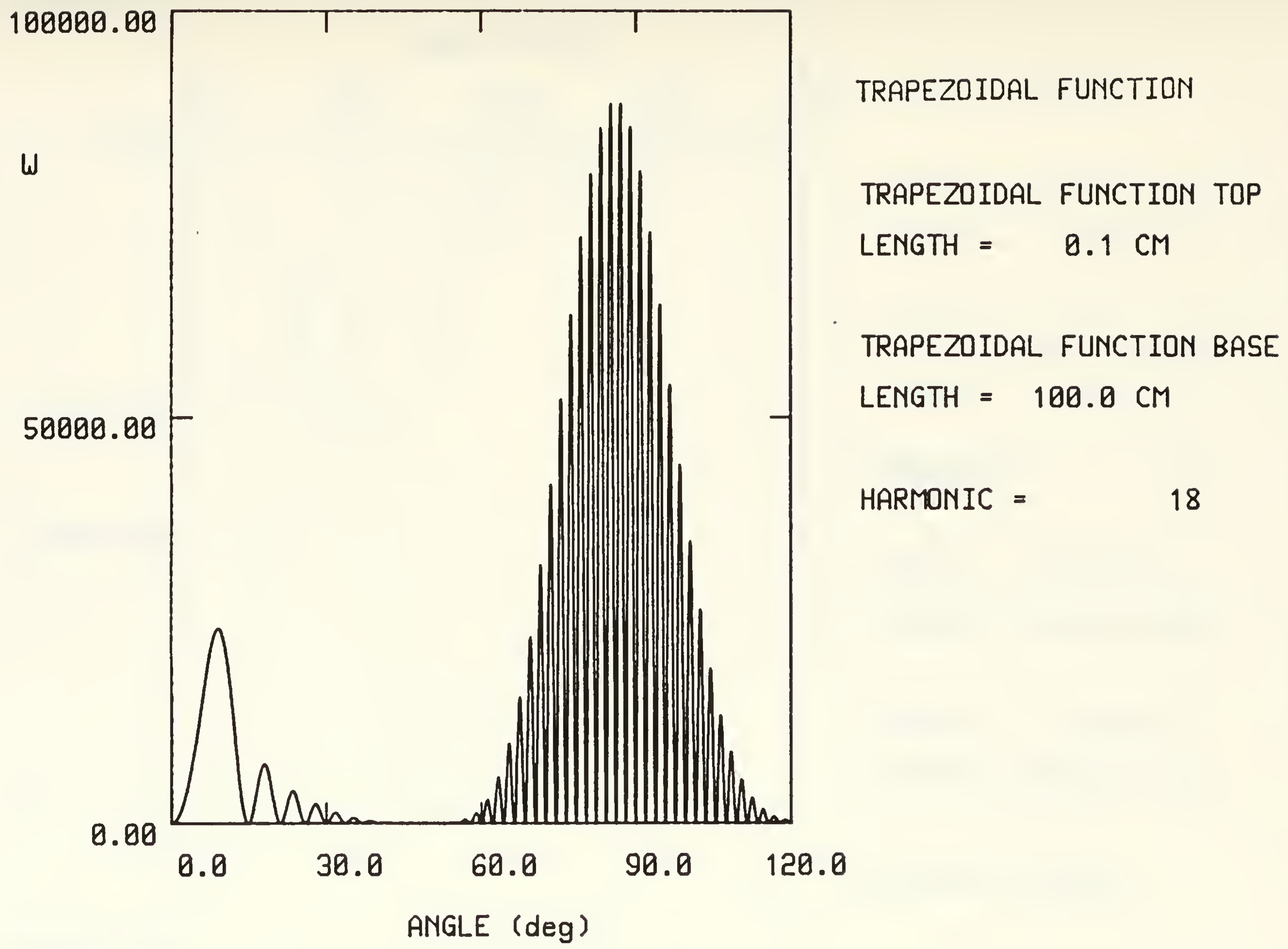
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM

HARMONIC = 18

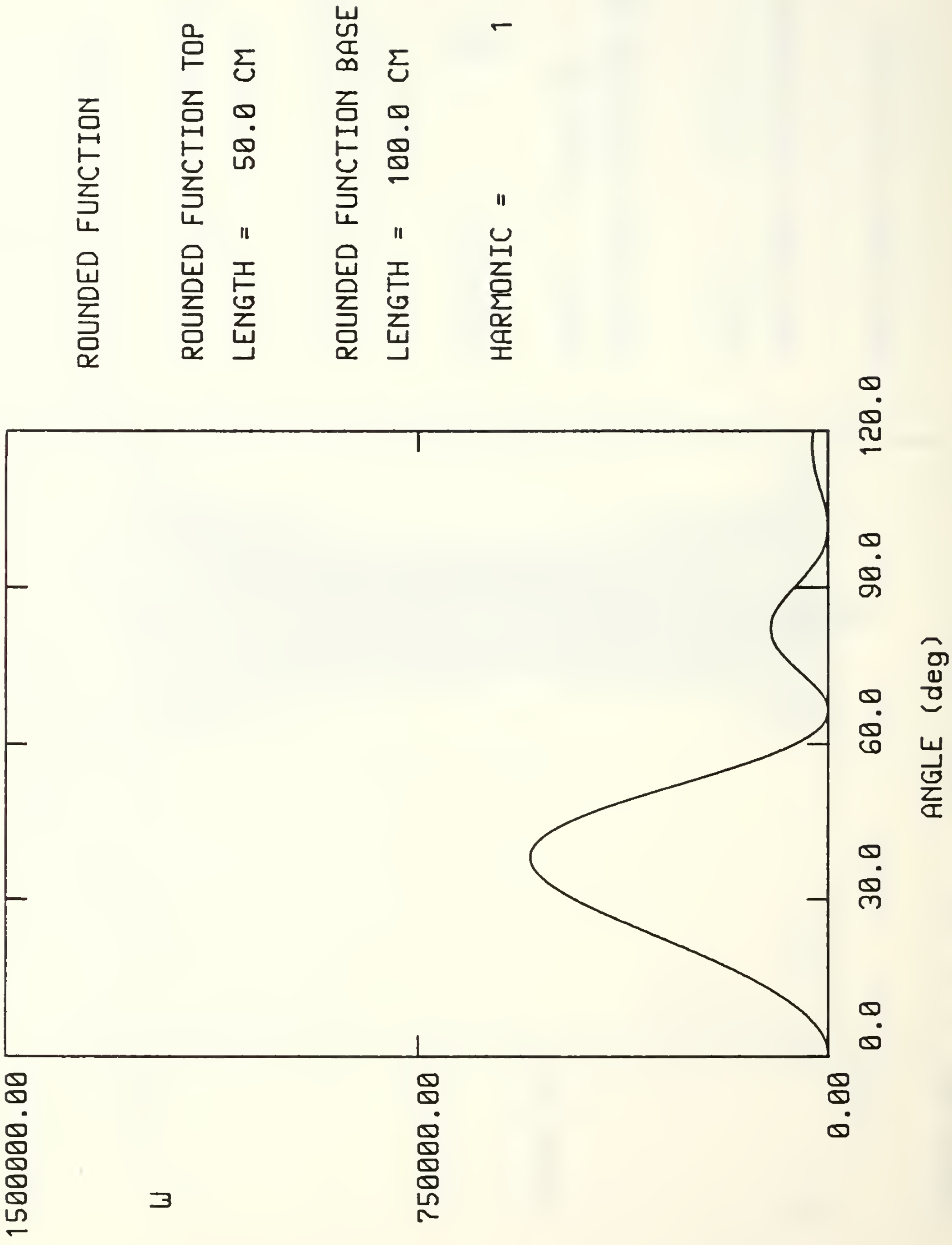

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 20.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18

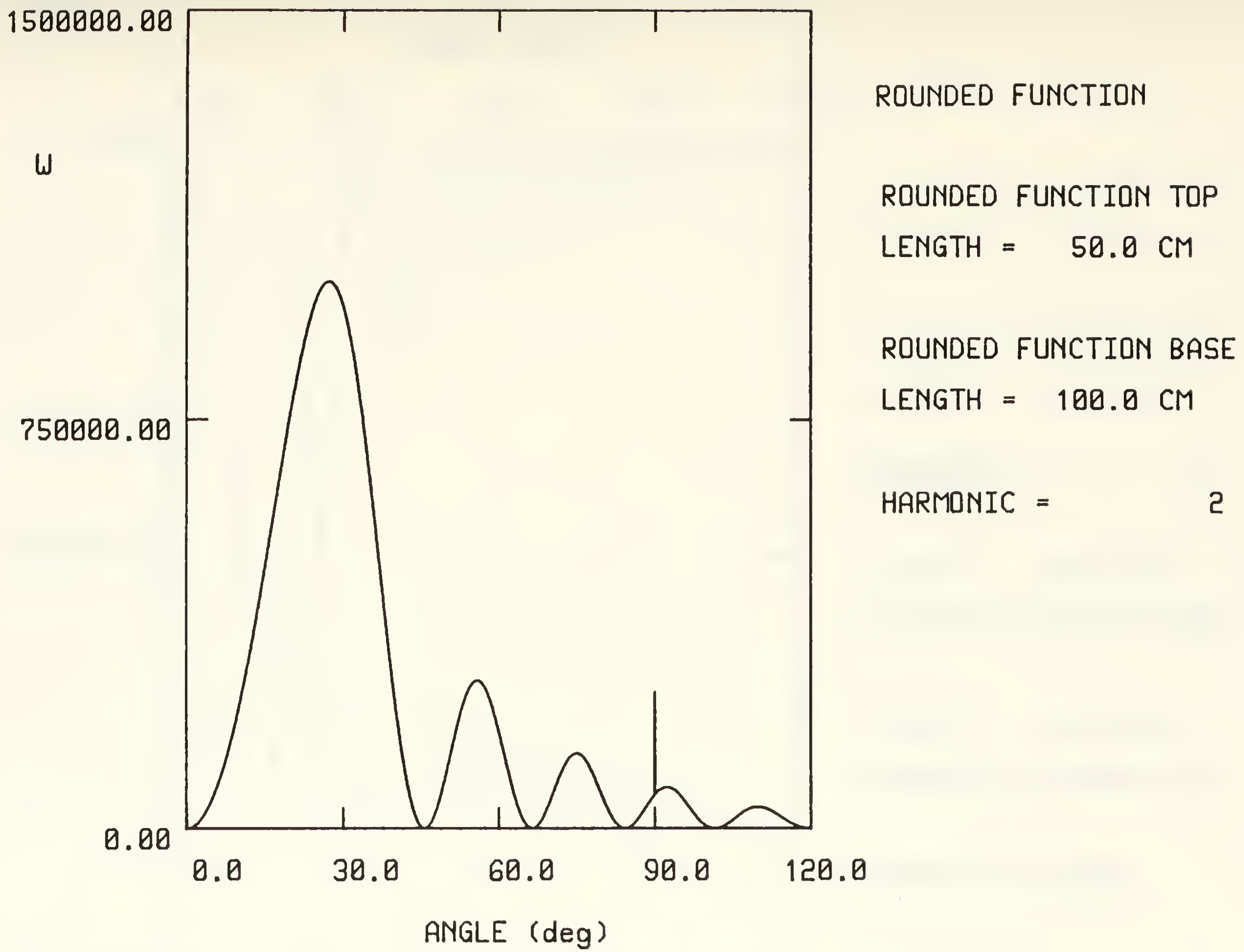

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 10.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18

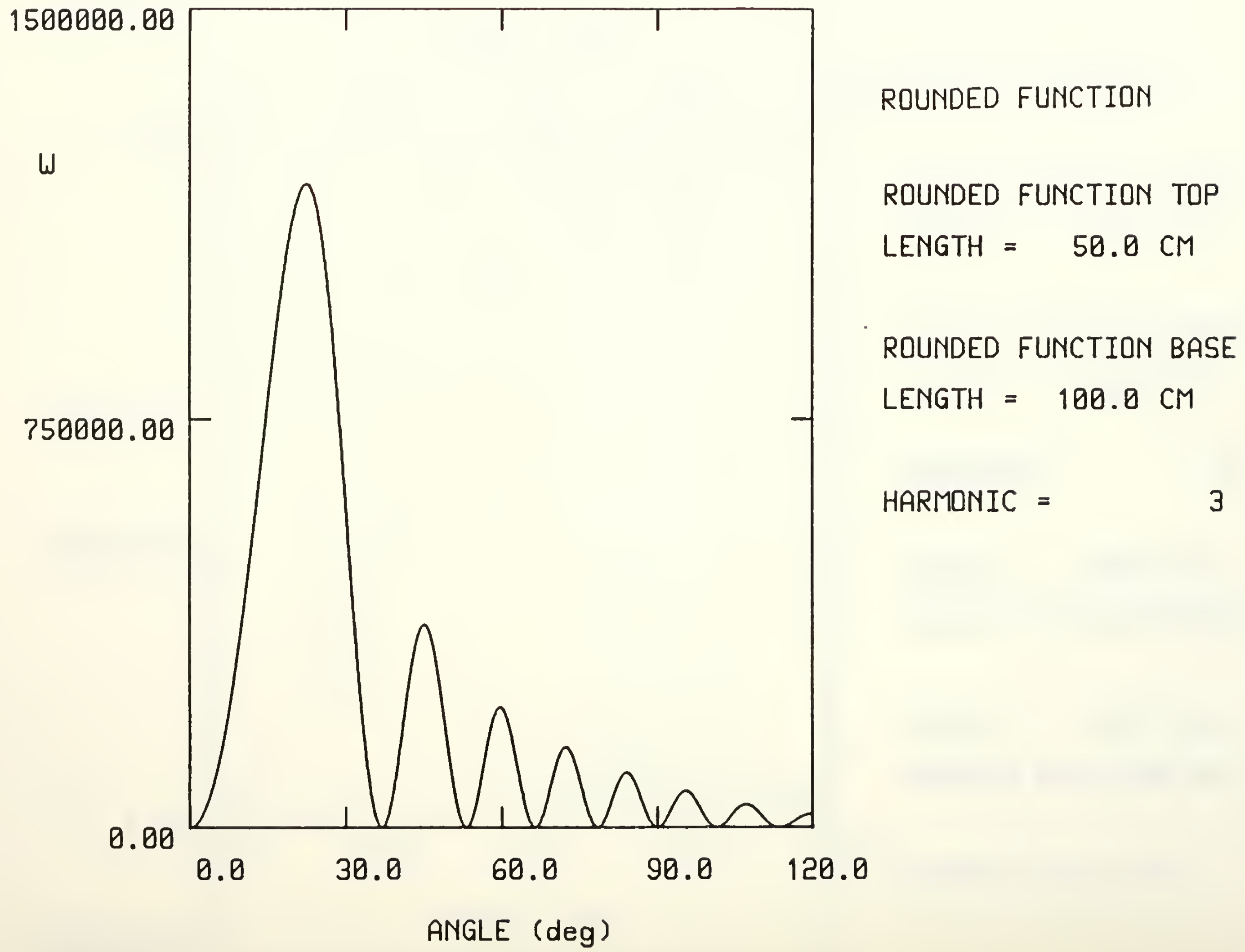

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 1.0 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18

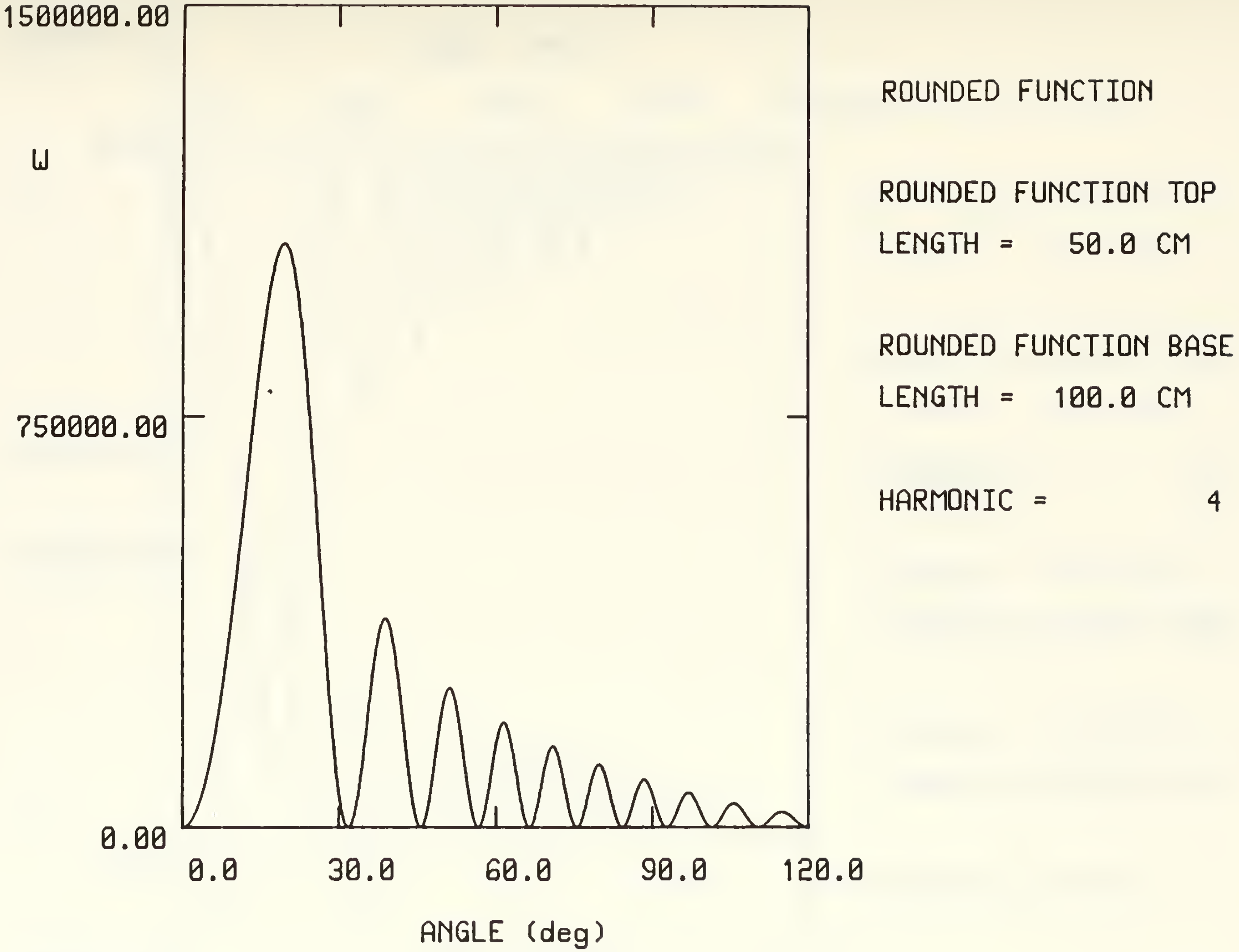

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
TRAPEZOIDAL FUNCTION
TRAPEZOIDAL FUNCTION TOP
LENGTH = 0.1 CM
TRAPEZOIDAL FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18

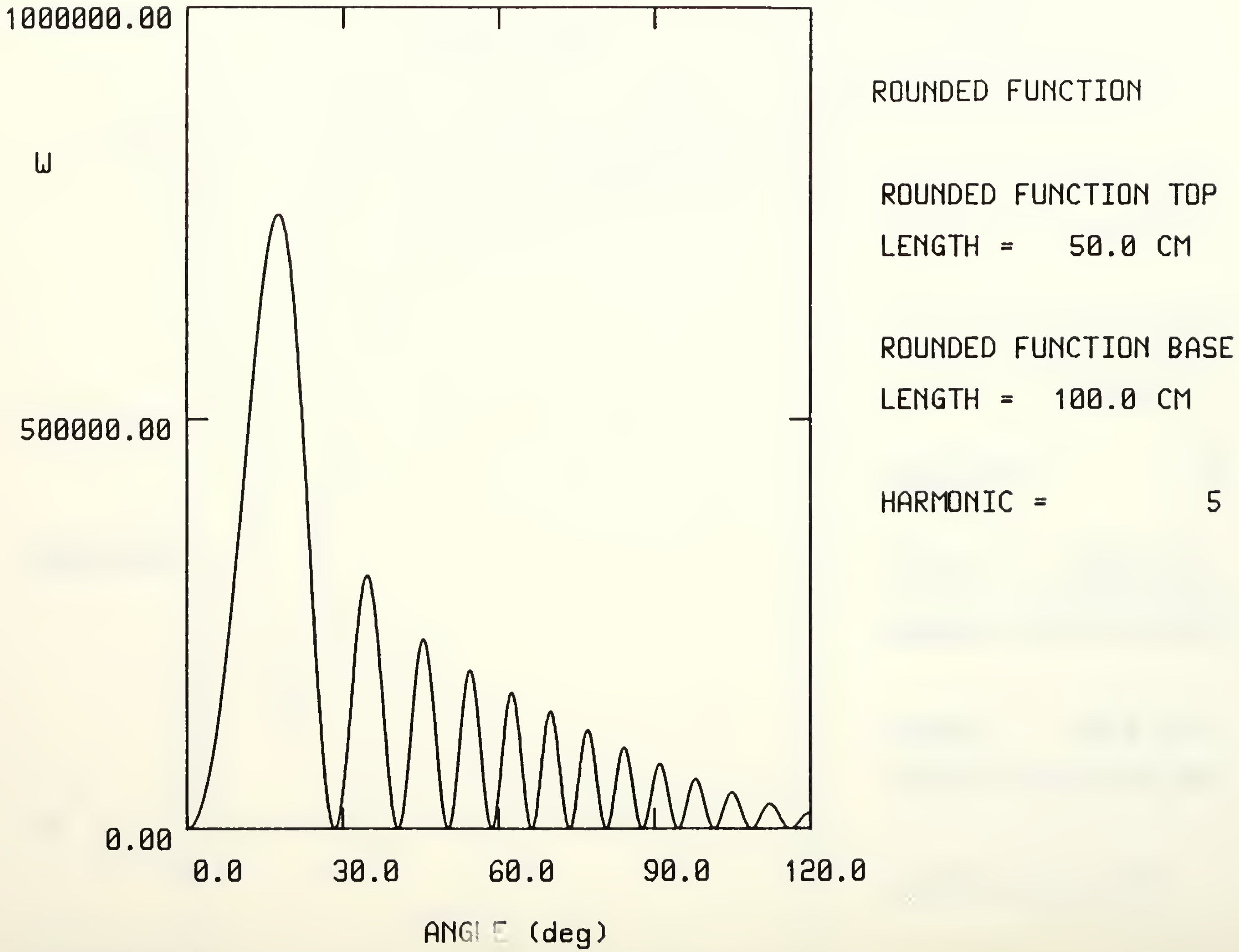
APPENDIX D: ROUNDED FUNCTION

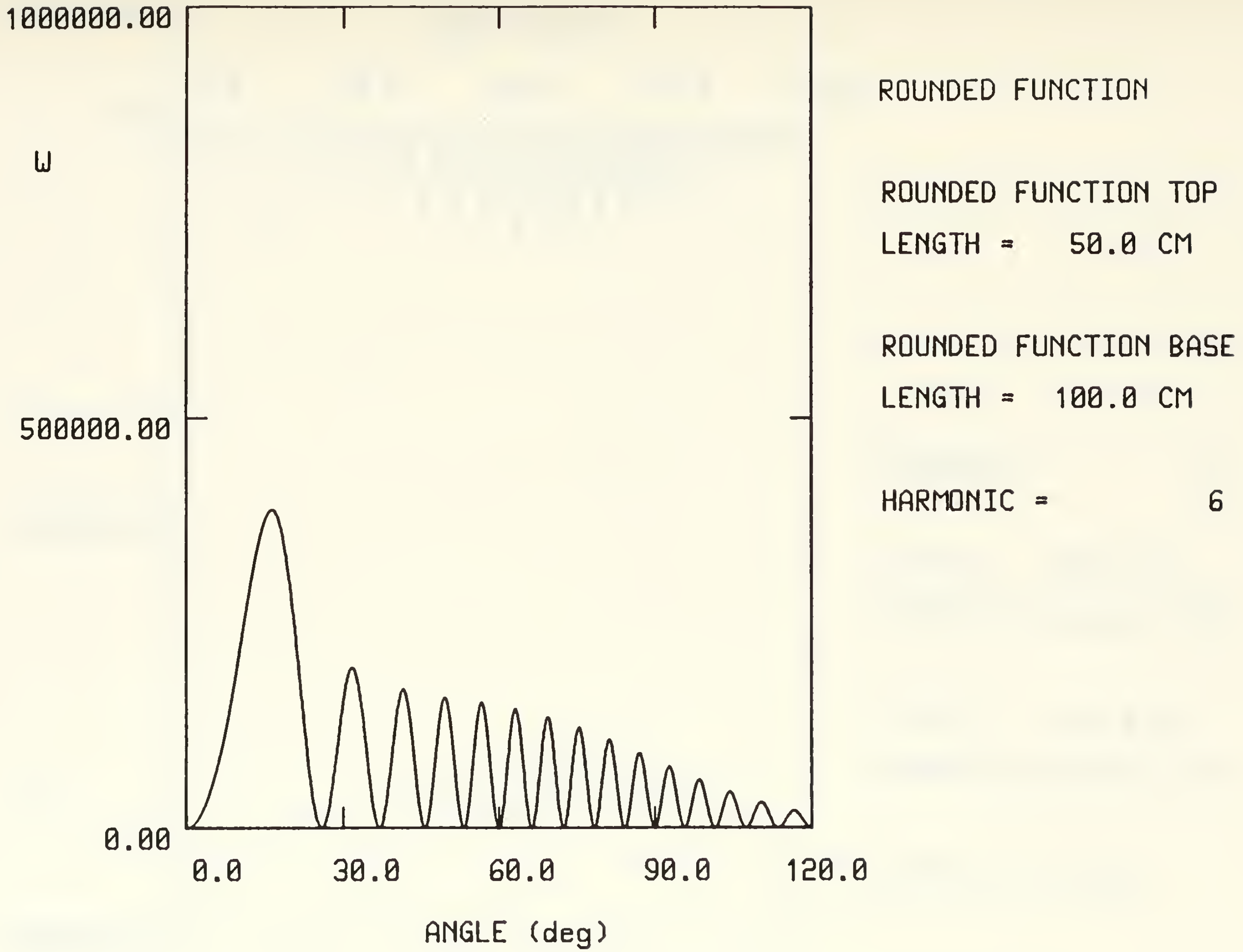


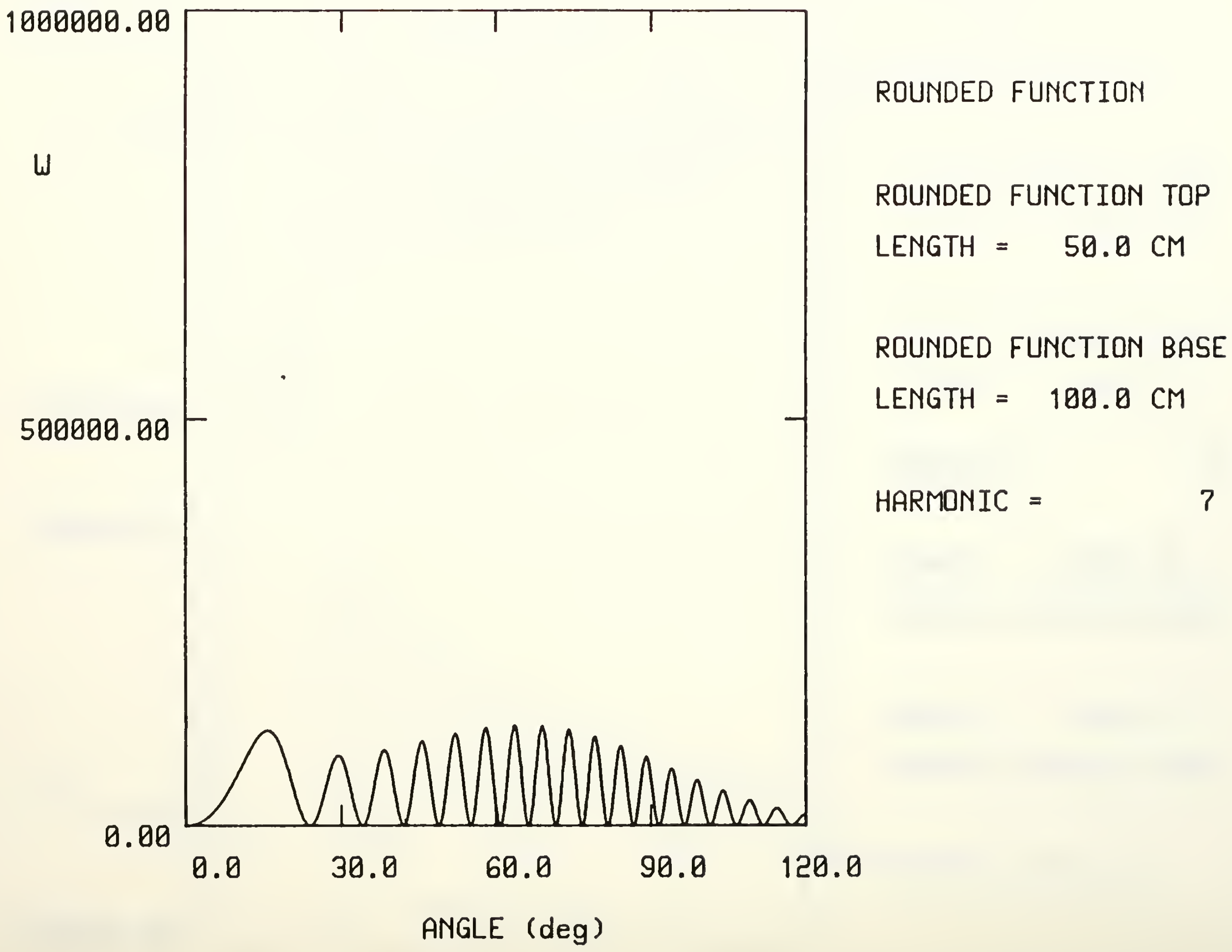

1500000.00
W
750000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 2

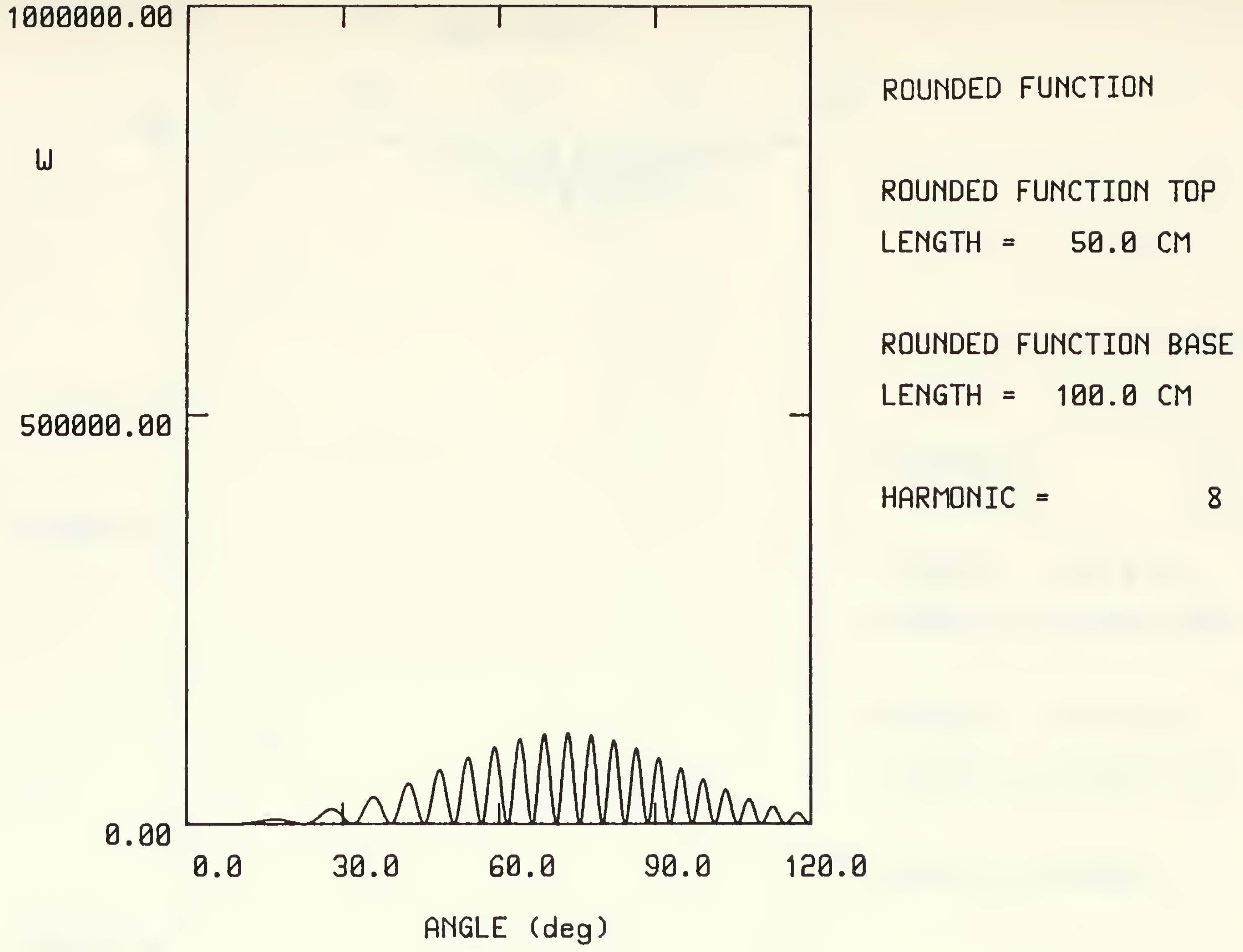

1500000.00
750000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 3

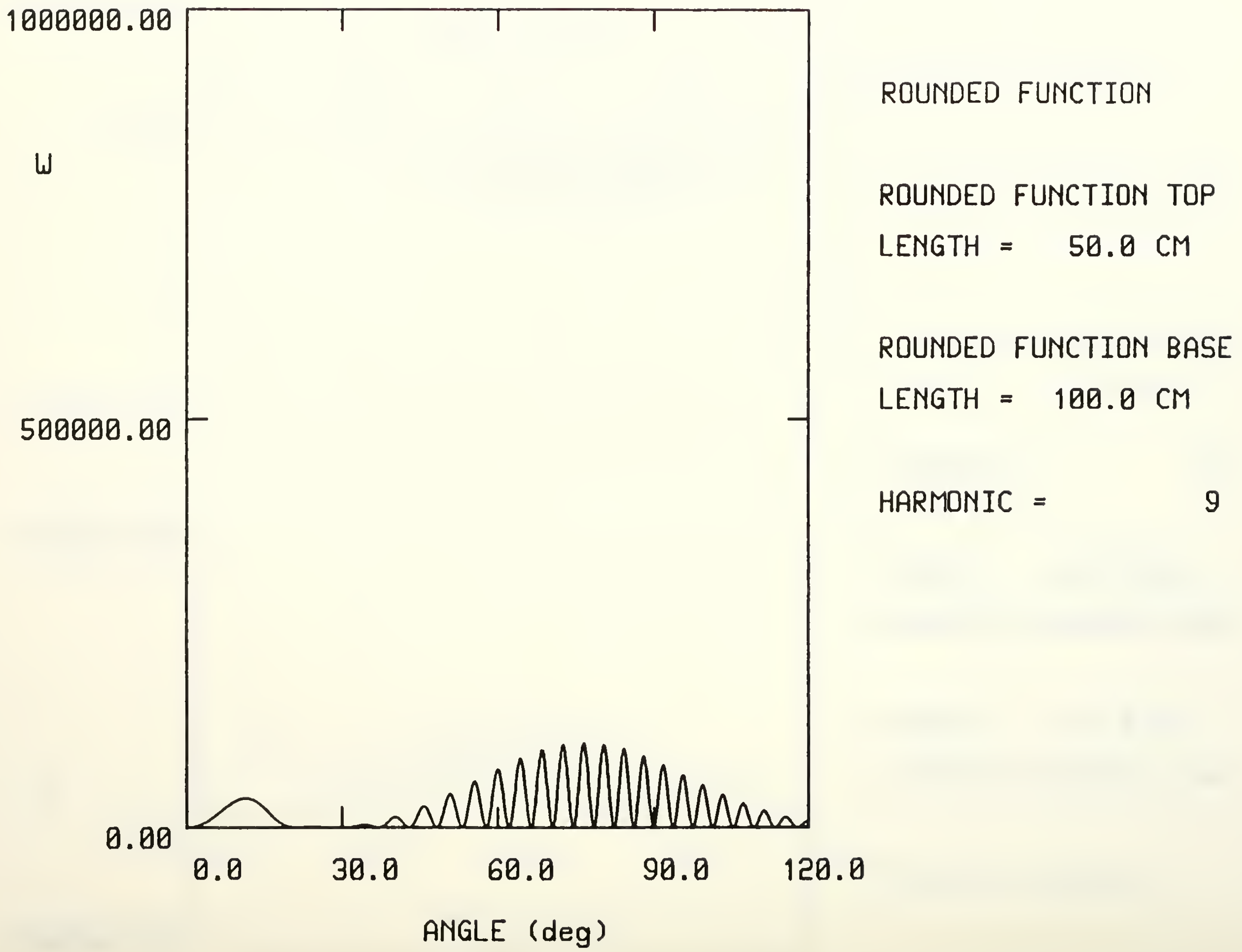

1500000.00
W
750000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 4

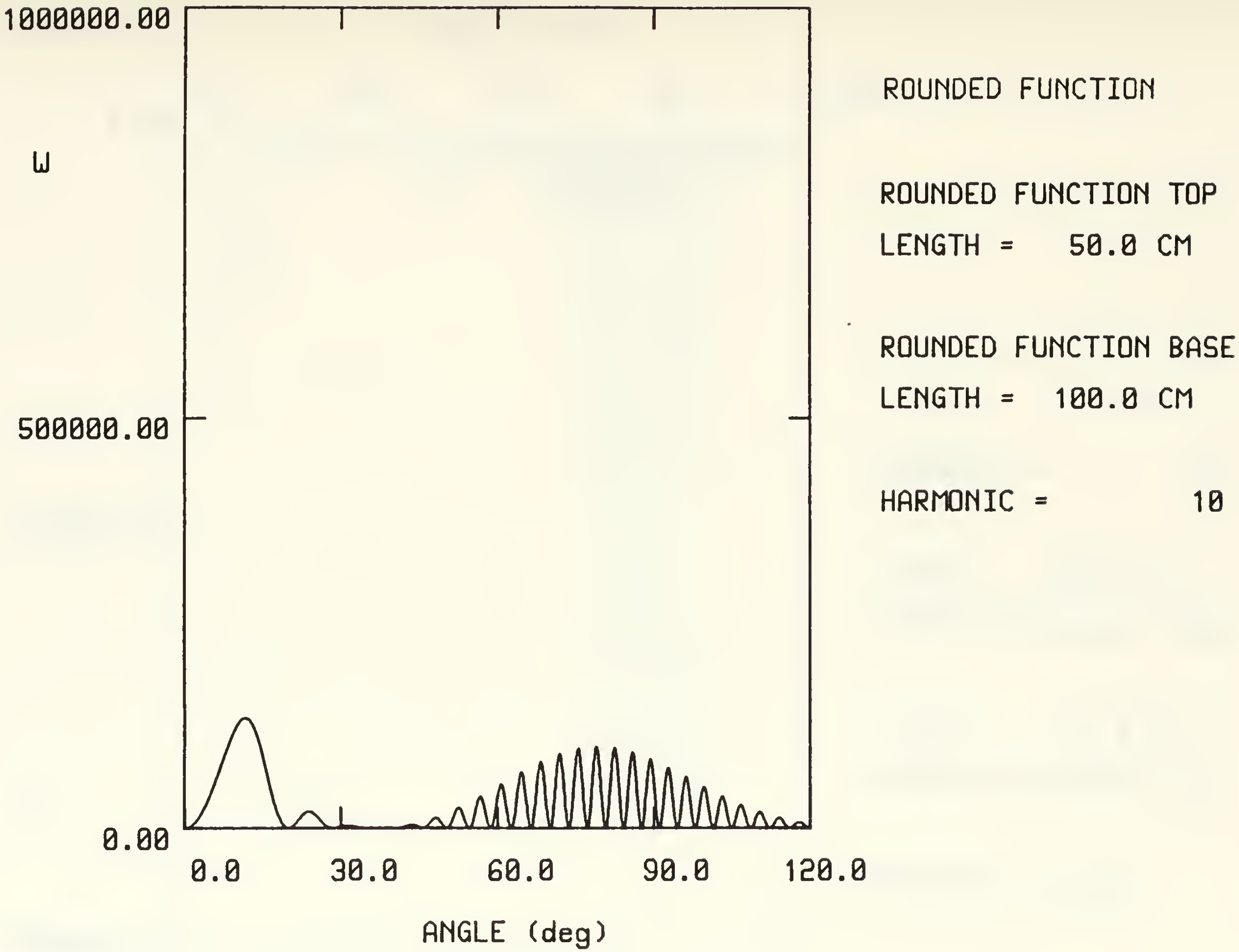

1000000.00
500000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 5

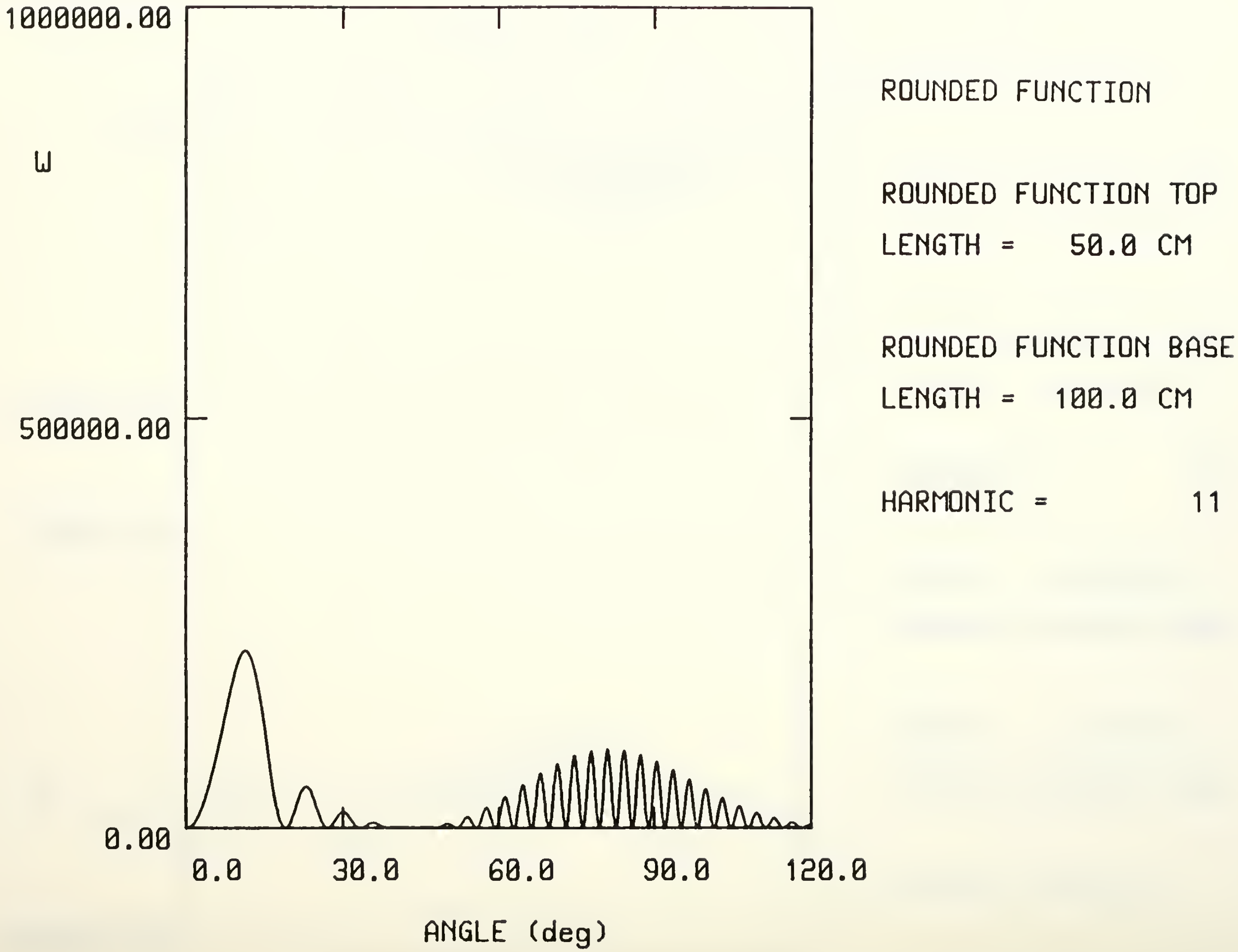

1000000.00
500000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 6

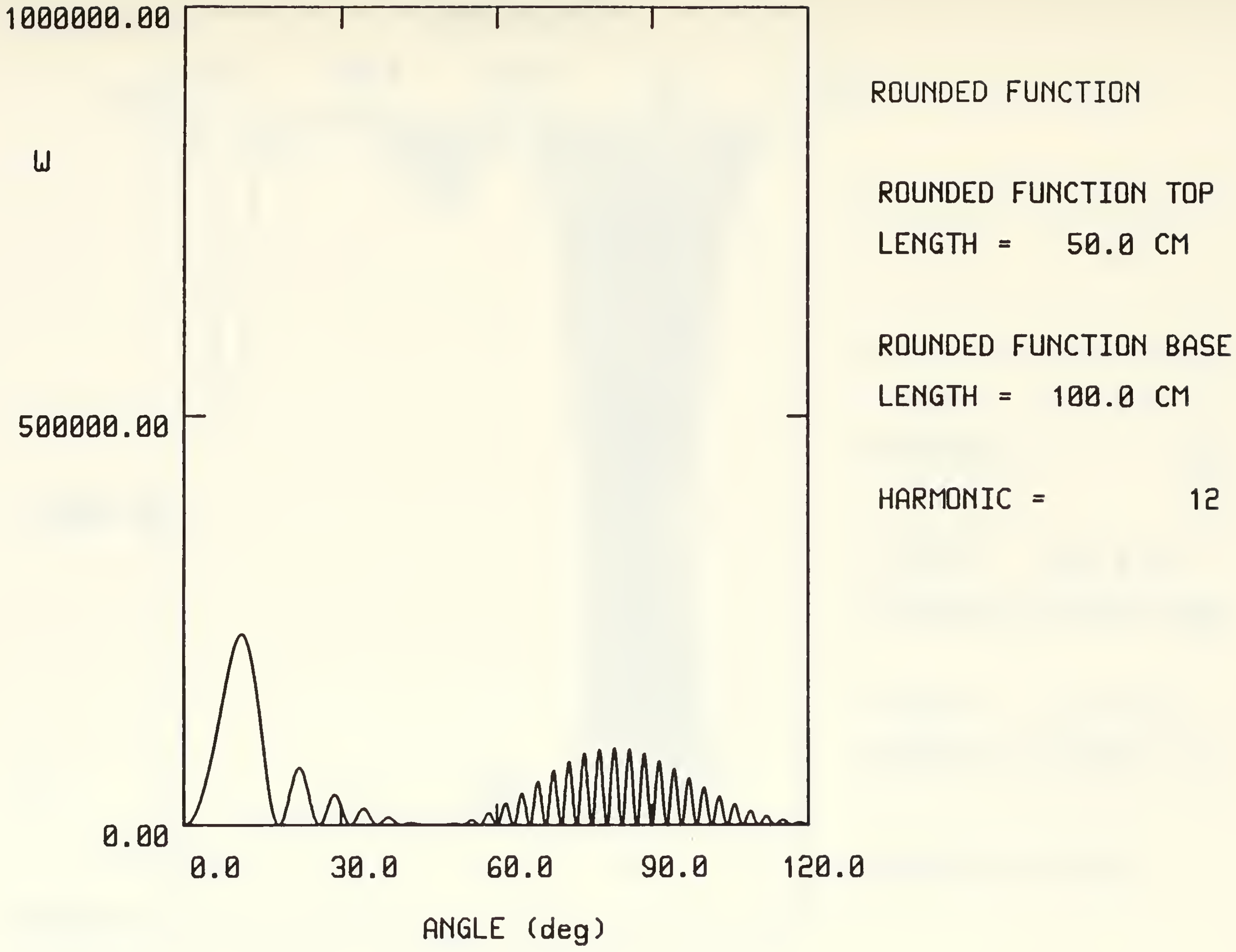

1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 7

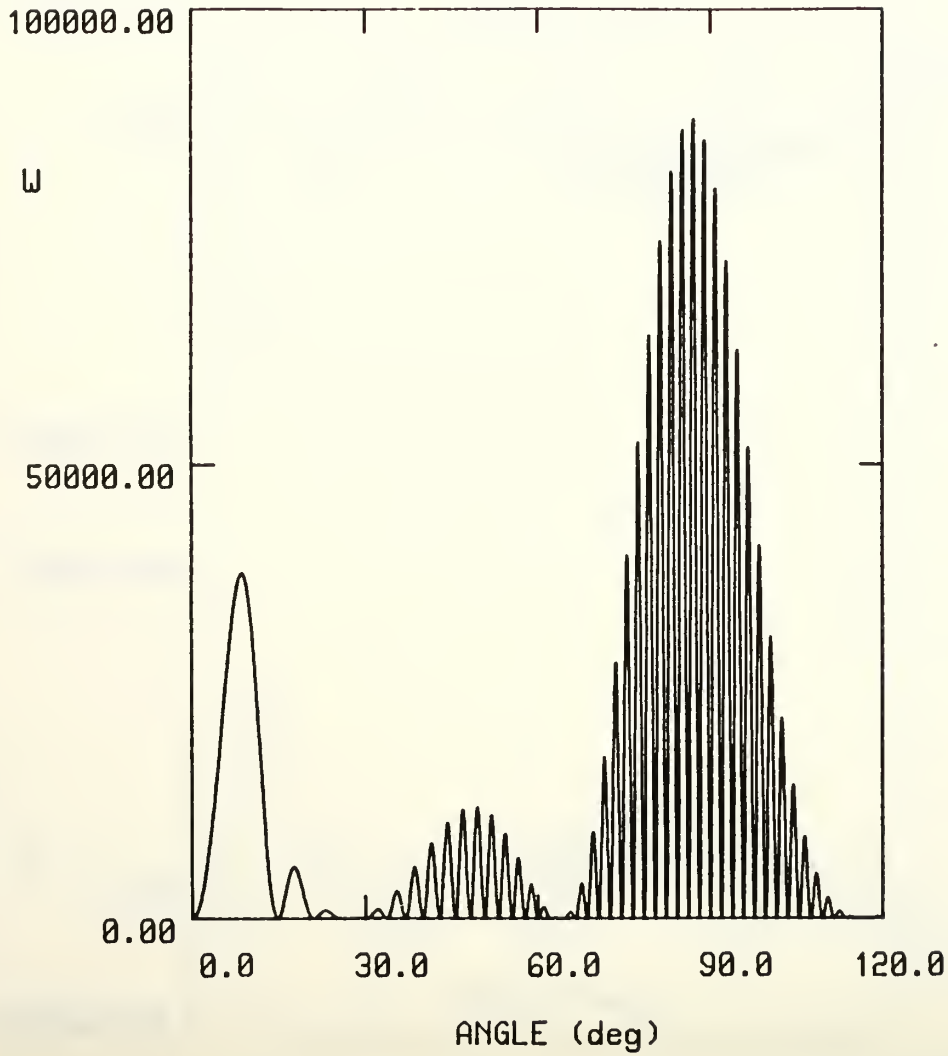

1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 8


1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 9


1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 10


1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 11


1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 12

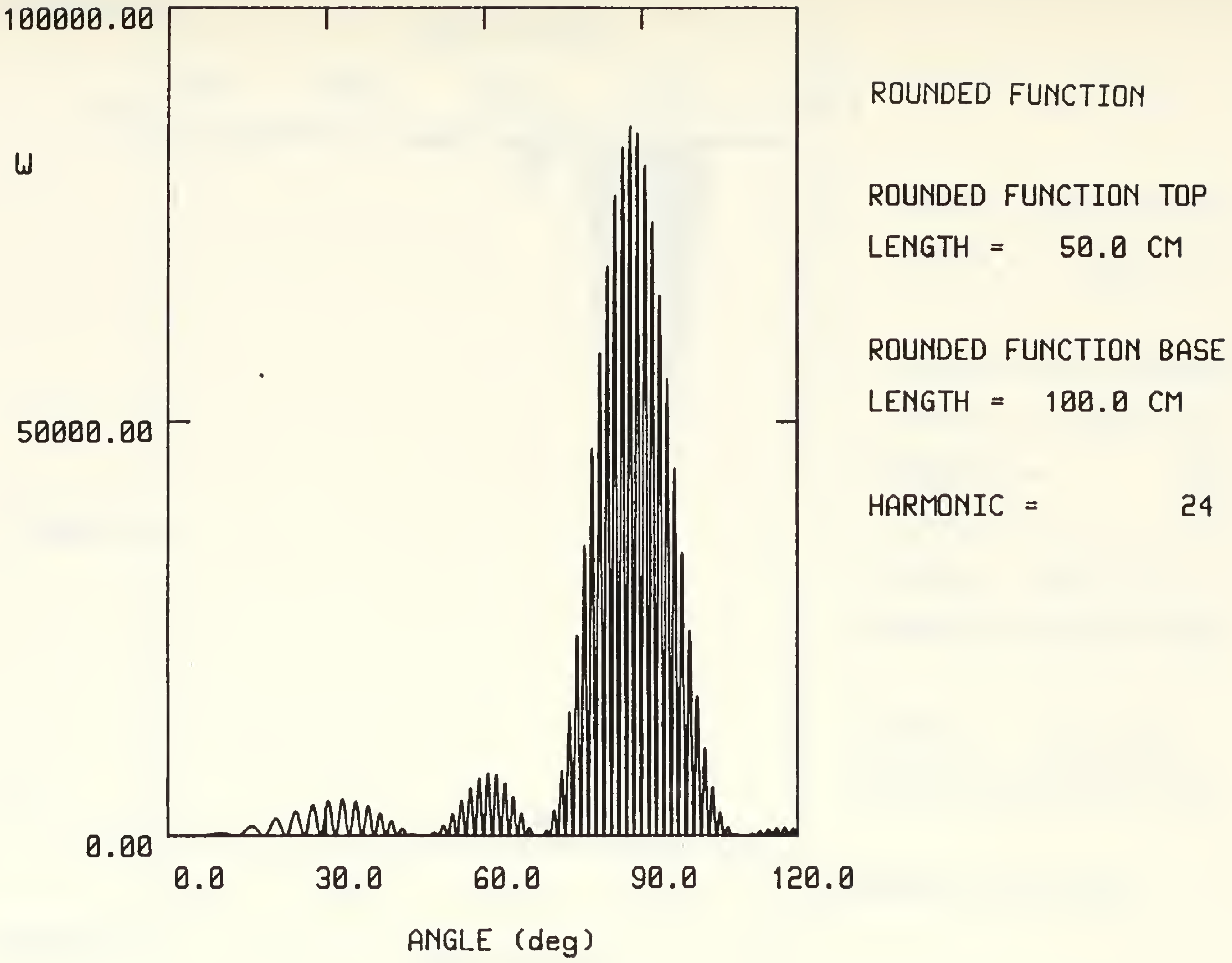


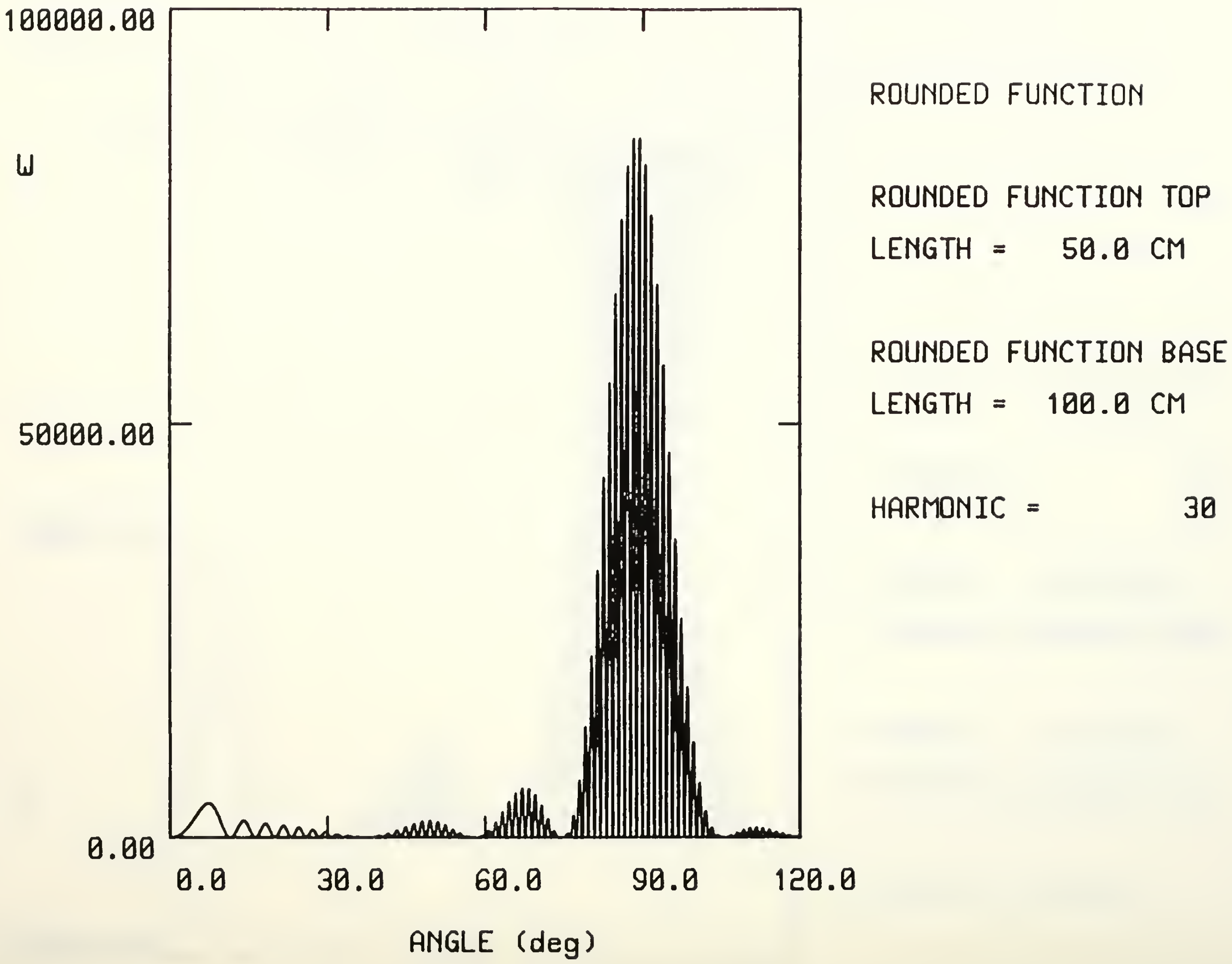
ROUNDED FUNCTION

ROUNDED FUNCTION TOP
LENGTH = 50.0 CM

ROUNDED FUNCTION BASE
LENGTH = 100.0 CM

HARMONIC = 18


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 24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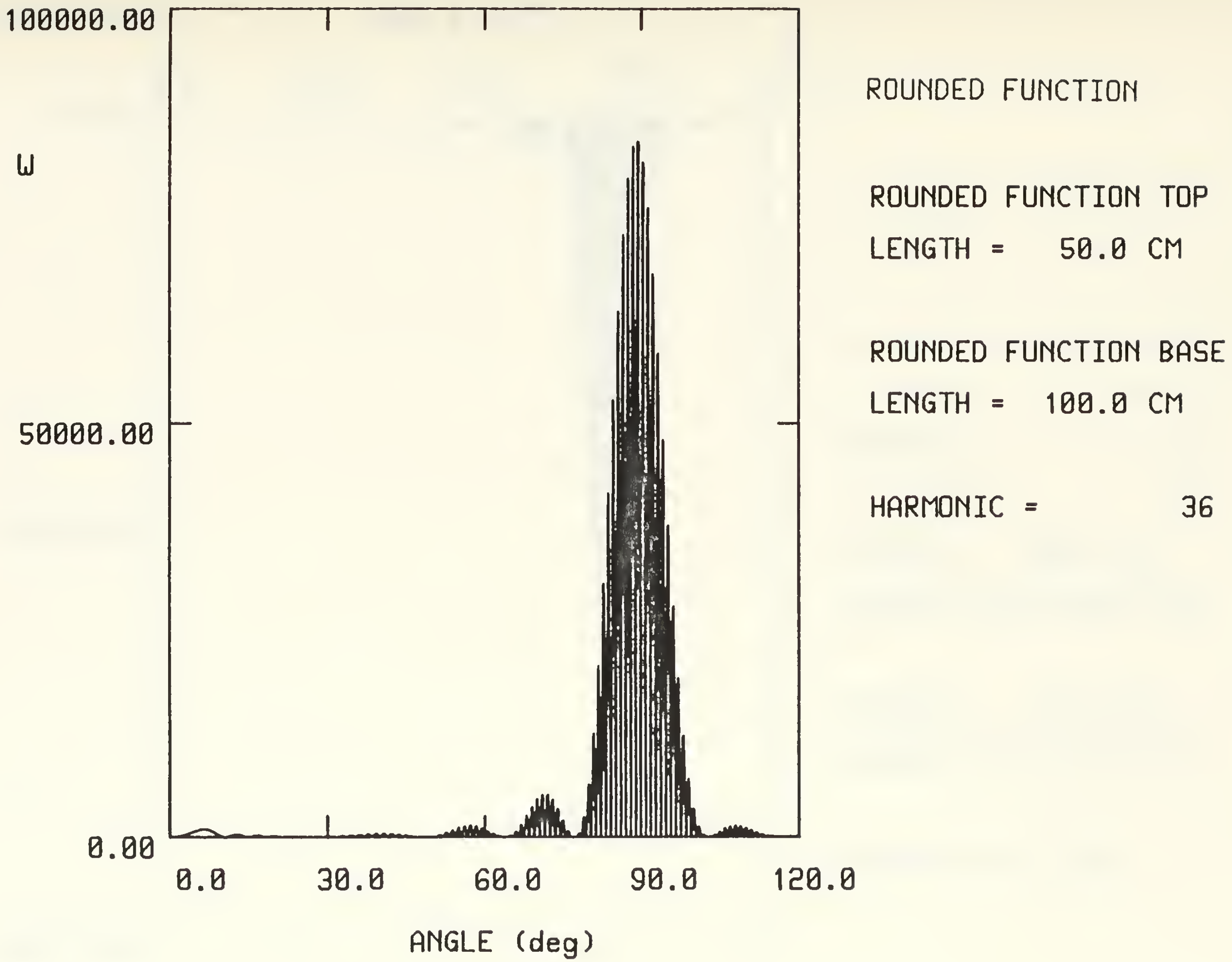
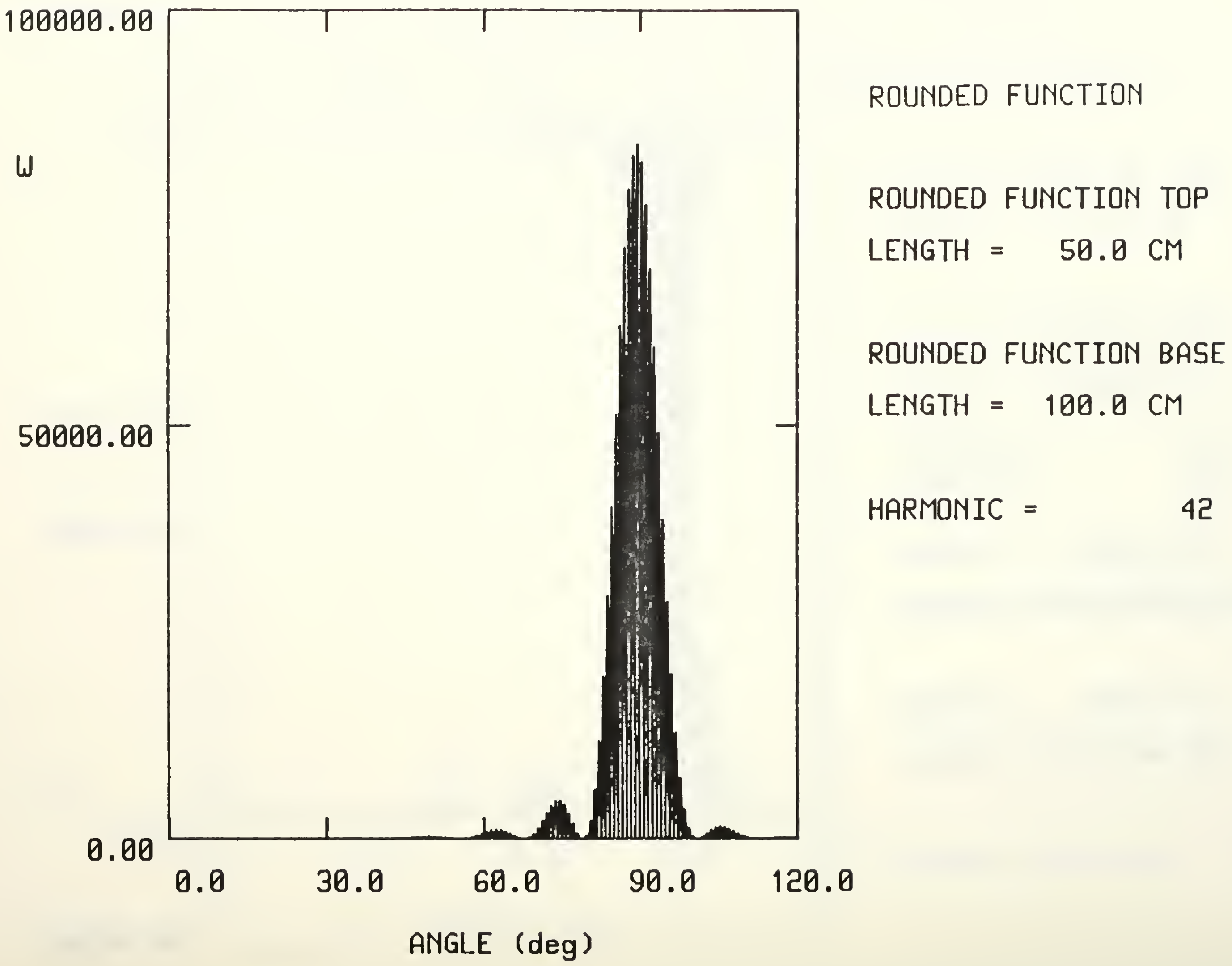
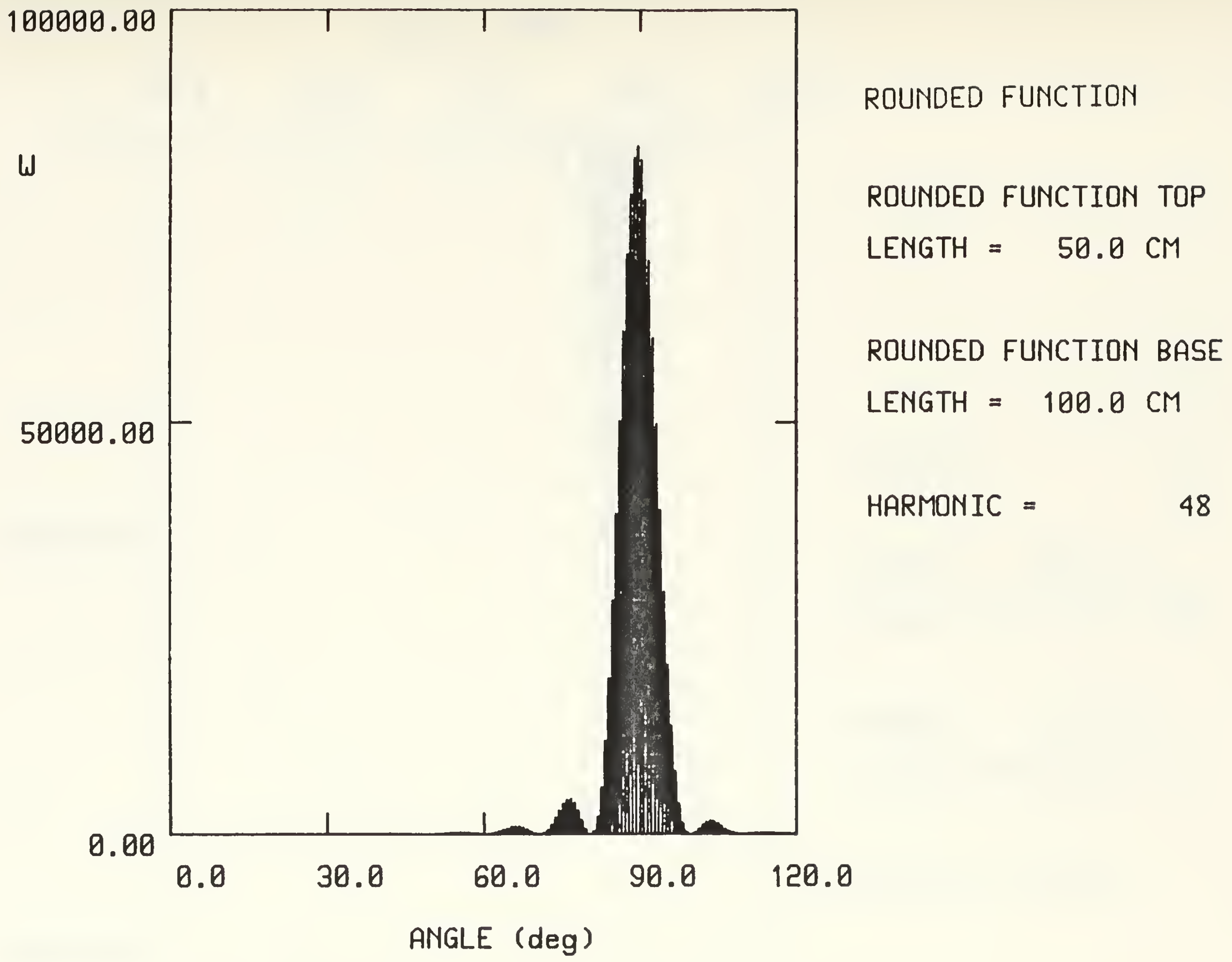
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 30


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 36


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 42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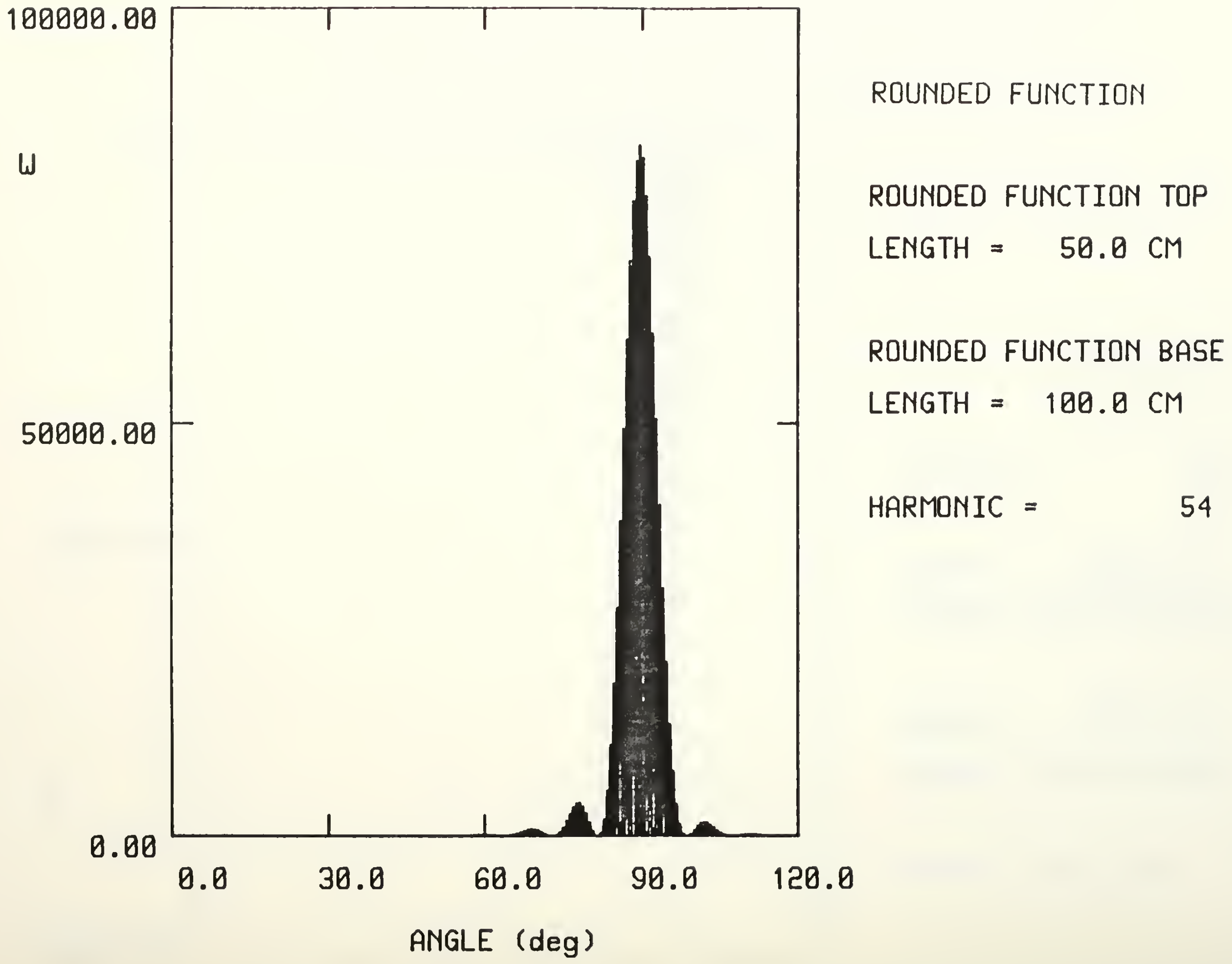
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 48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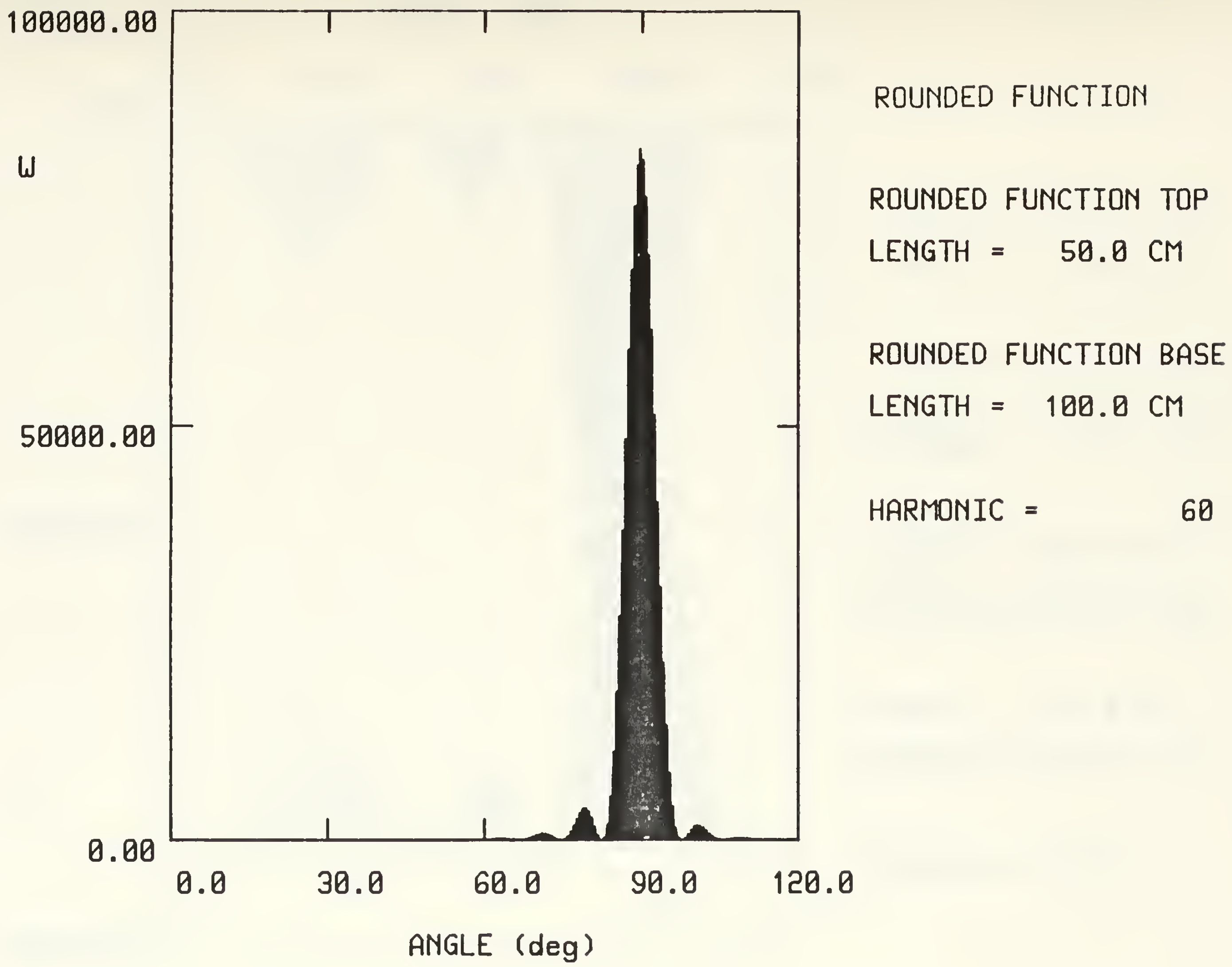
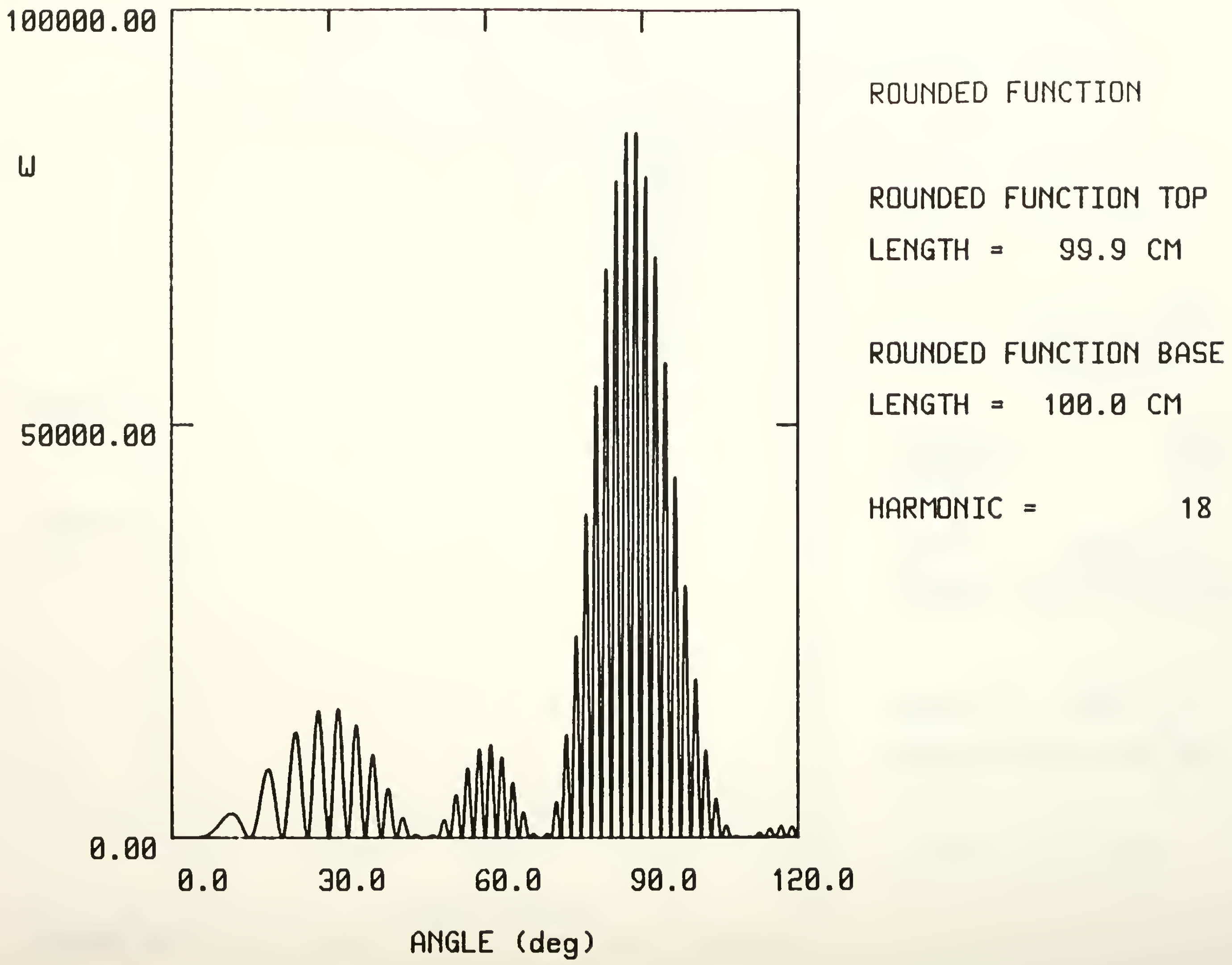
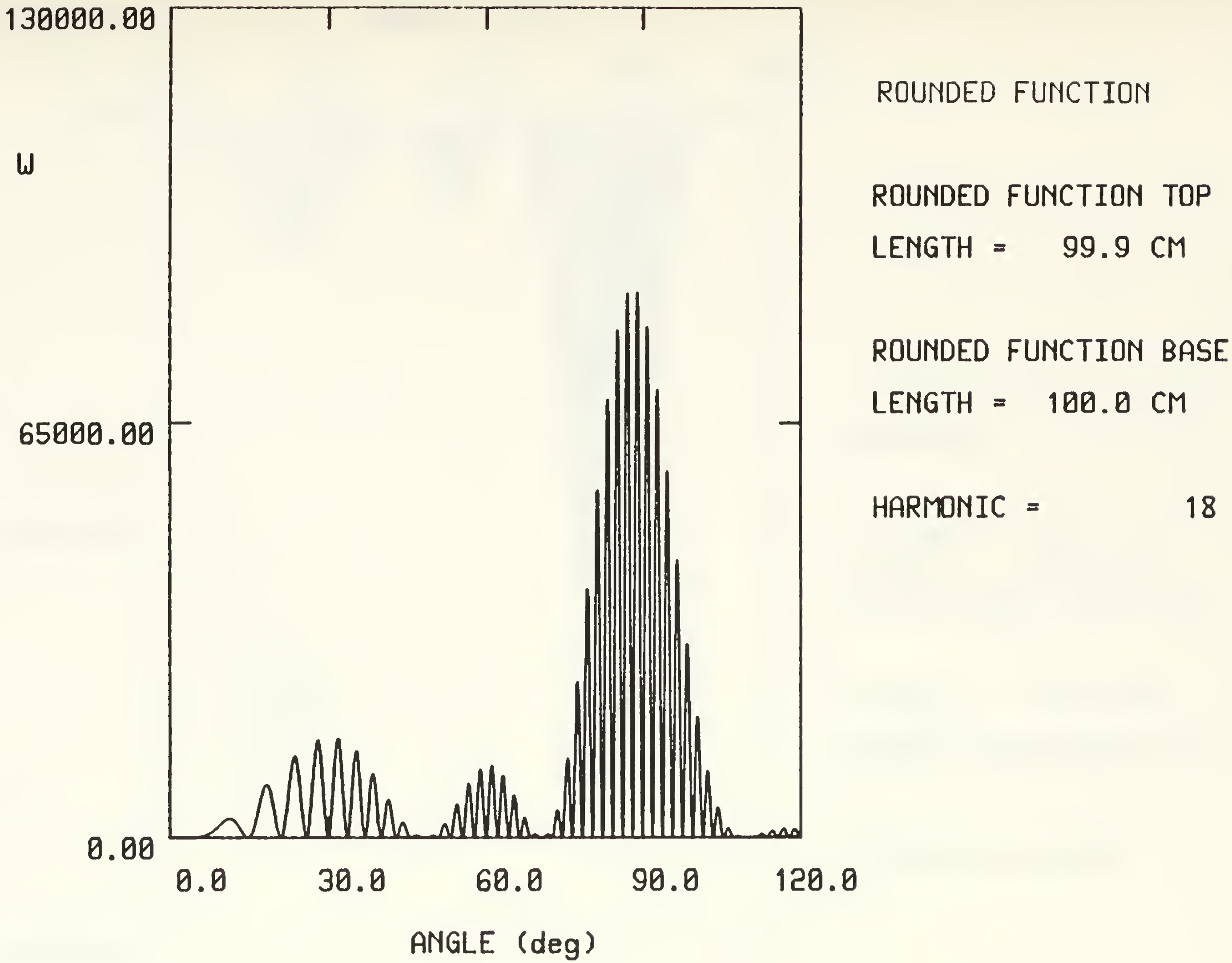
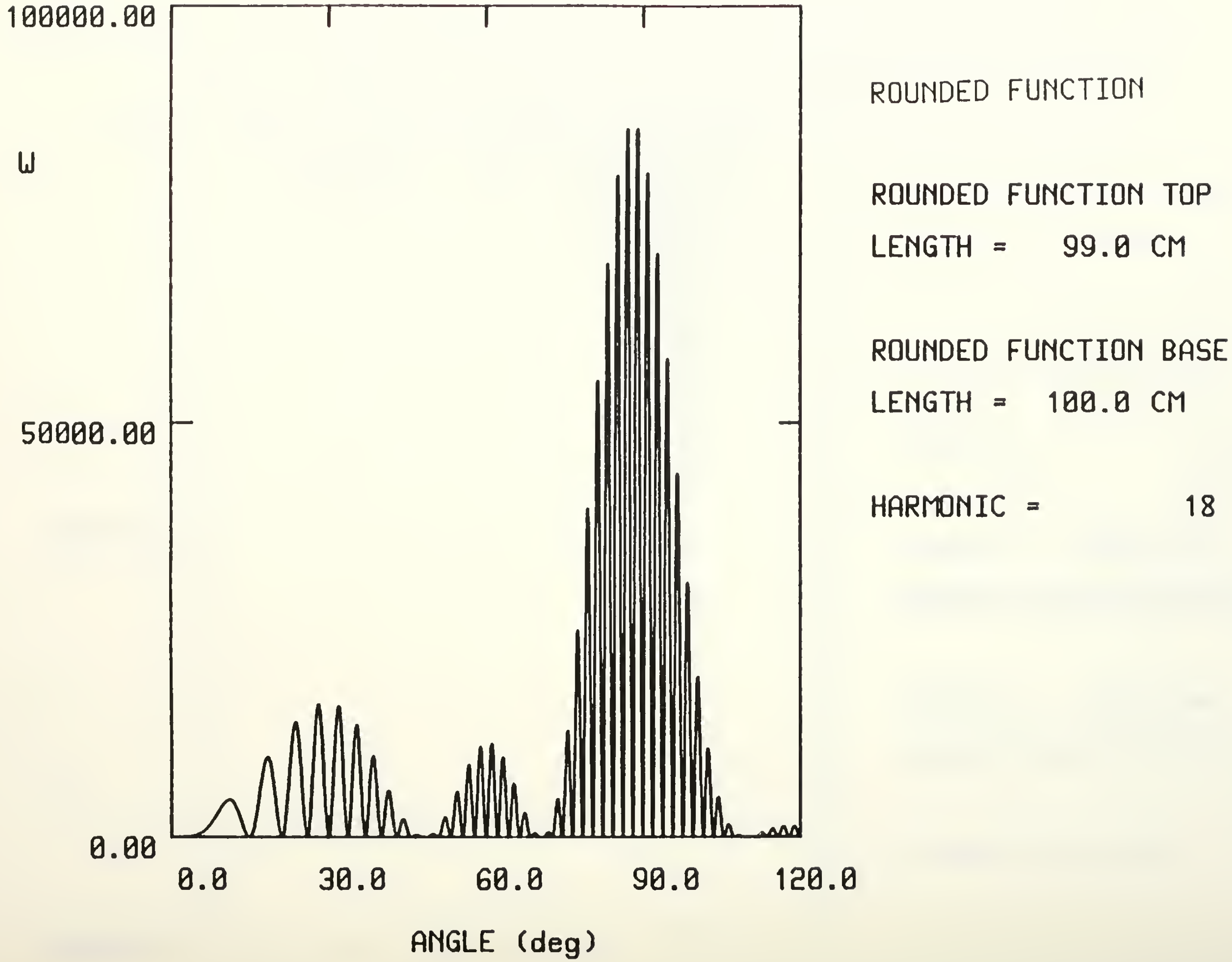
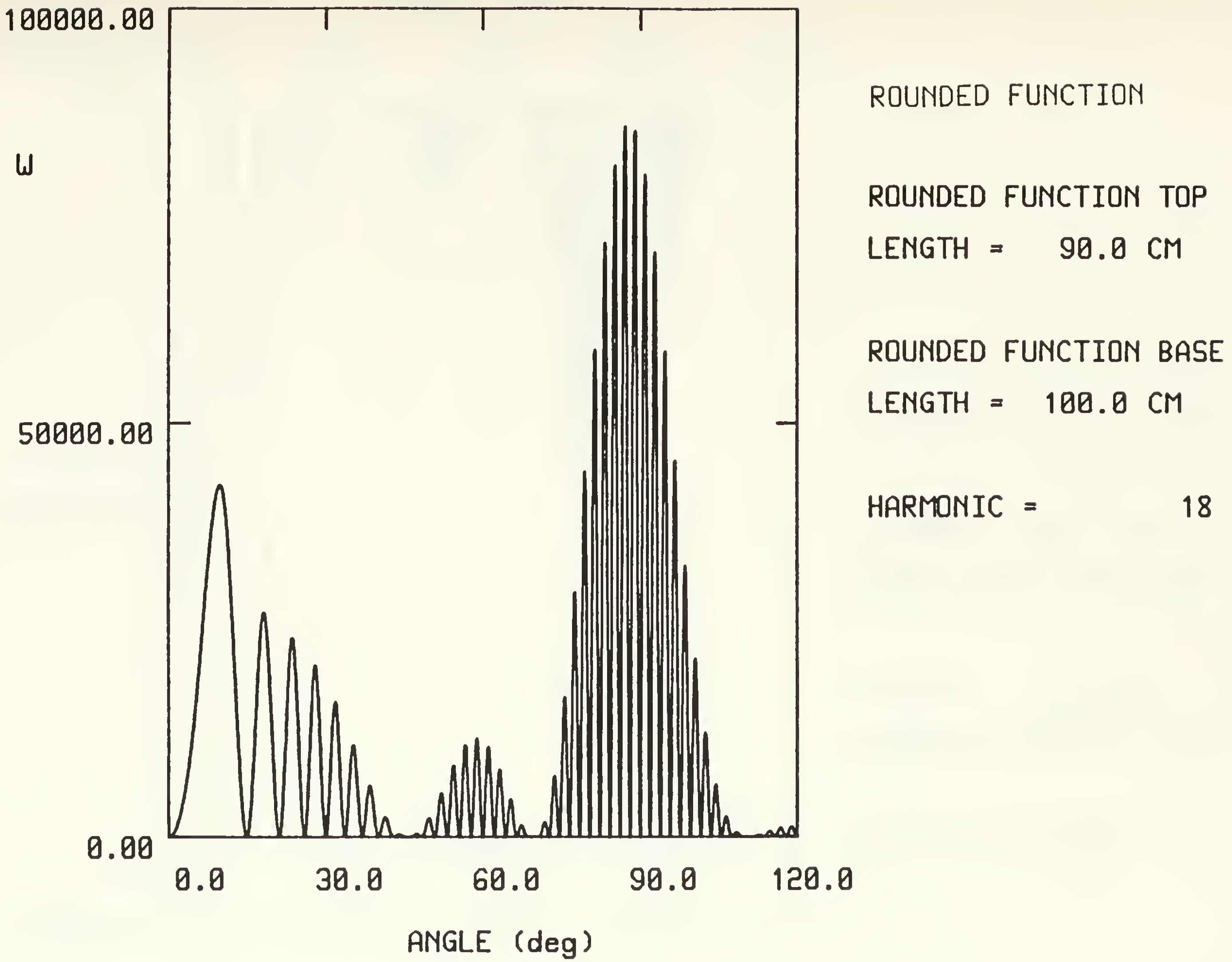
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 54


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 60


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 99.9 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18


130000.00
W
65000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 99.9 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 99.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18


100000.00
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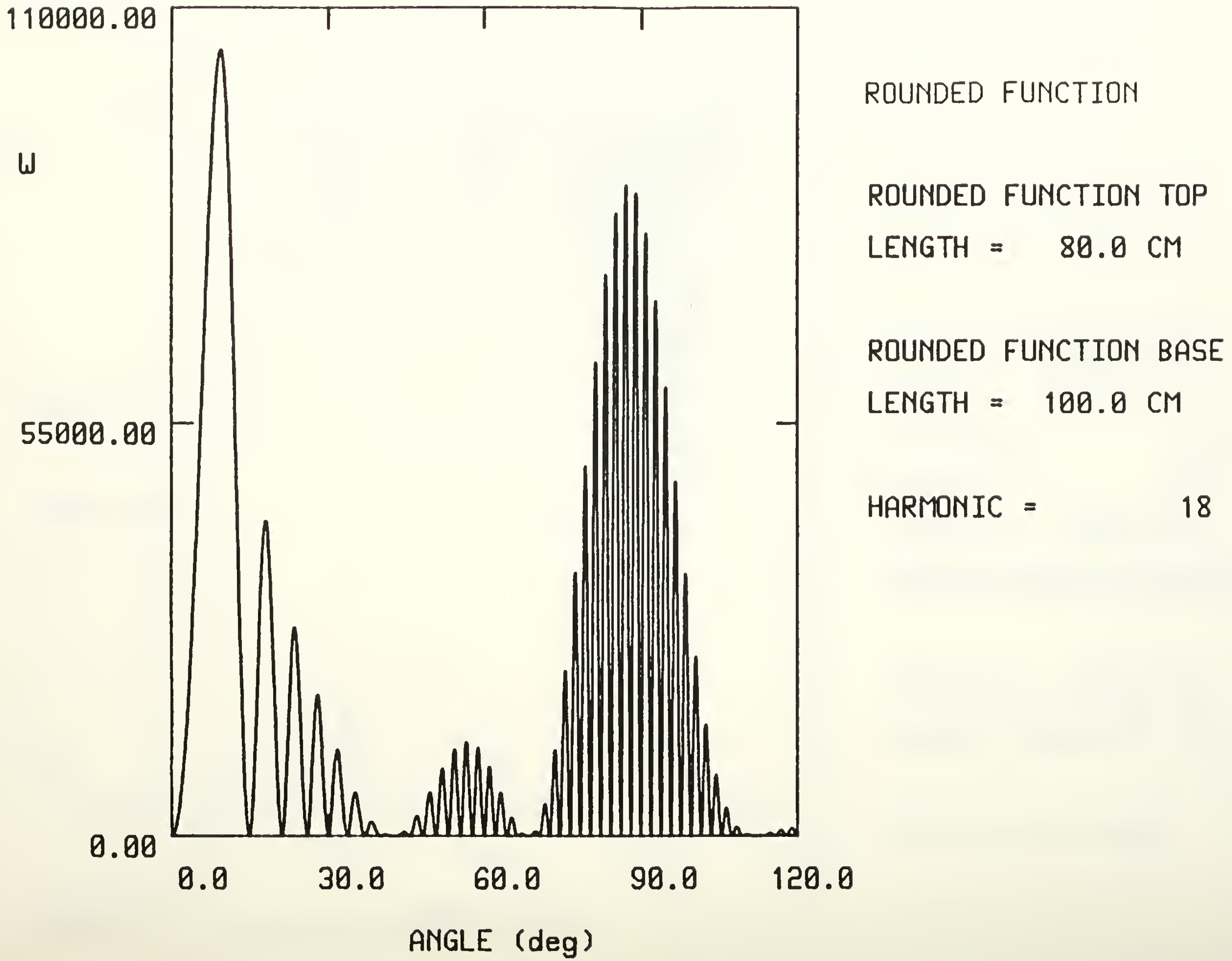
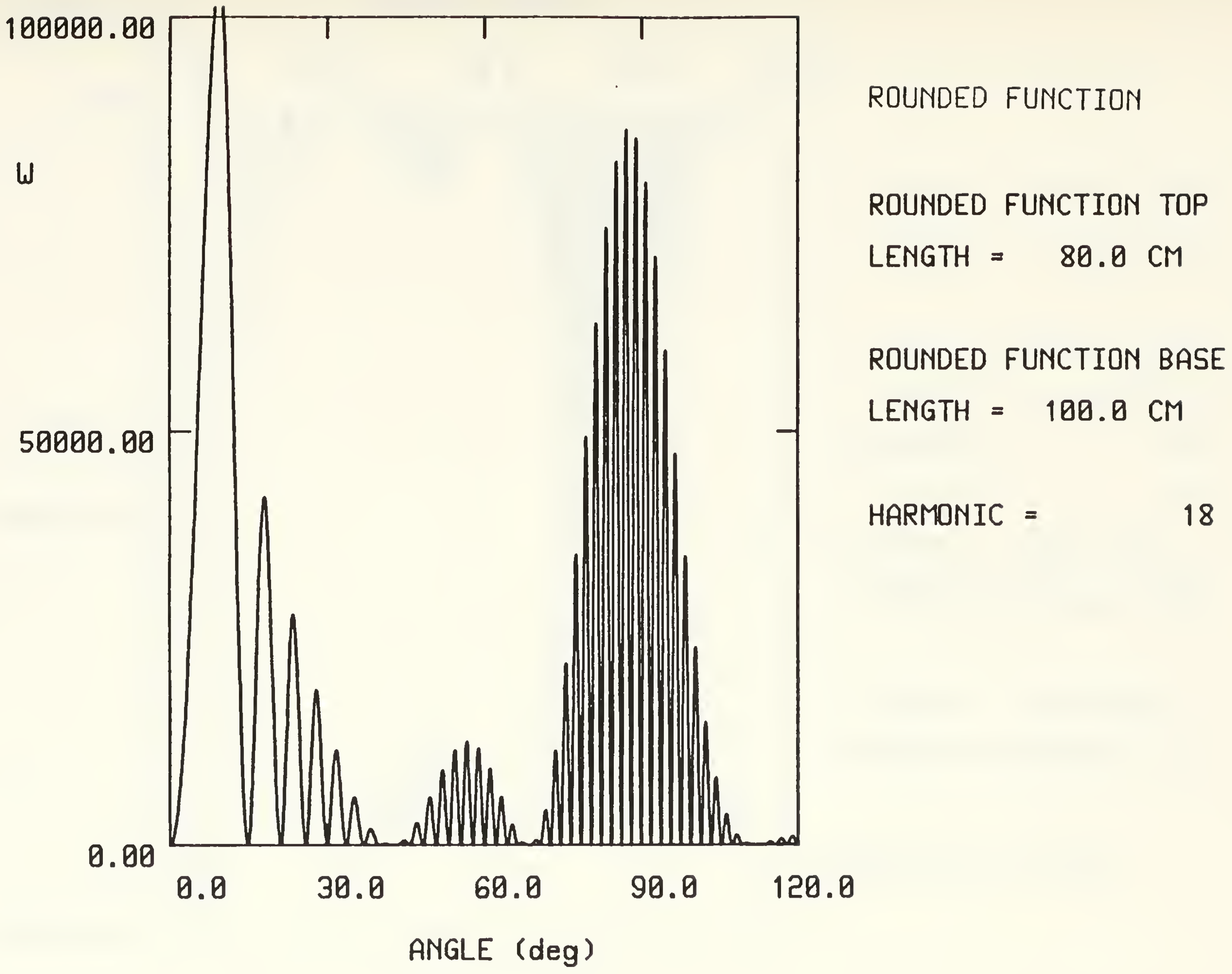
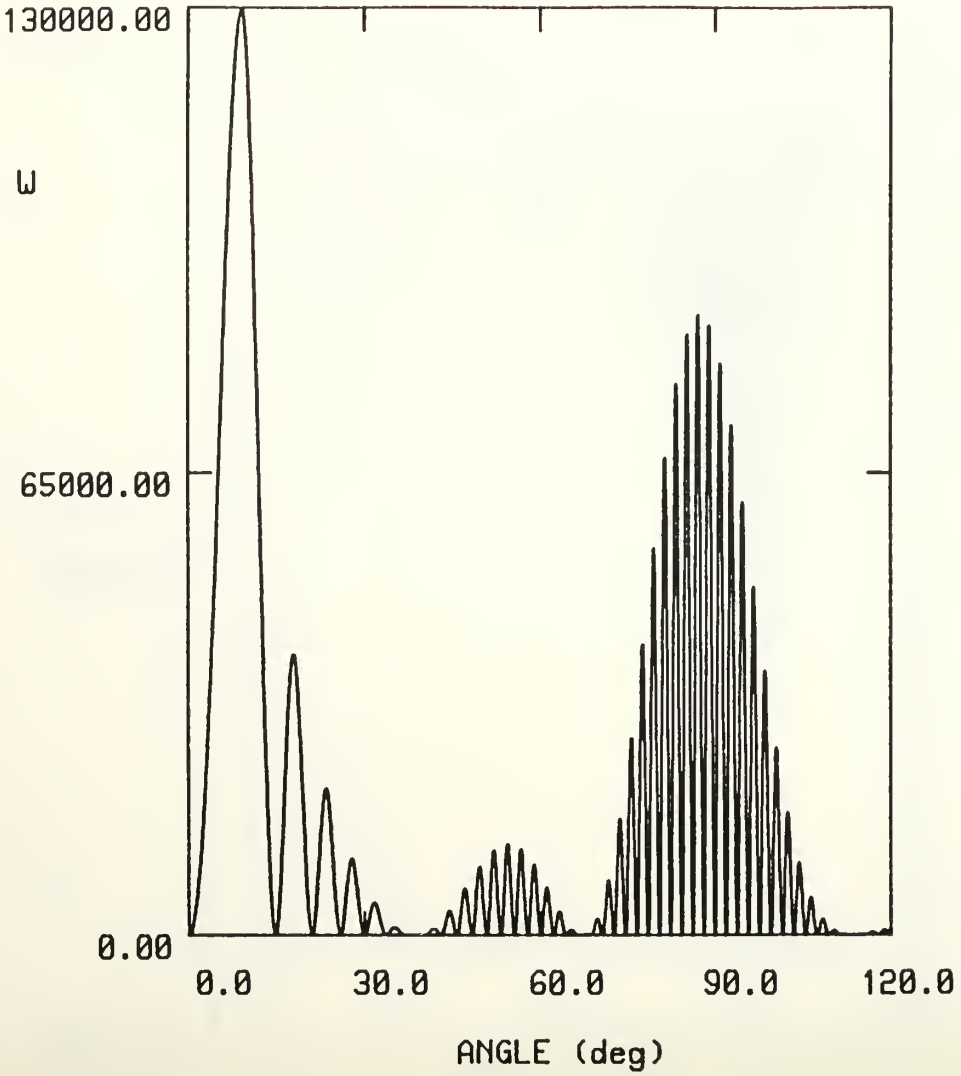
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 90.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18


110000.00
W
55000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 80.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18


100000.00
50000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 80.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18

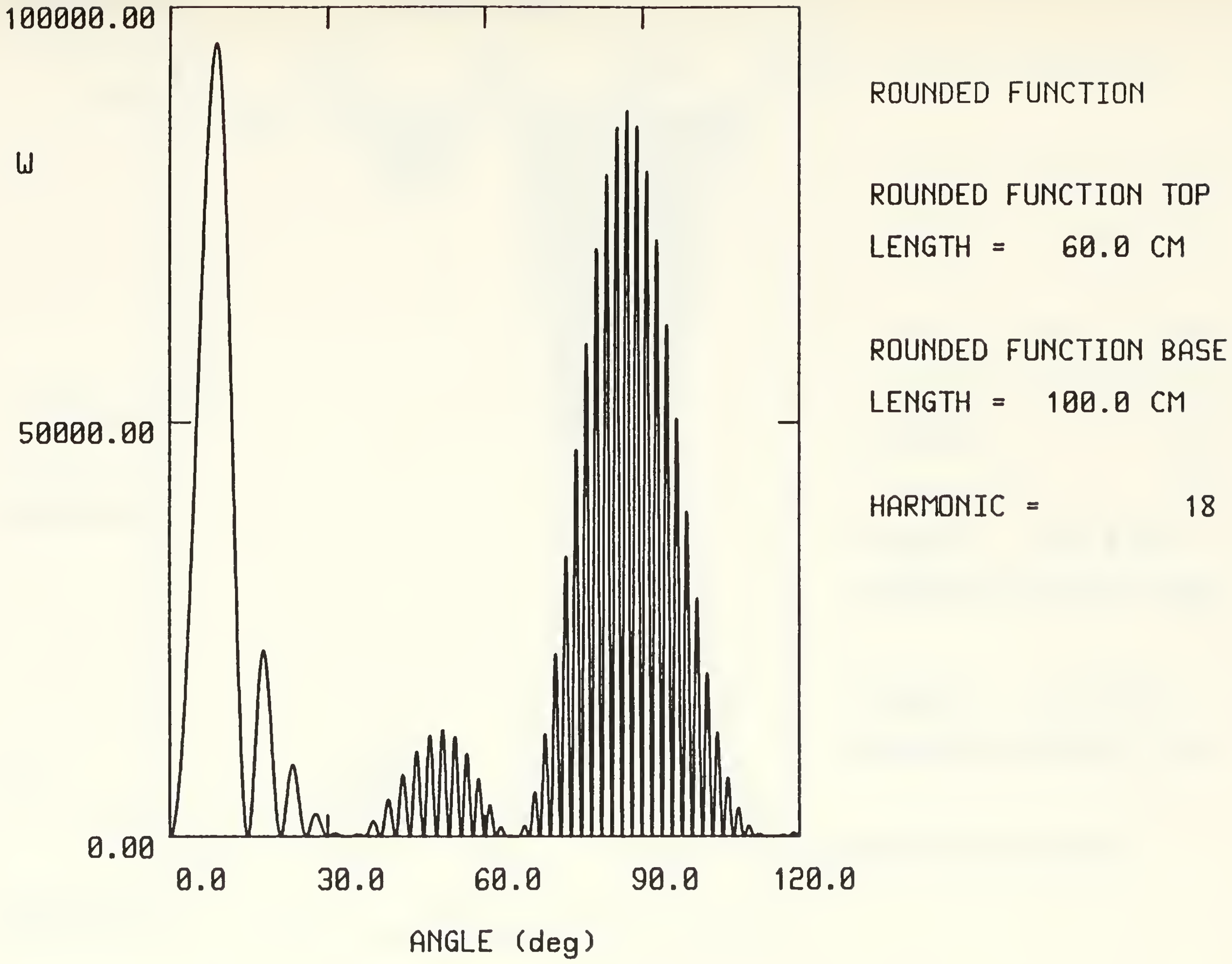


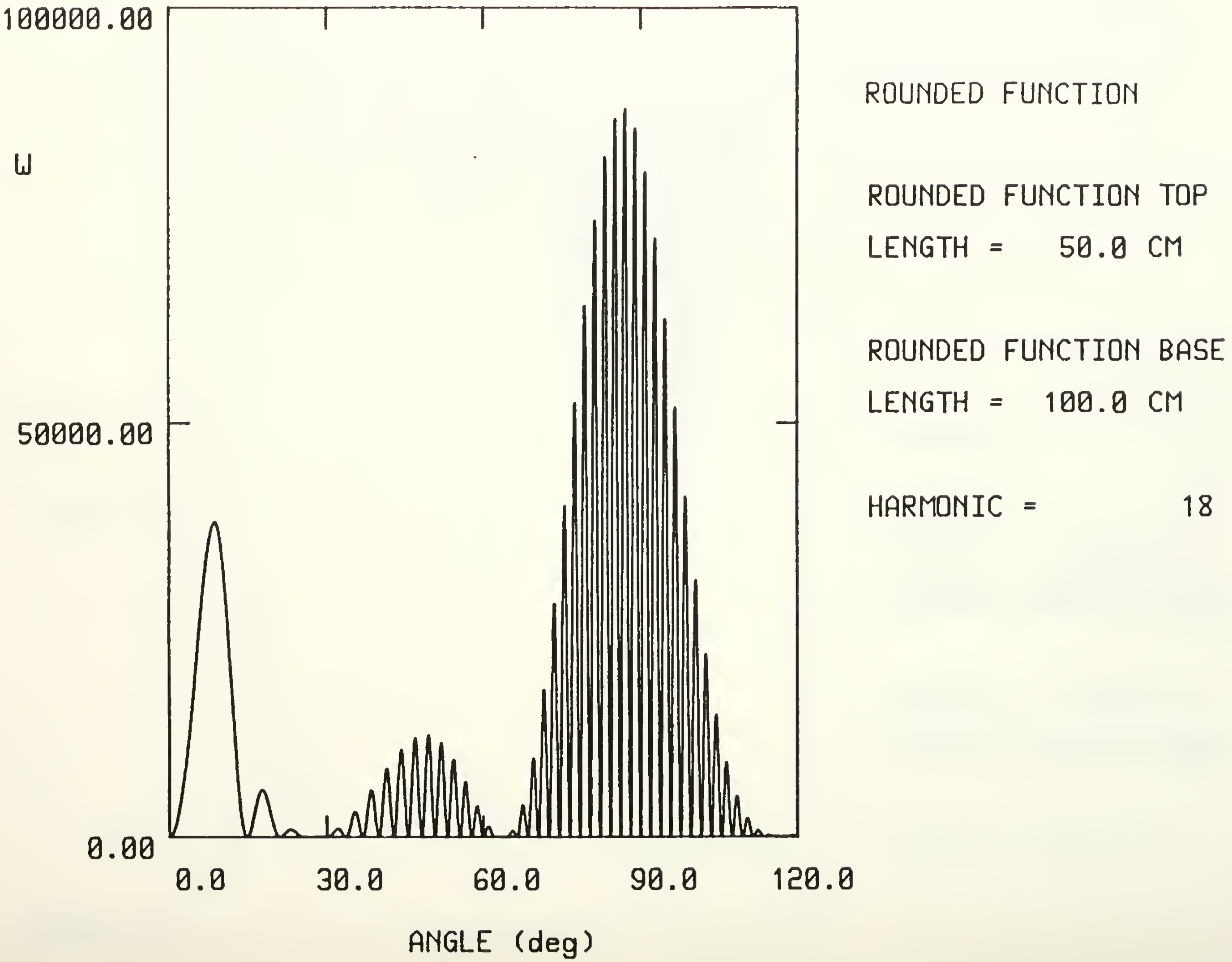
ROUNDED FUNCTION

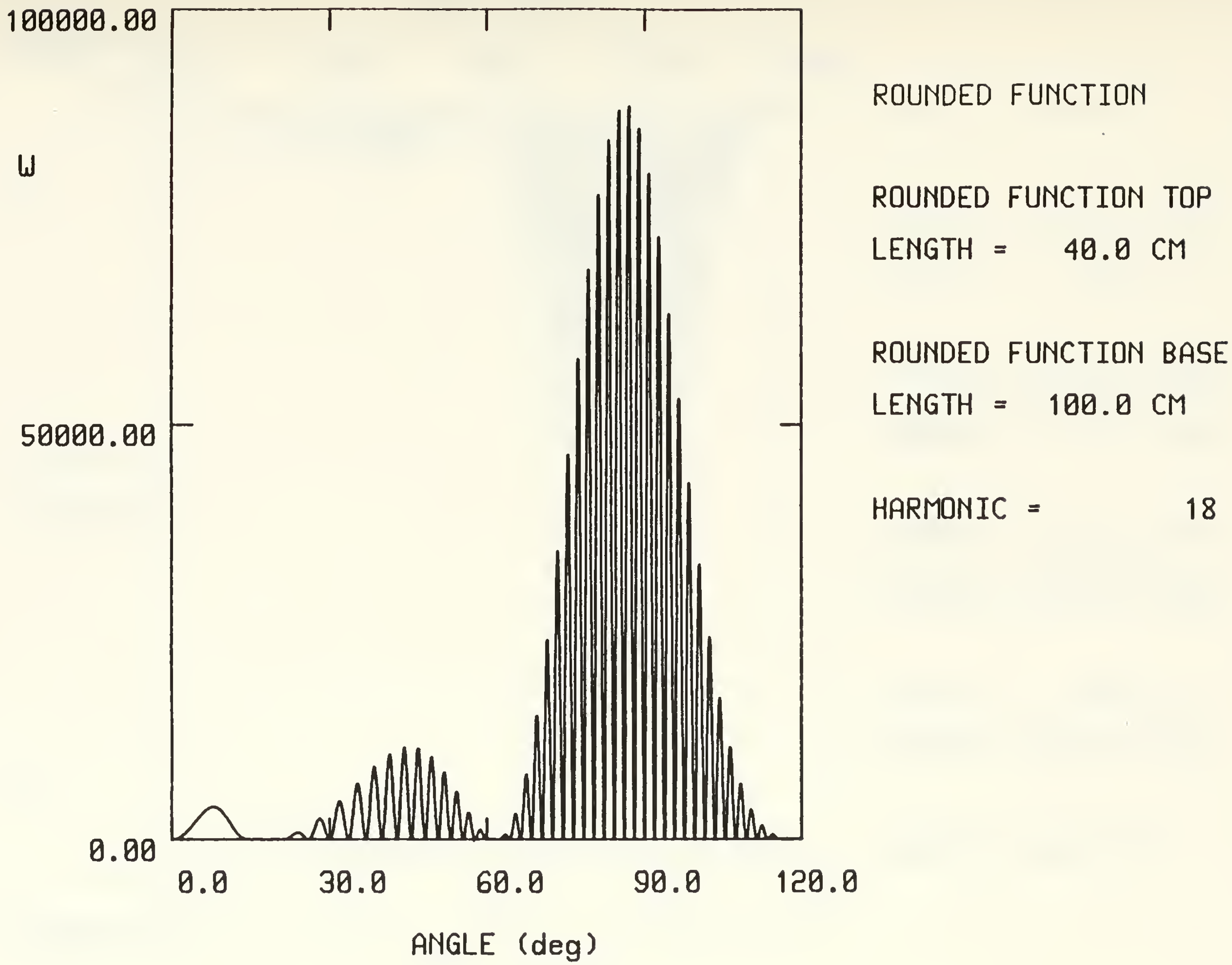
ROUNDED FUNCTION TOP
LENGTH = 70.0 CM

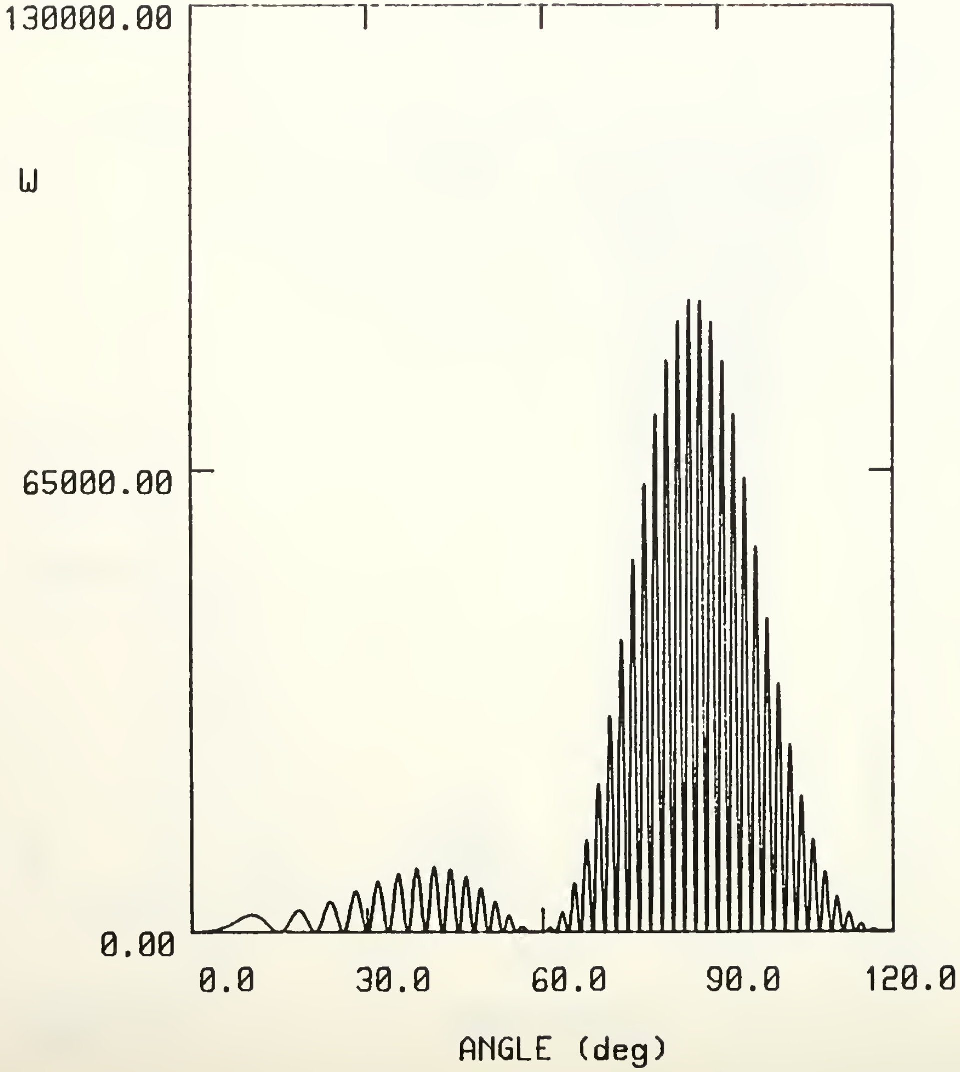
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM

HARMONIC = 18


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 60.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 50.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 40.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18

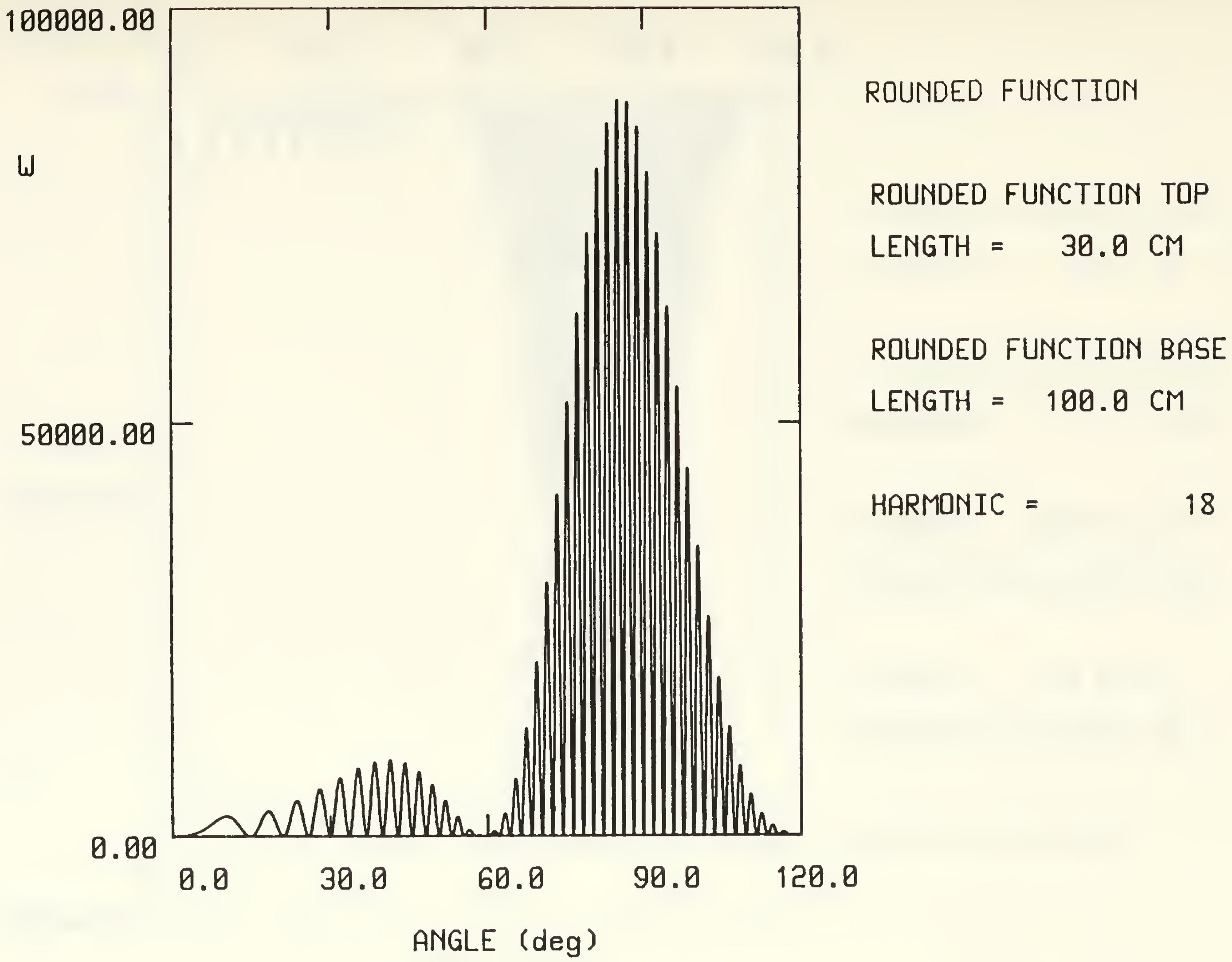


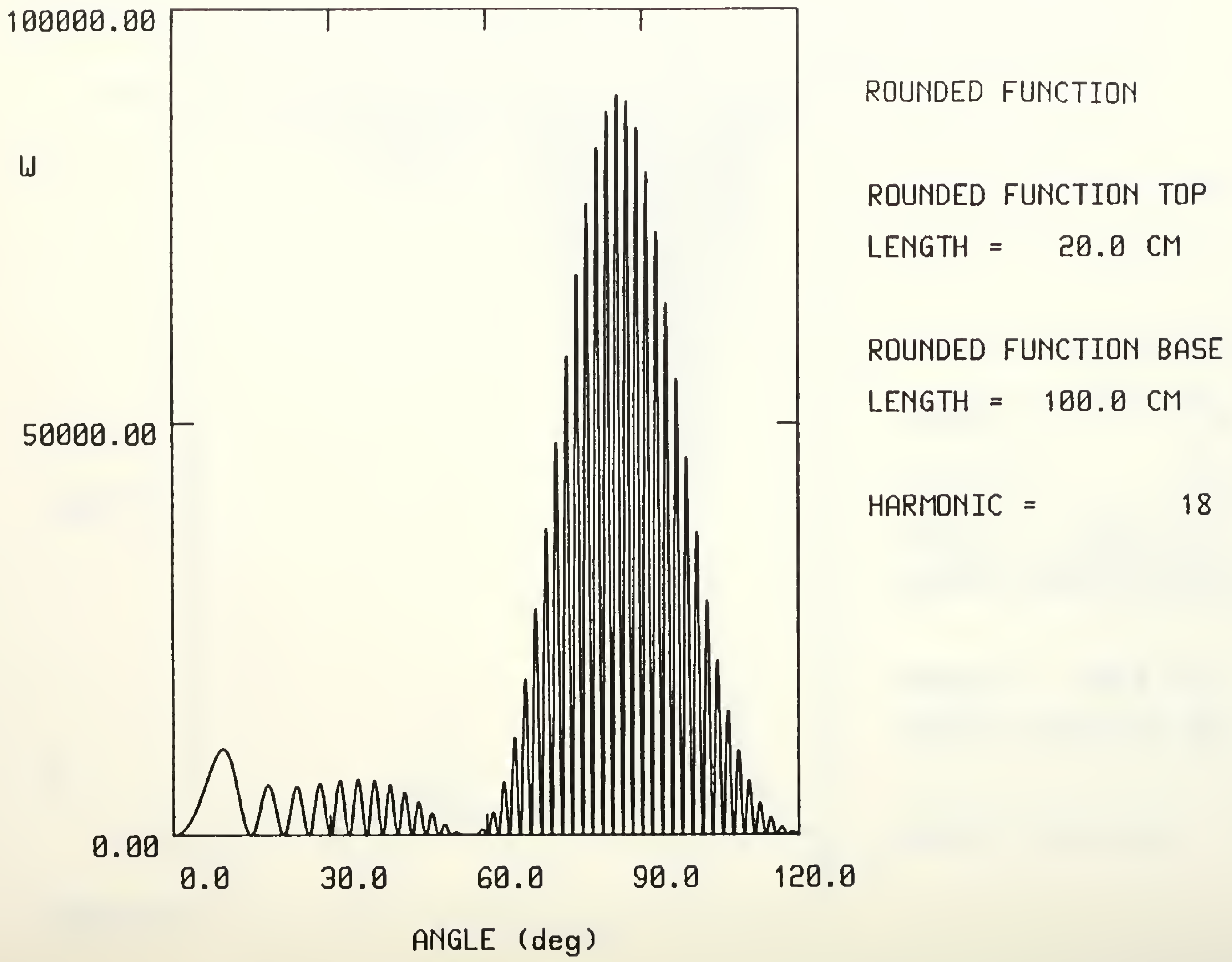
ROUNDED FUNCTION

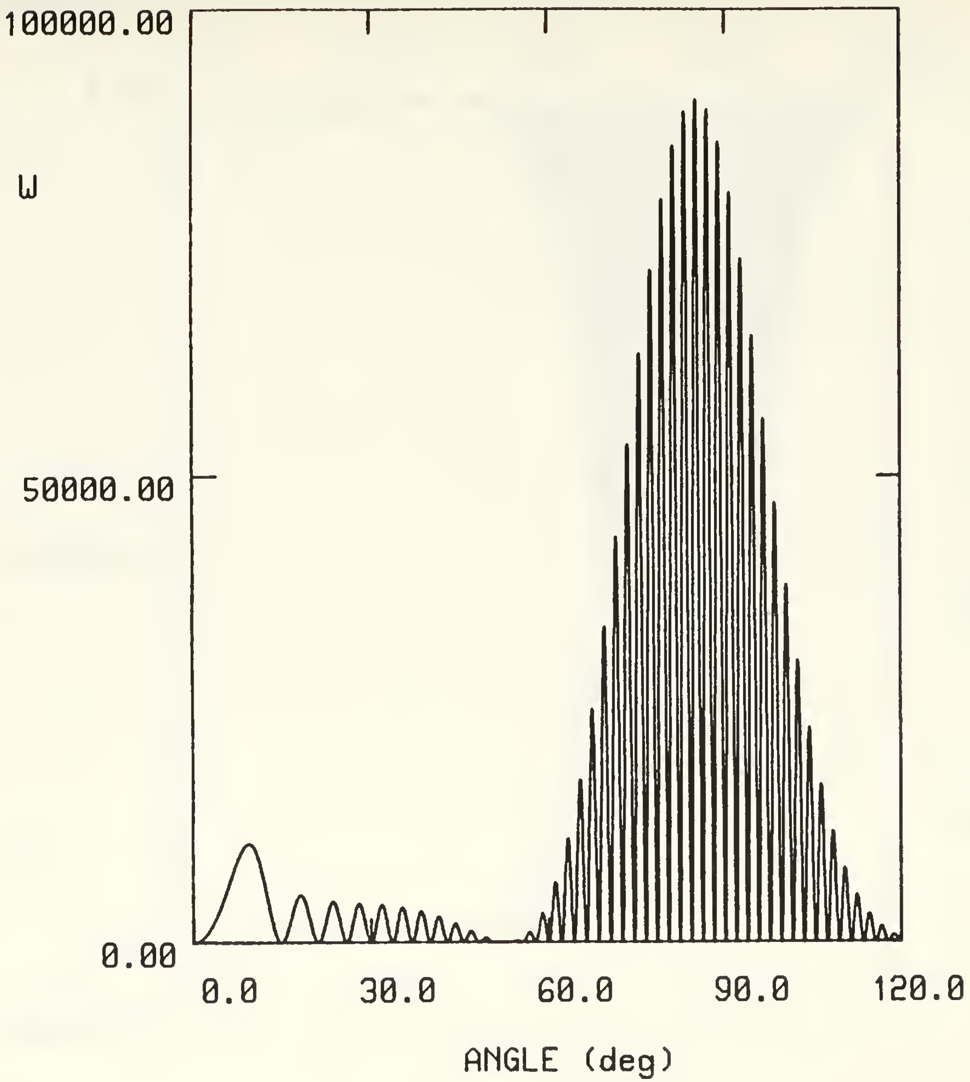
ROUNDED FUNCTION TOP
LENGTH = 30.0 CM

ROUNDED FUNCTION BASE
LENGTH = 100.0 CM

HARMONIC = 18


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 30.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 20.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18

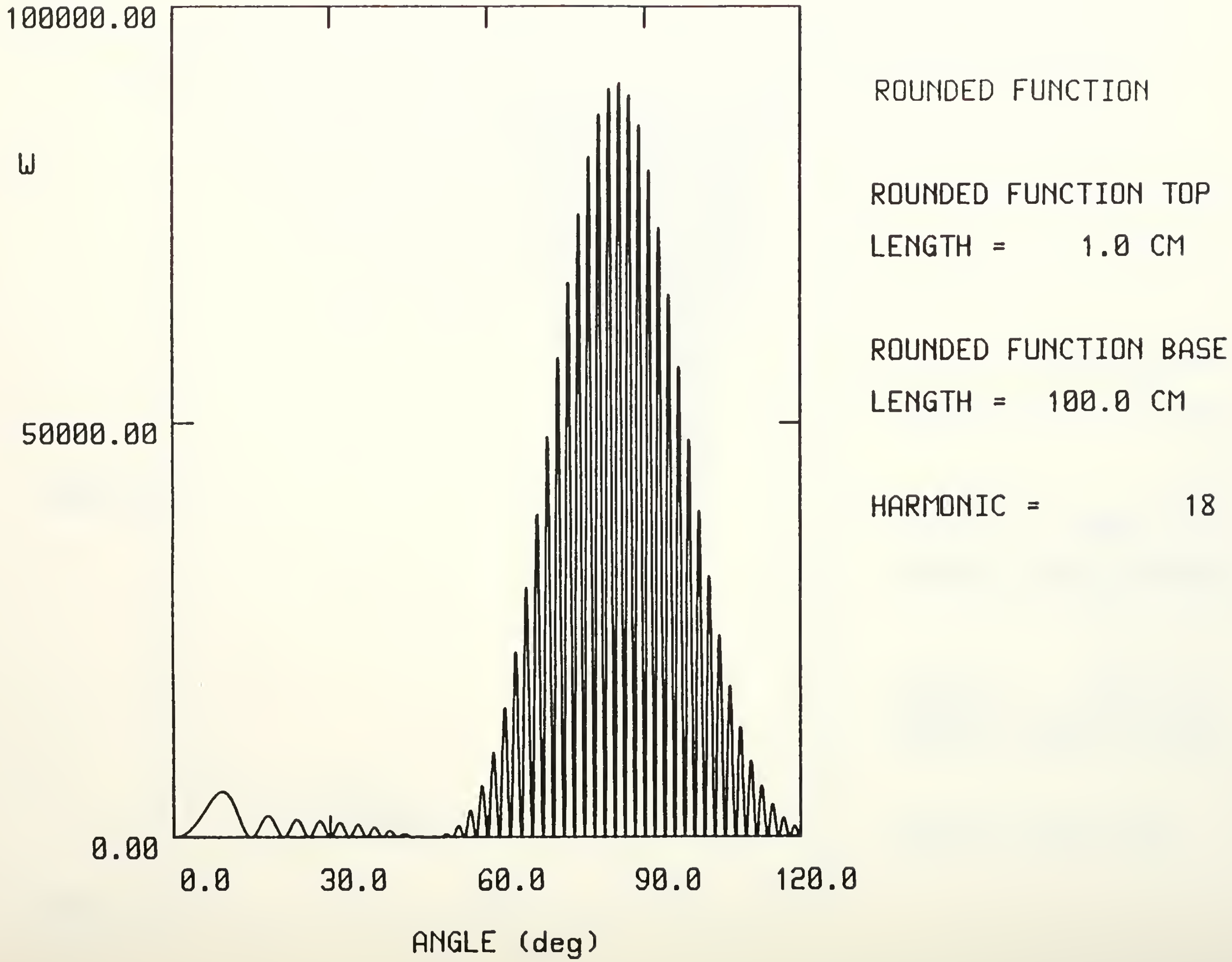


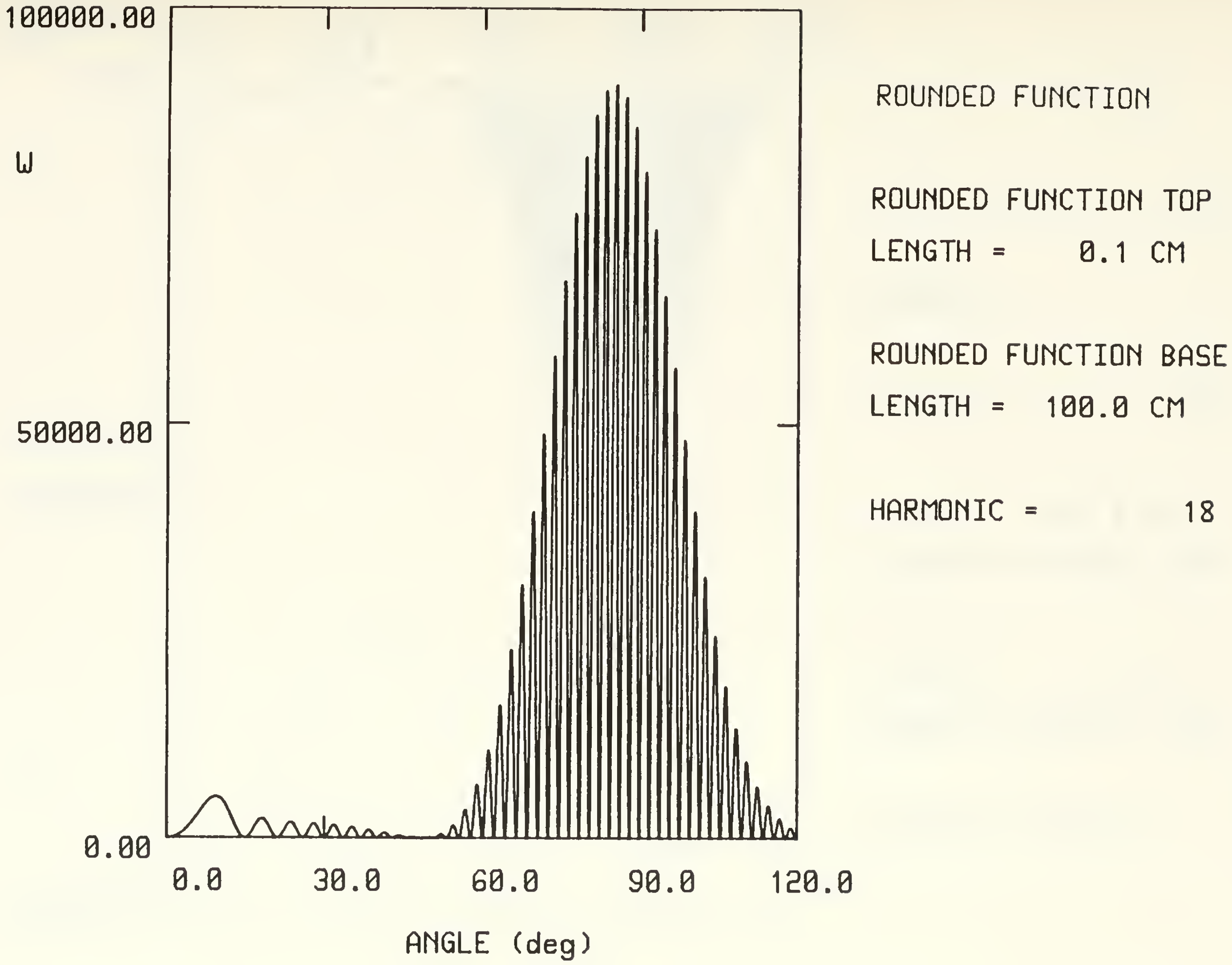
ROUNDED FUNCTION

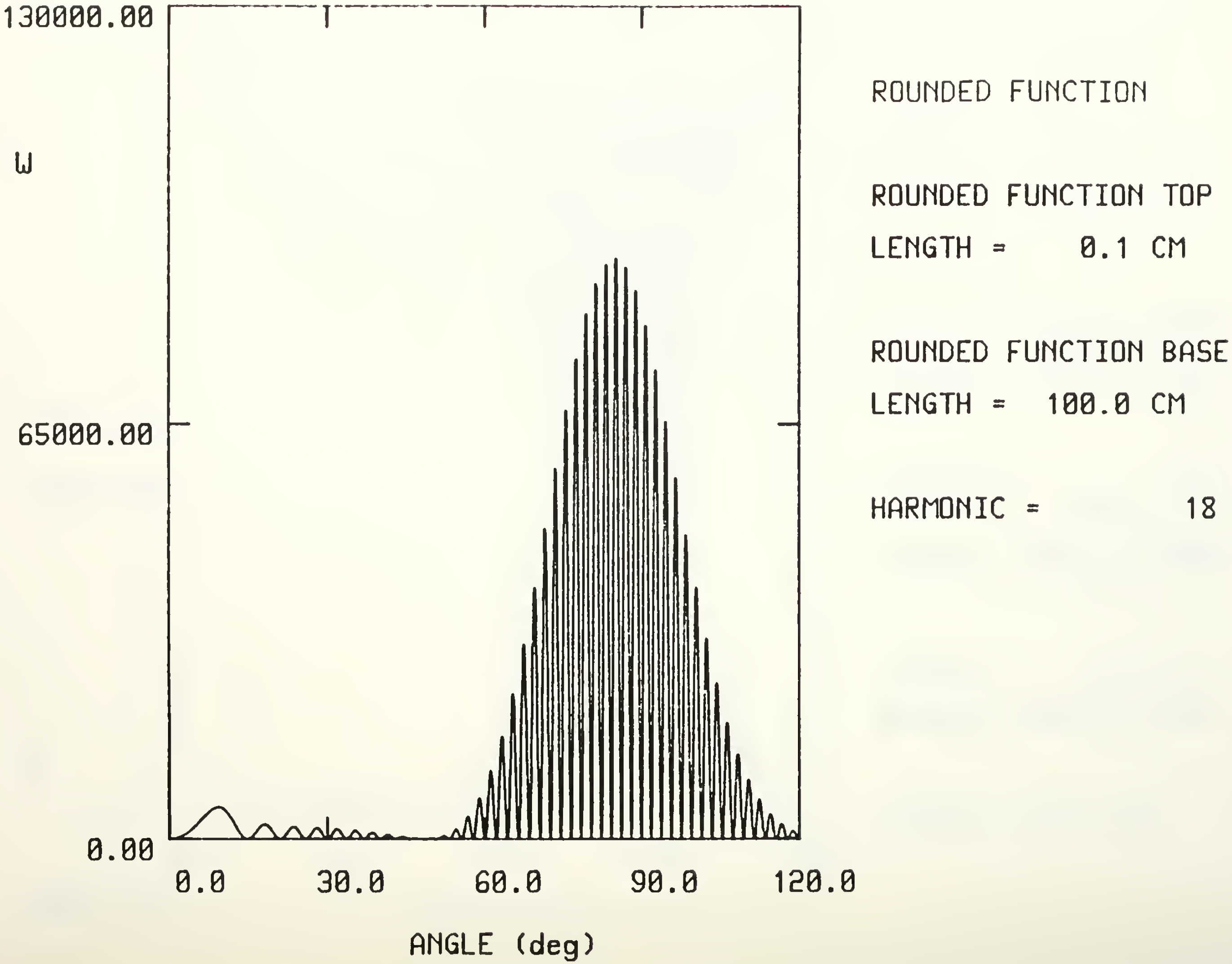
ROUNDED FUNCTION TOP
LENGTH = 10.0 CM

ROUNDED FUNCTION BASE
LENGTH = 100.0 CM

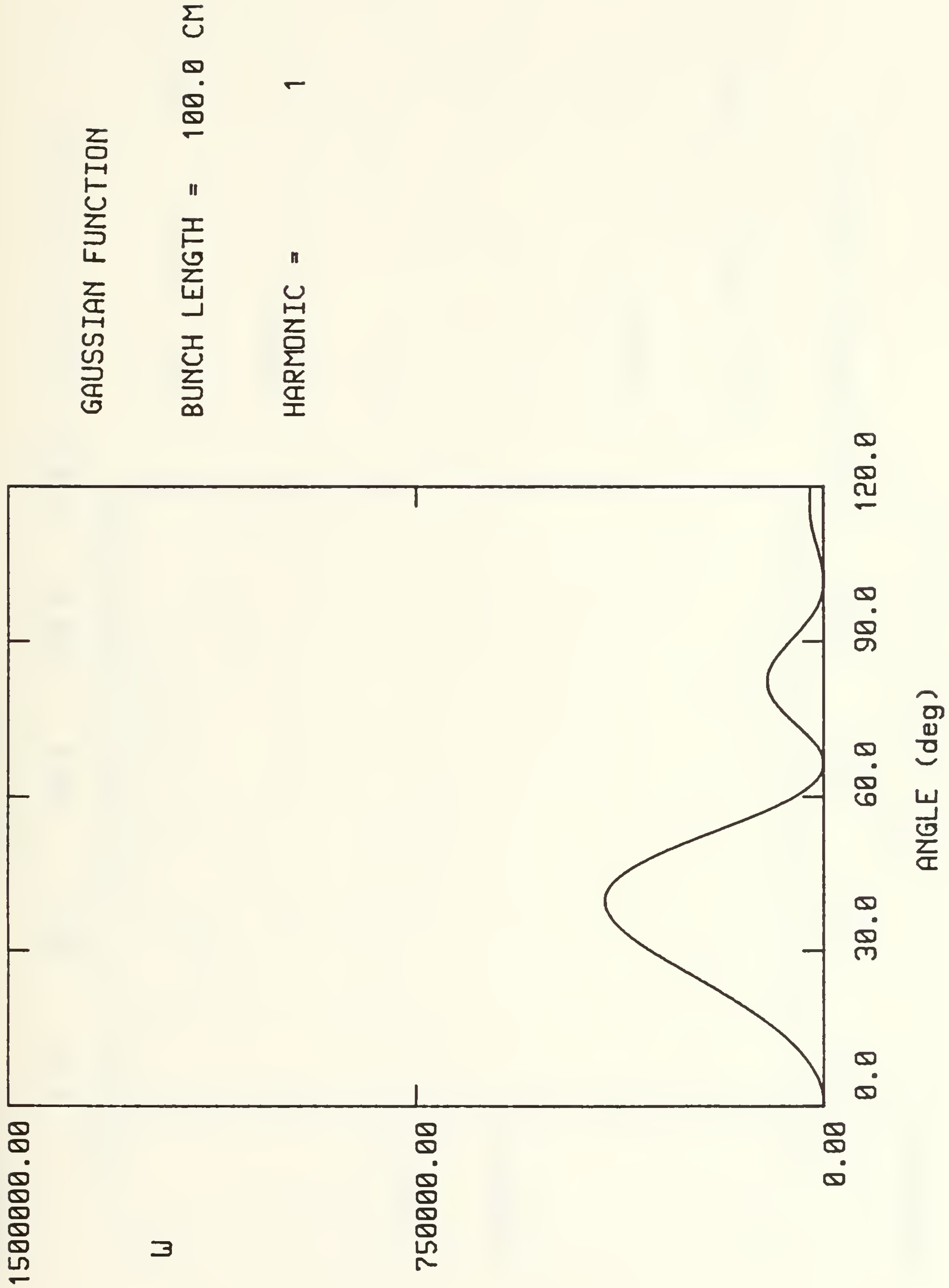
HARMONIC = 18

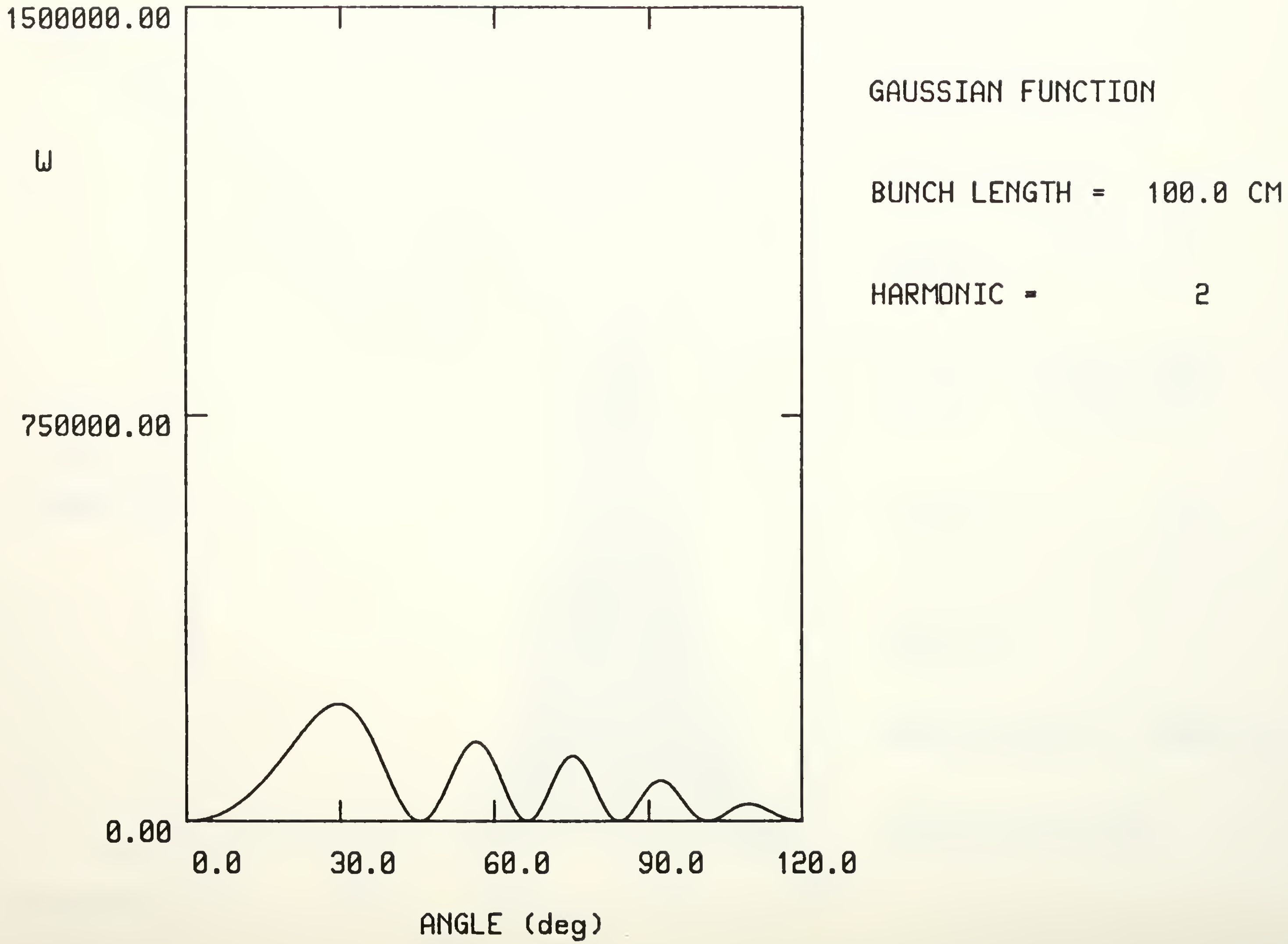

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 1.0 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18

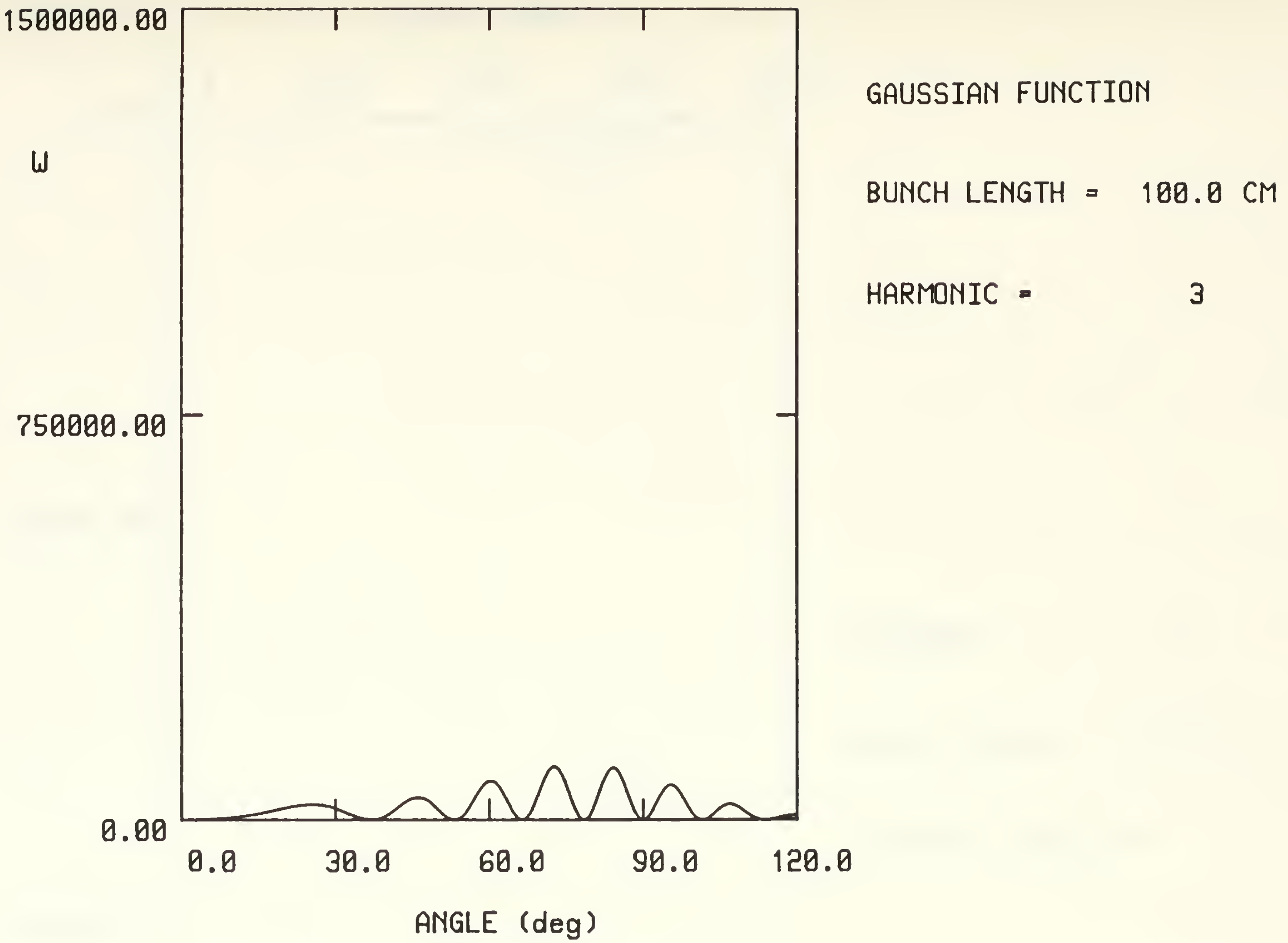

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 0.1 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18

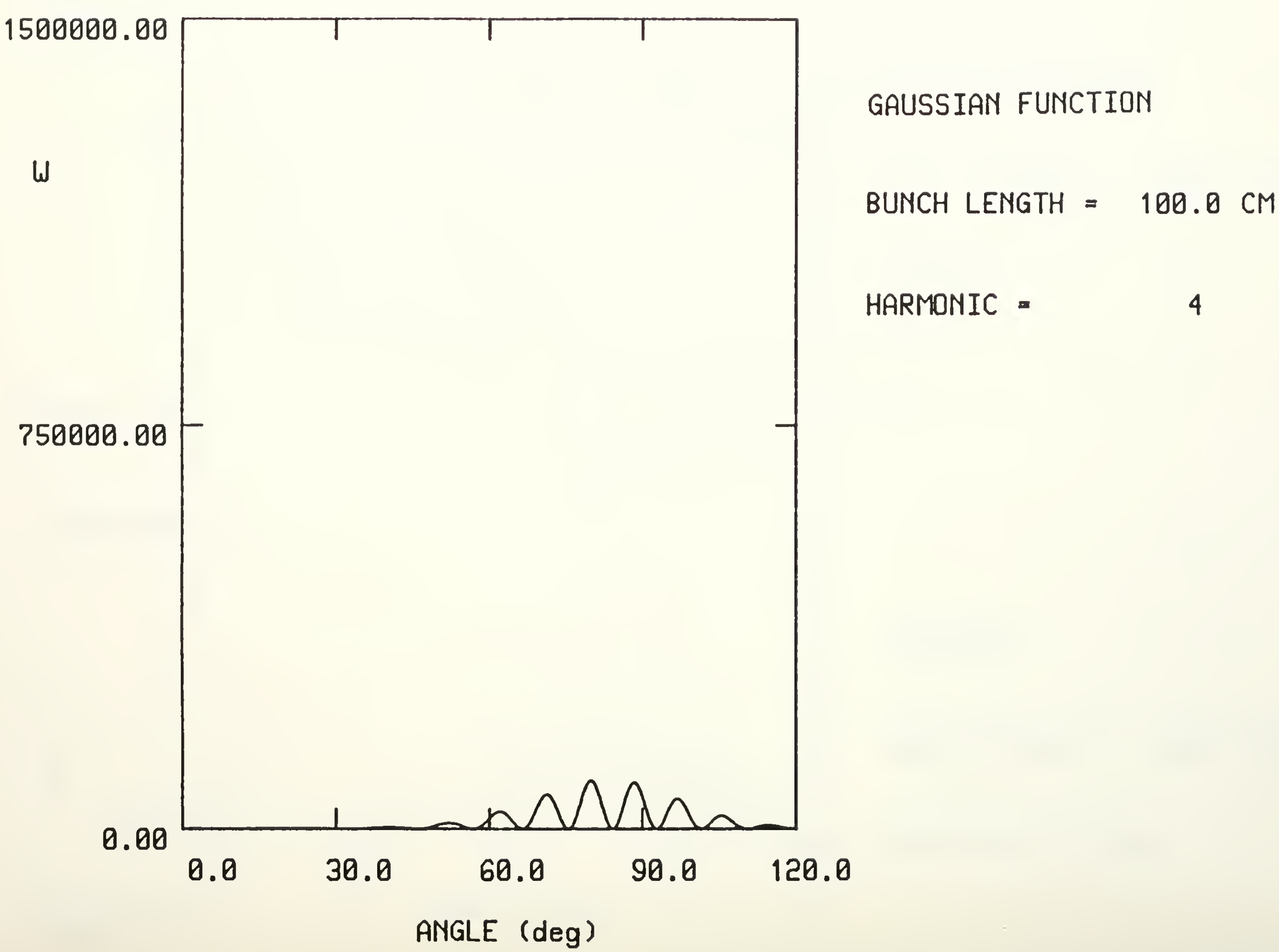

130000.00
W
65000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
ROUNDED FUNCTION
ROUNDED FUNCTION TOP
LENGTH = 0.1 CM
ROUNDED FUNCTION BASE
LENGTH = 100.0 CM
HARMONIC = 18

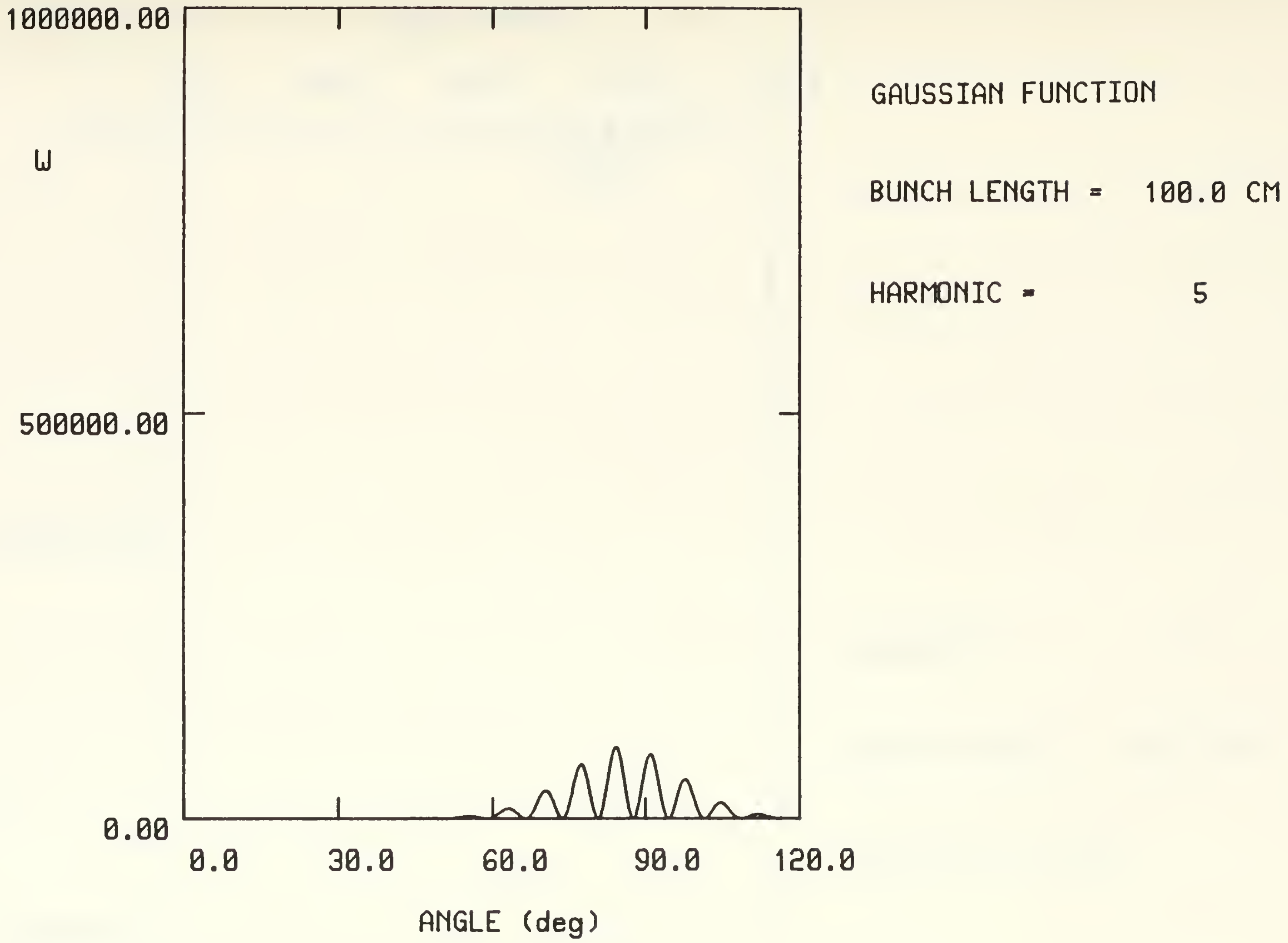
APPENDIX E: GAUSSIAN FUNCTION

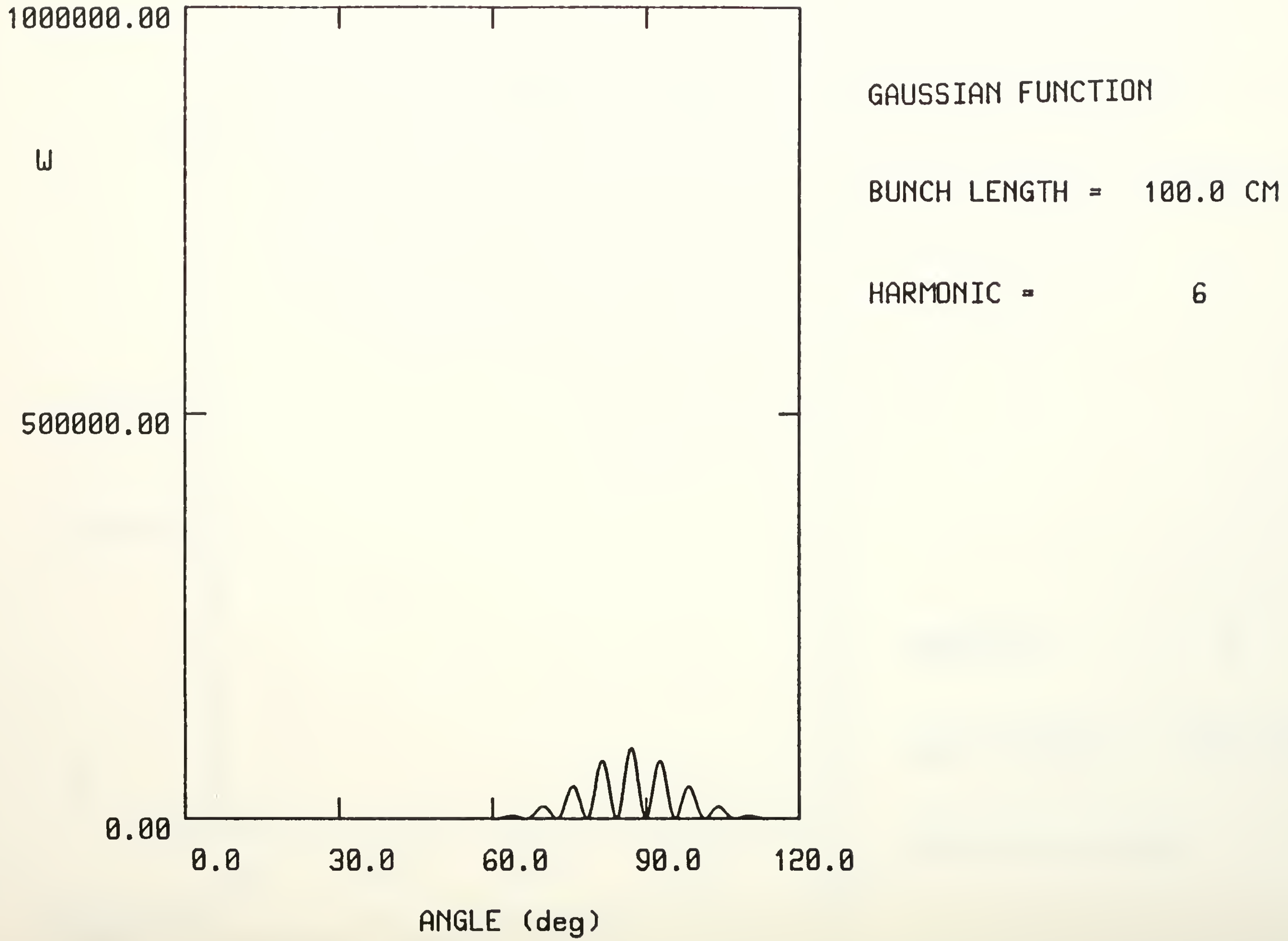


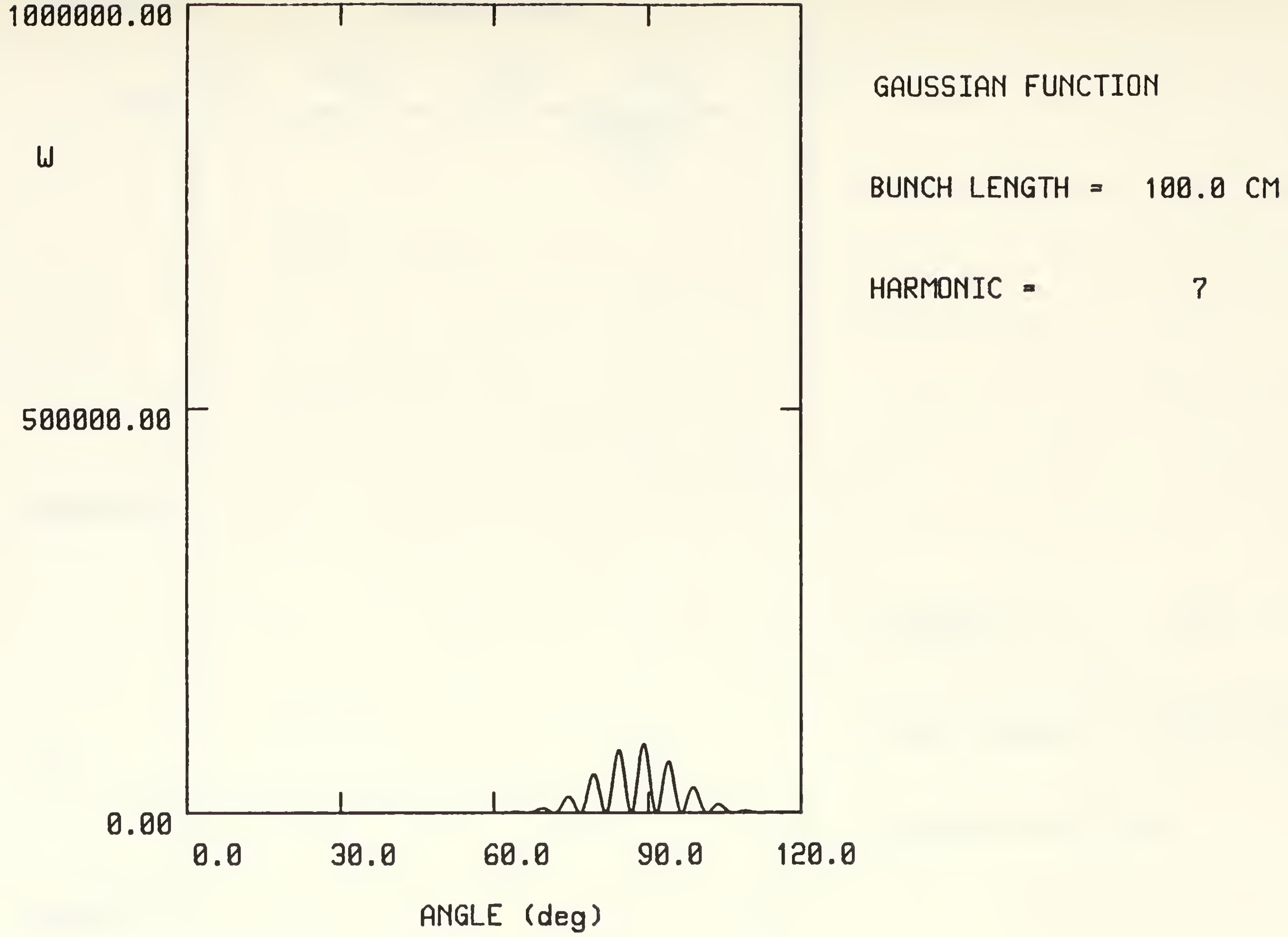

1500000.00
750000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 2

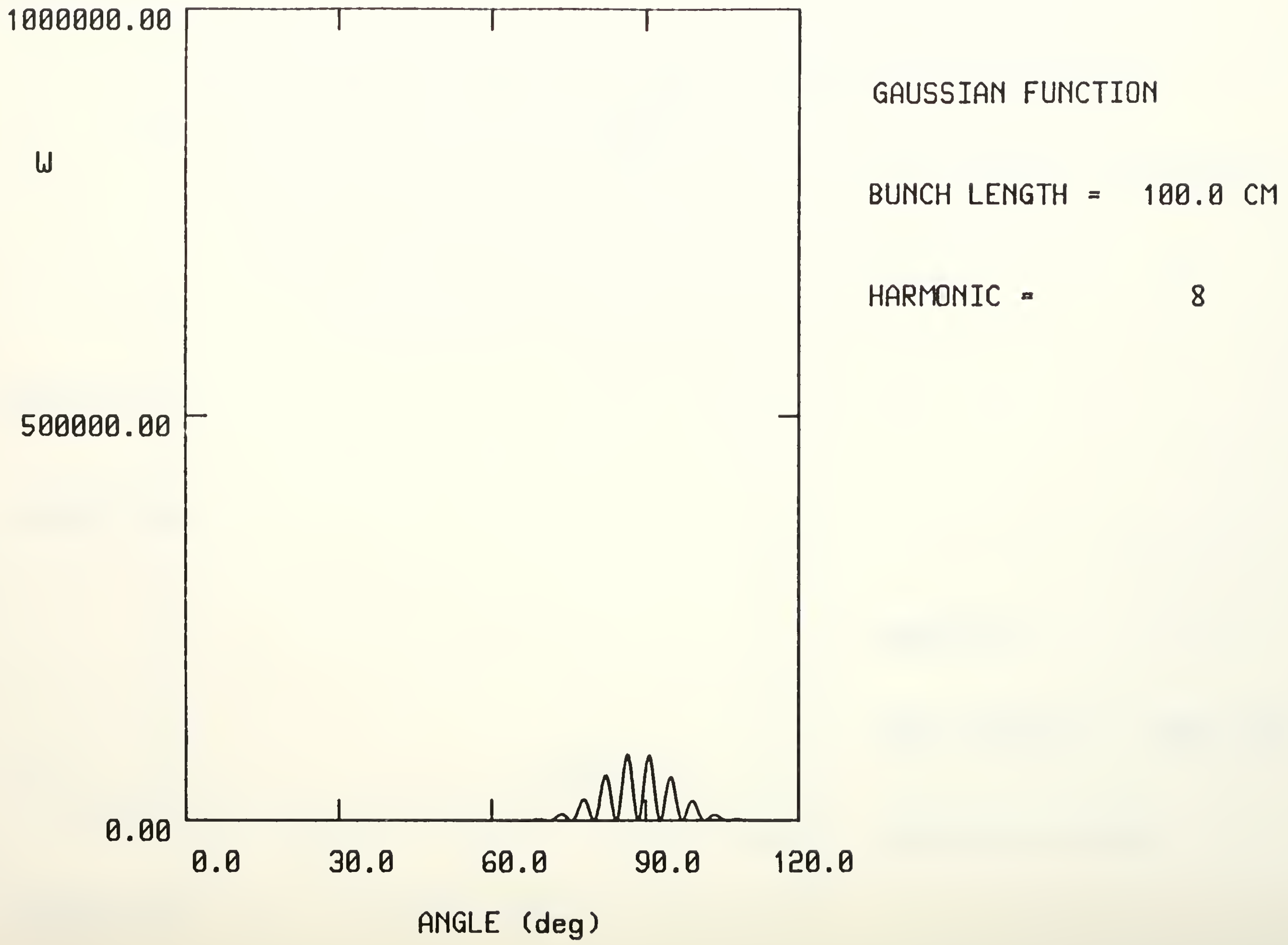

1500000.00
W
750000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 3

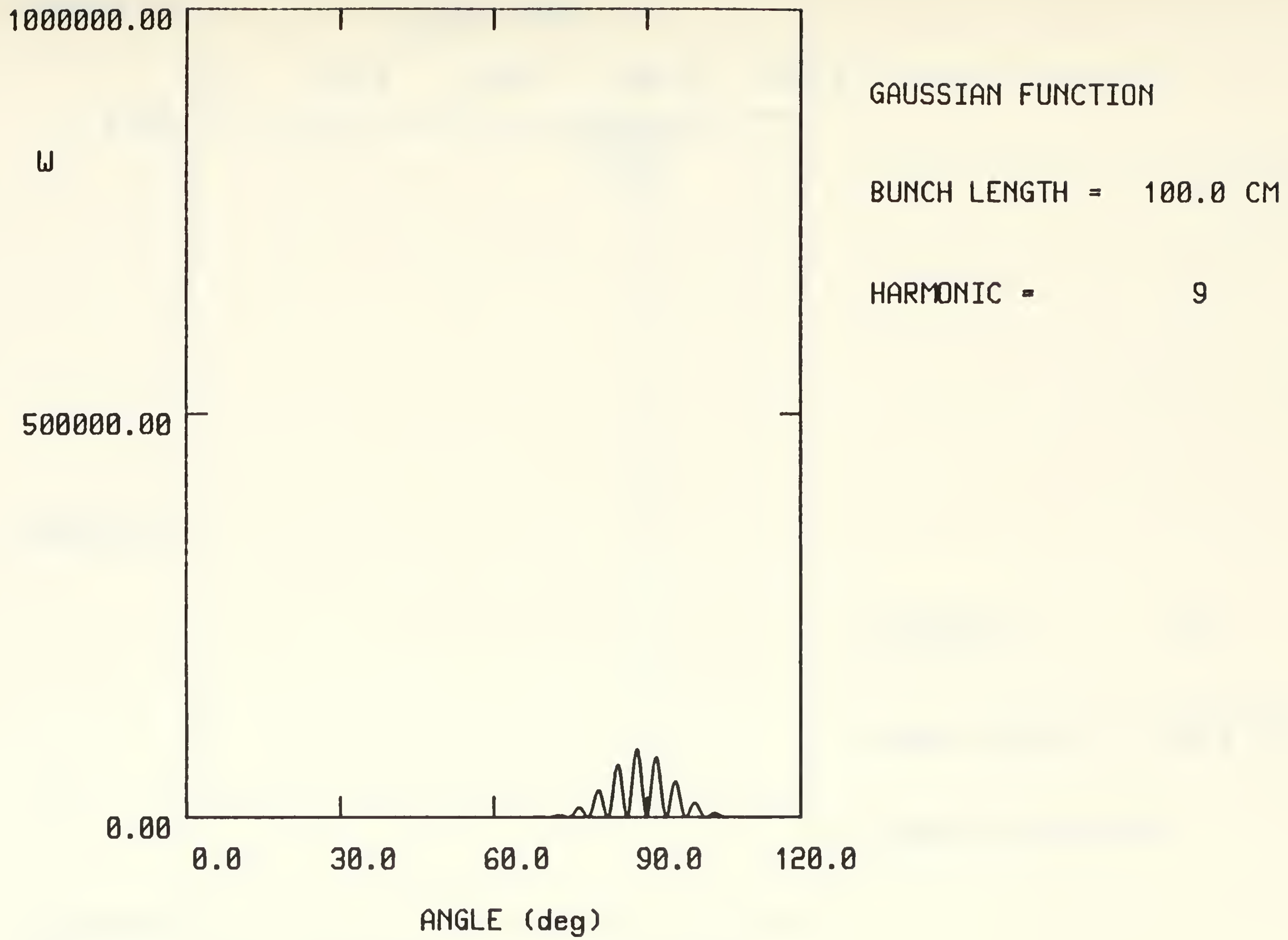

1500000.00
W
750000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 4

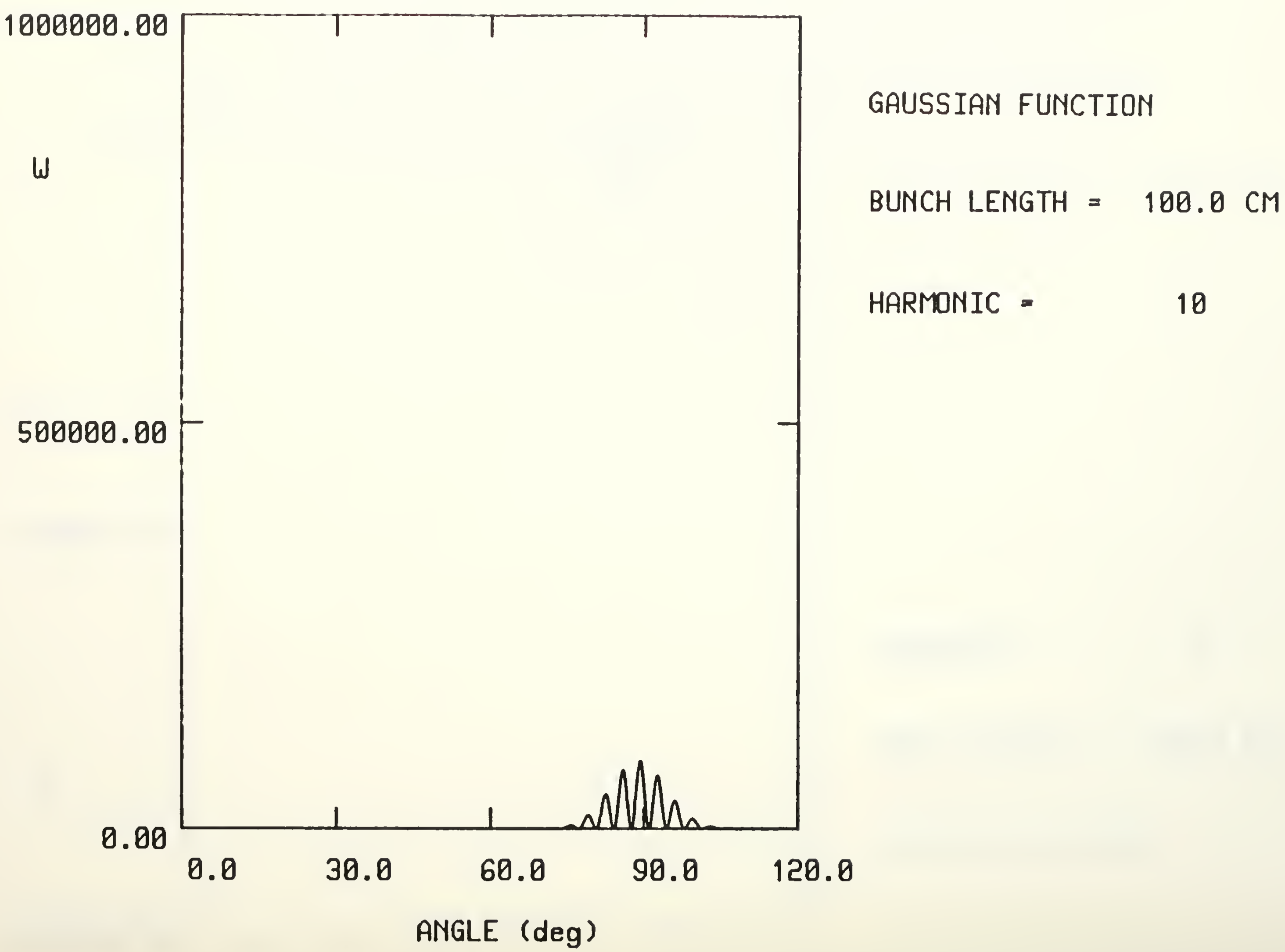

1000000.00
500000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 5

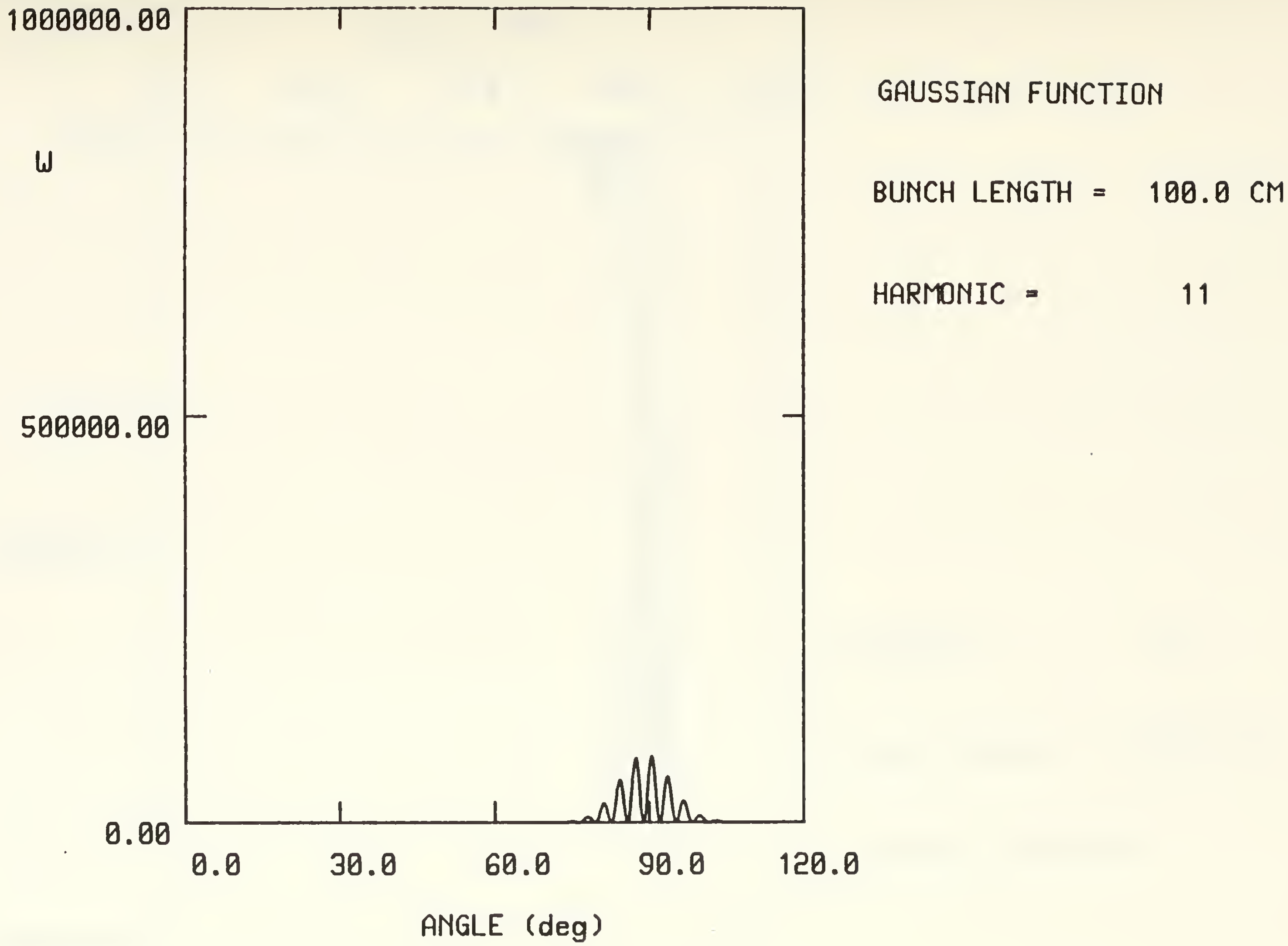

1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 6

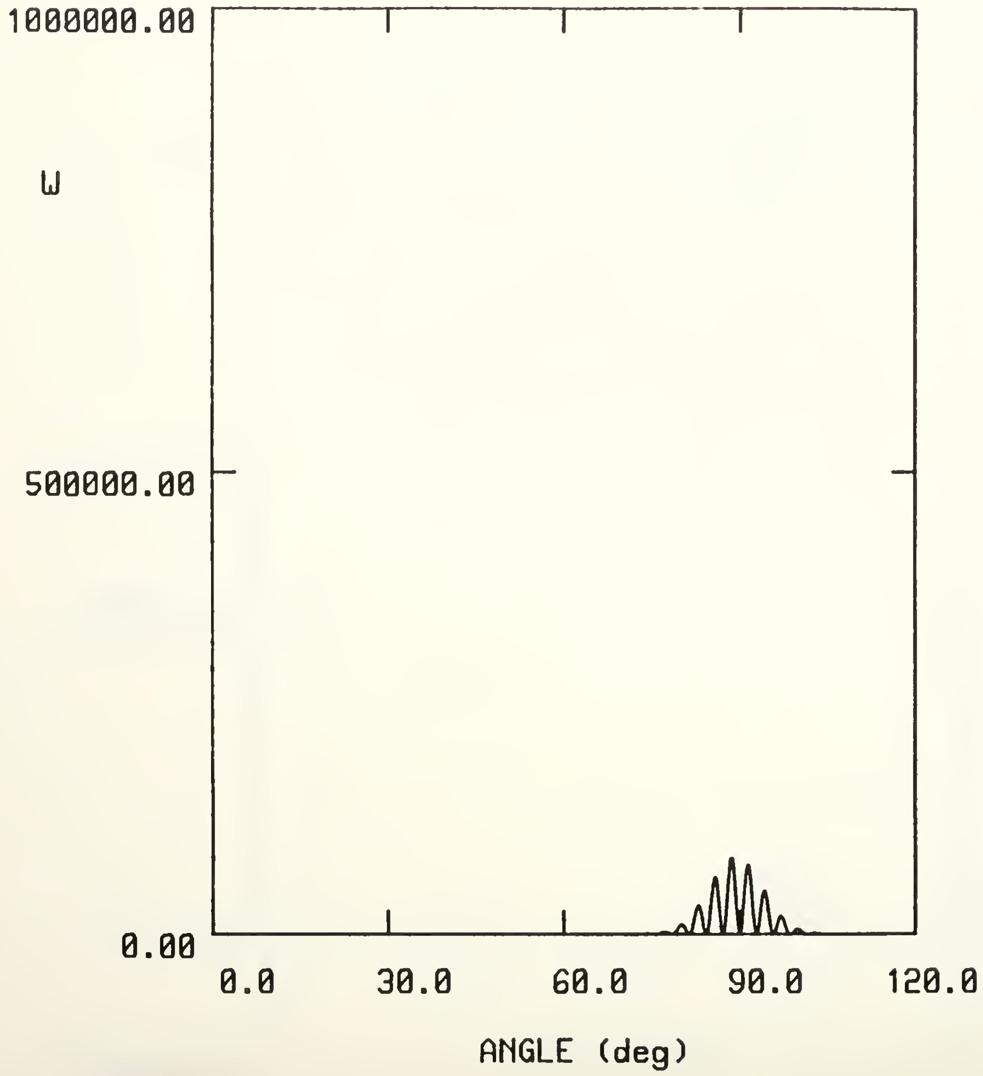

1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 7


1000000.00
500000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 8


1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 9


GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 10
W
1000000.00
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)

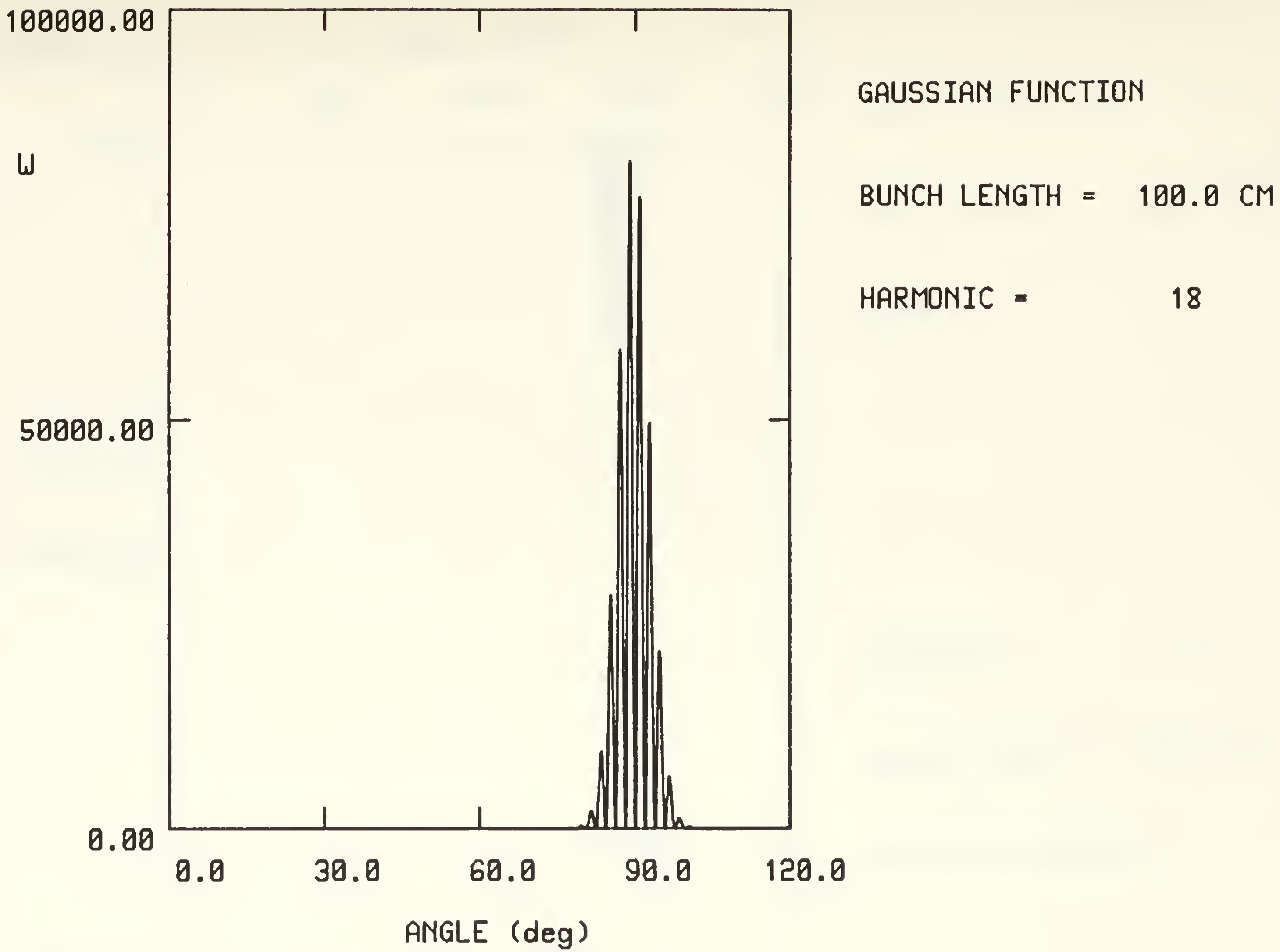

1000000.00
500000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 11

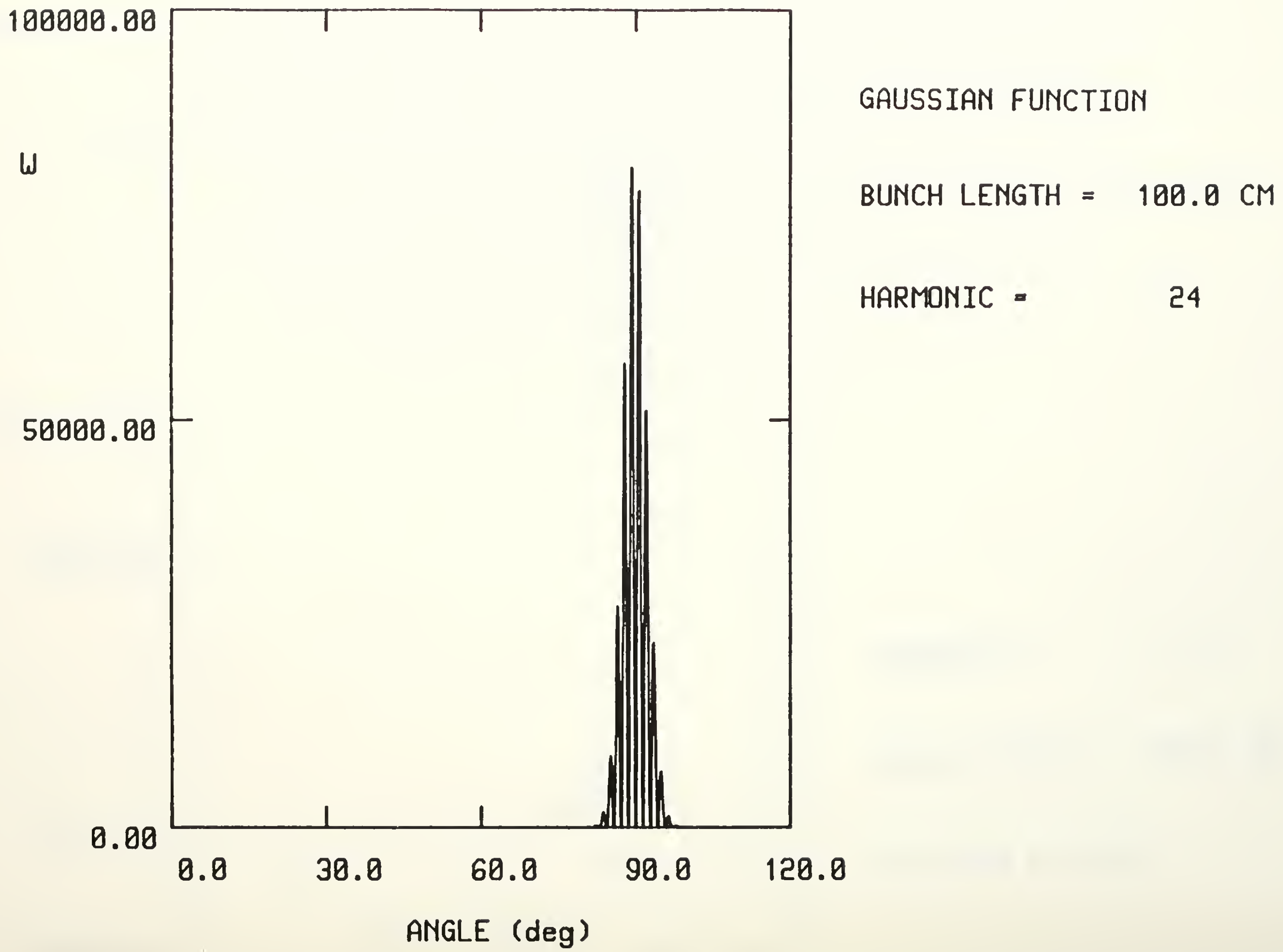


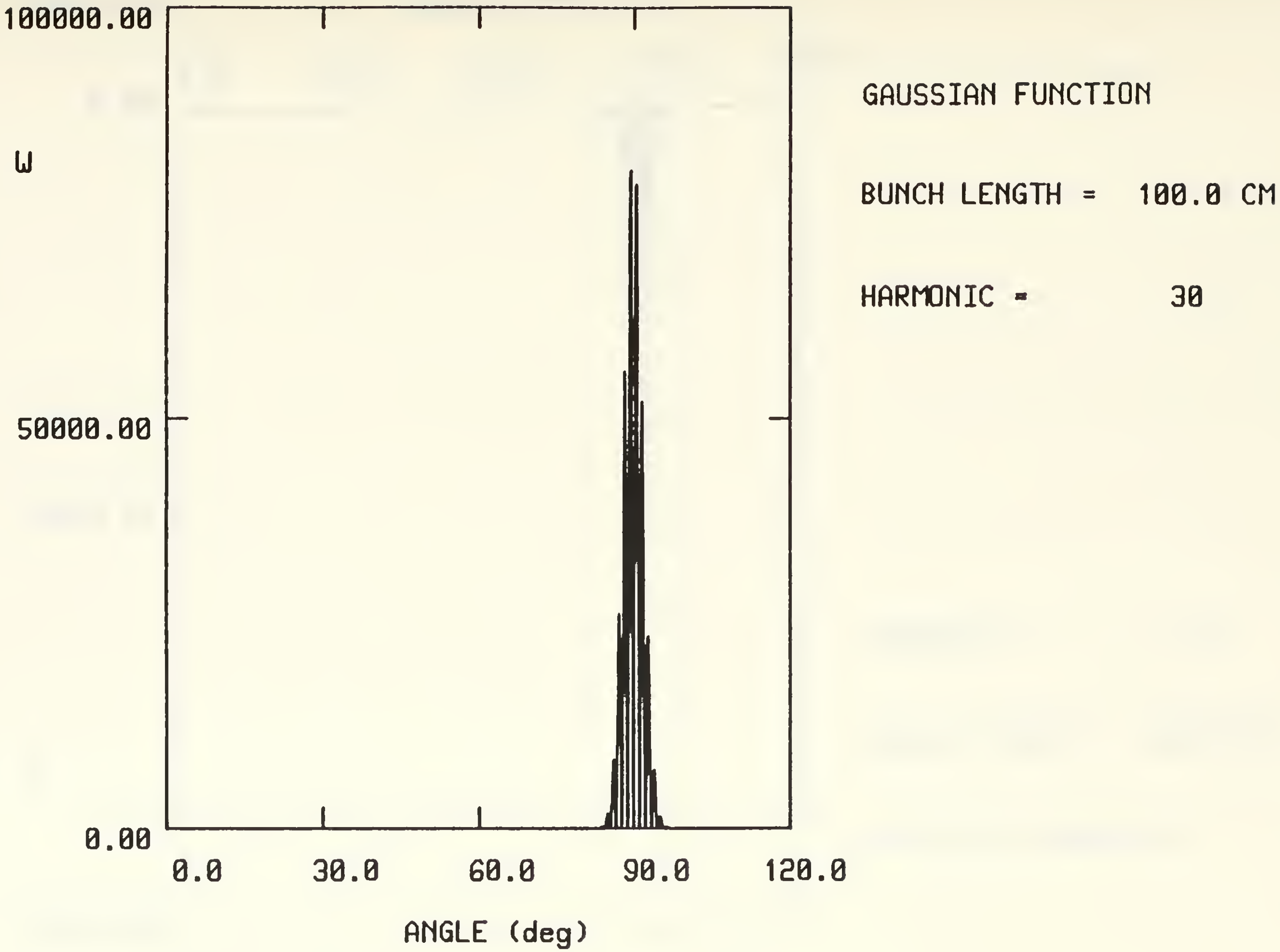
GAUSSIAN FUNCTION

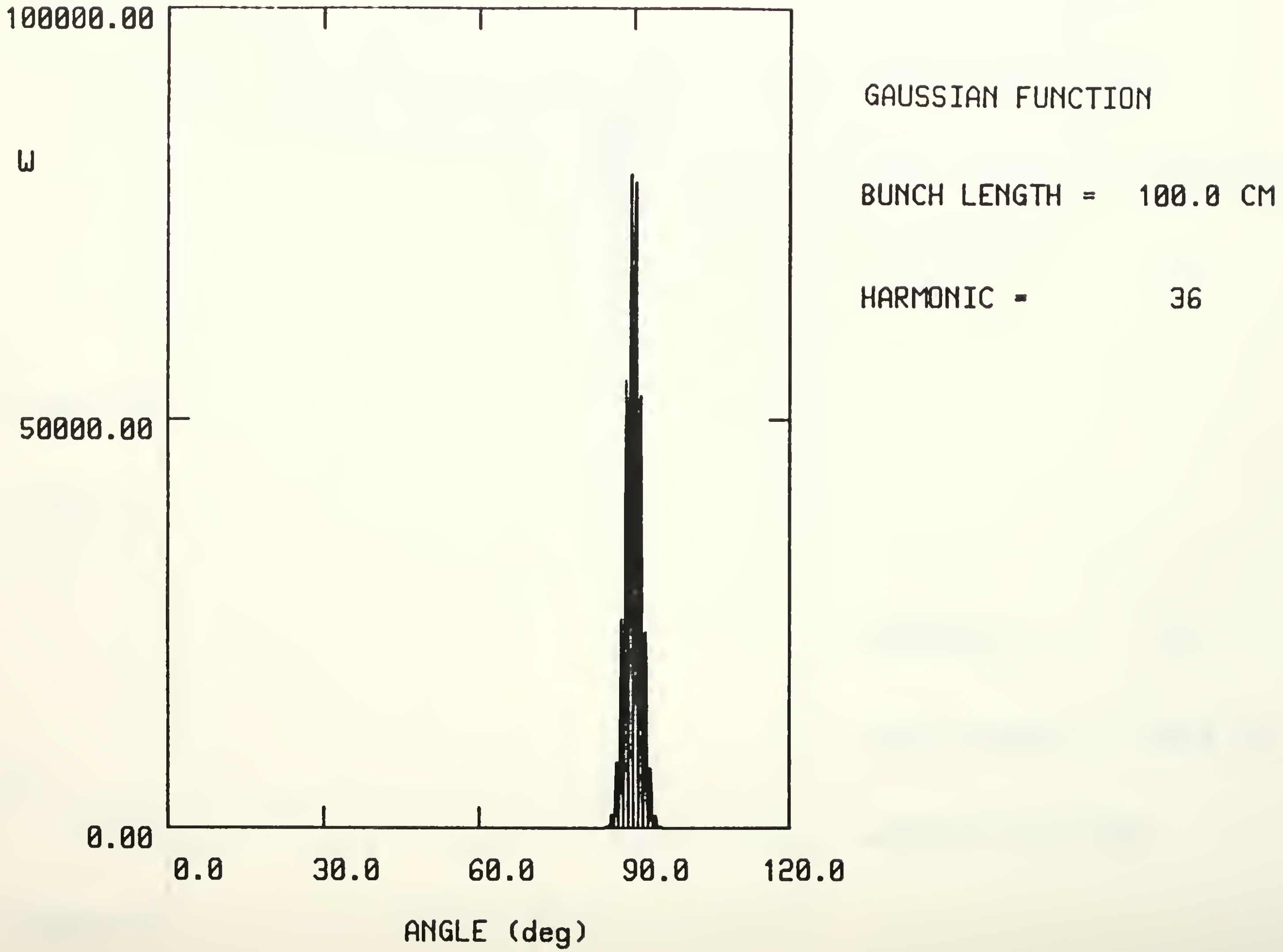
BUNCH LENGTH = 100.0 CM

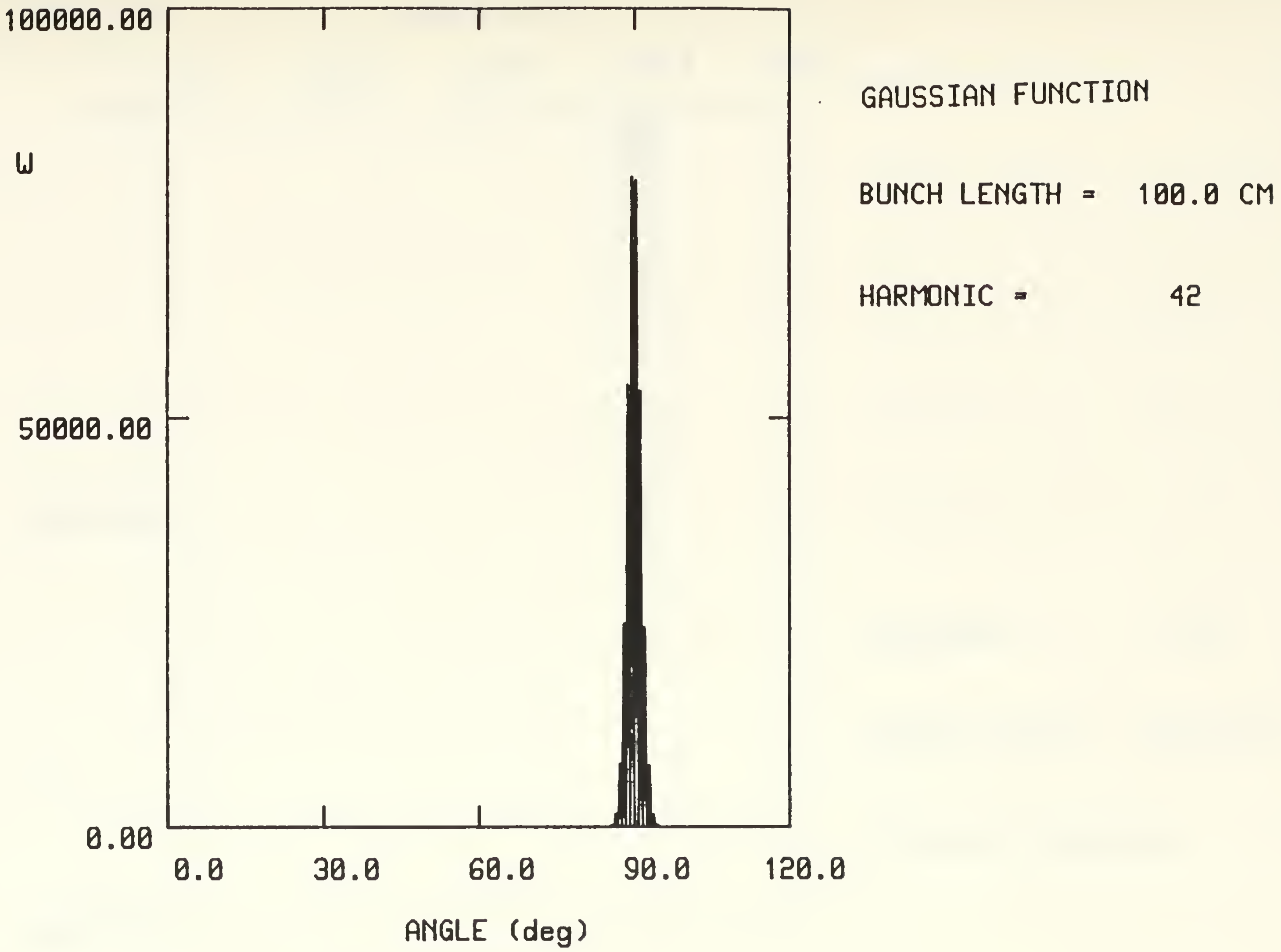
HARMONIC = 12

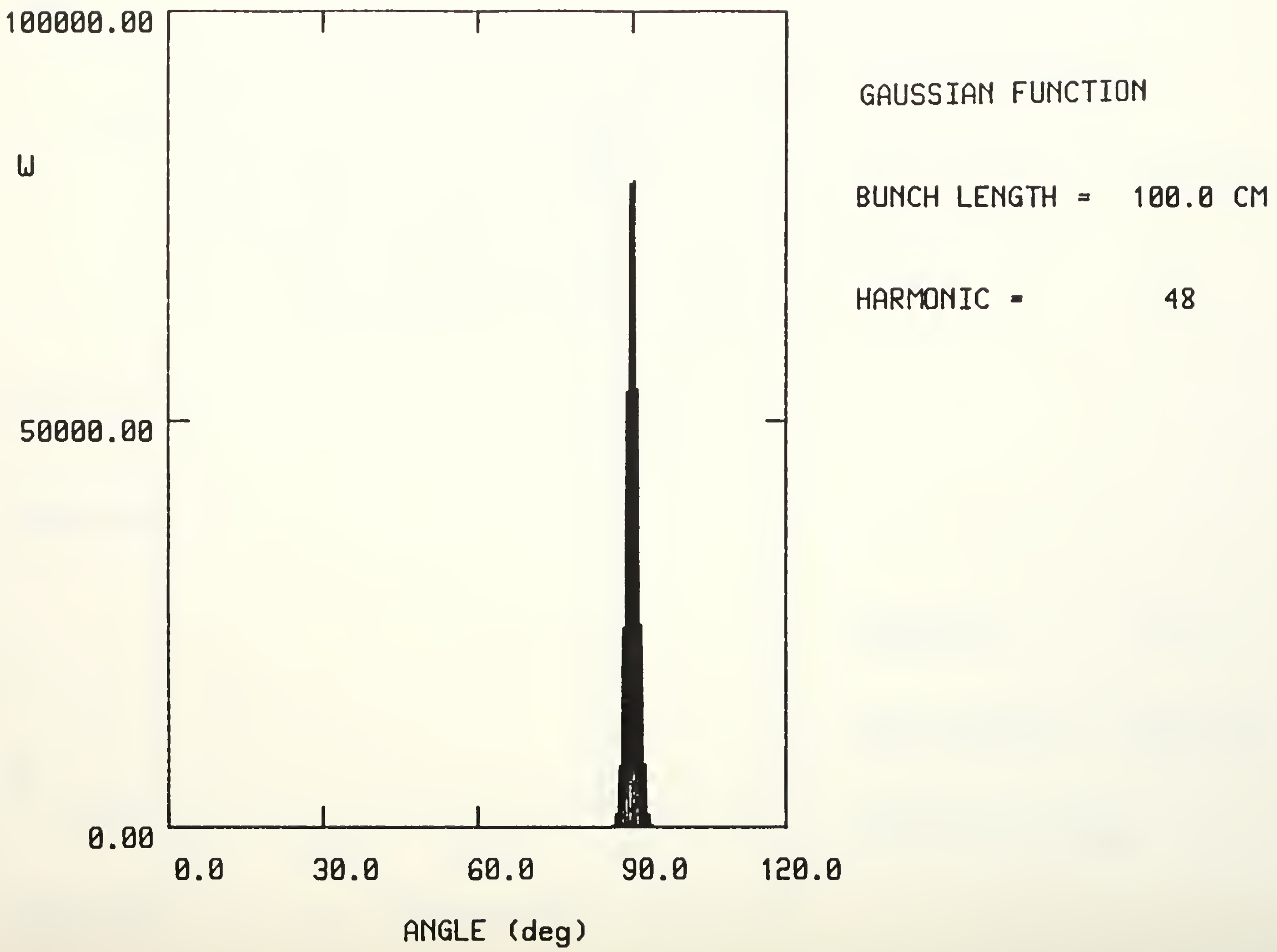

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 18

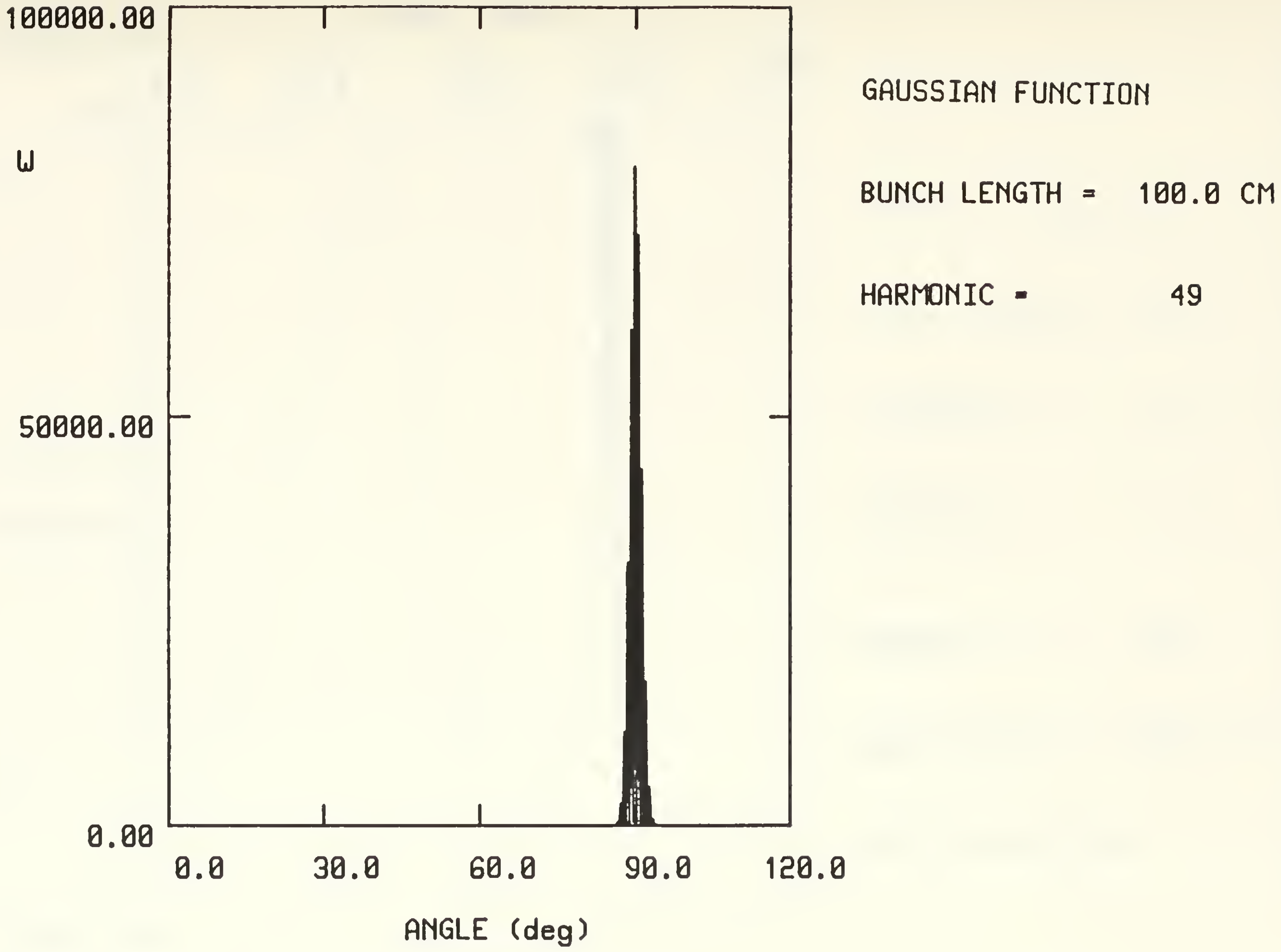

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 24

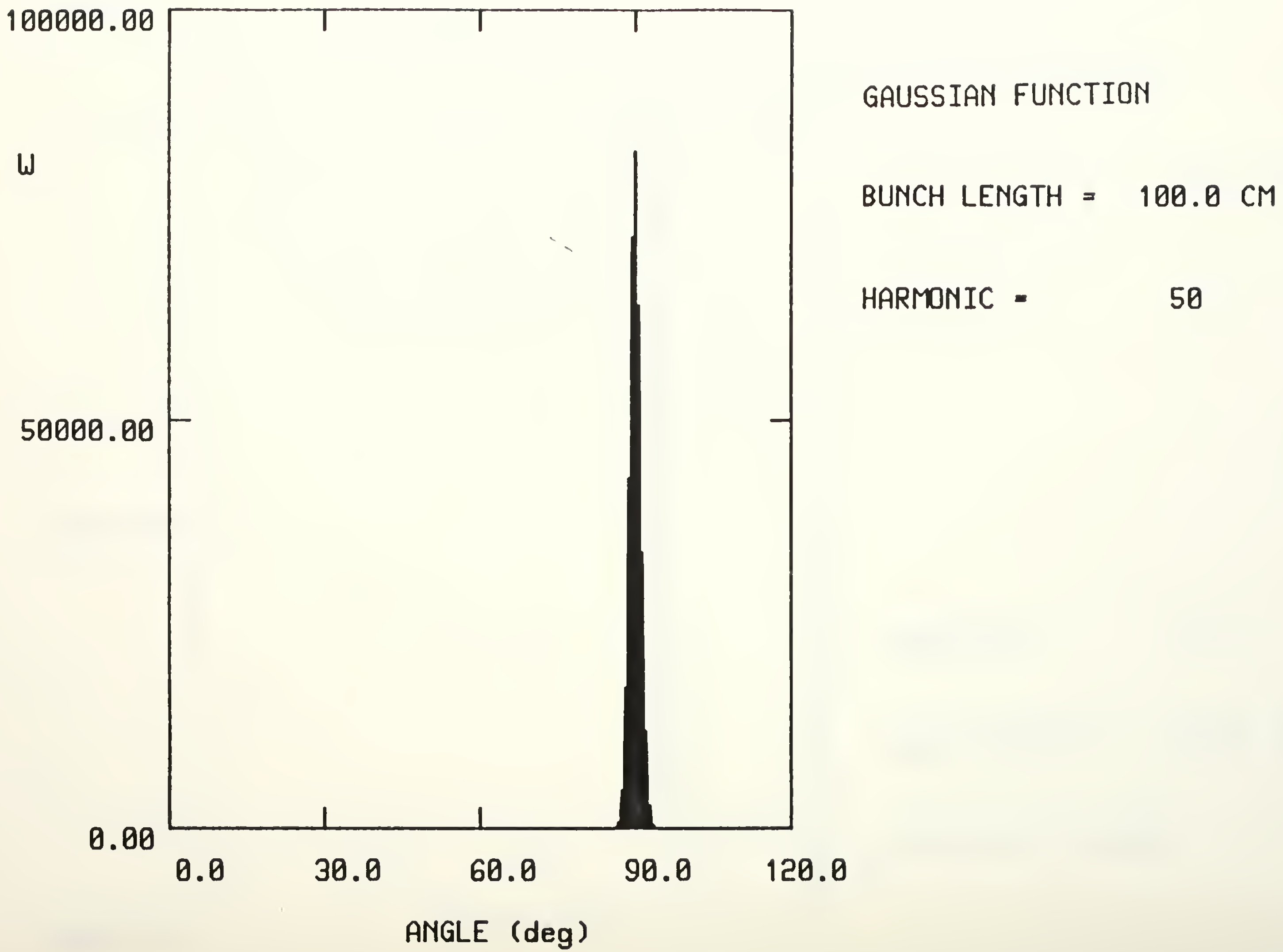

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 30


100000.00
50000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 36

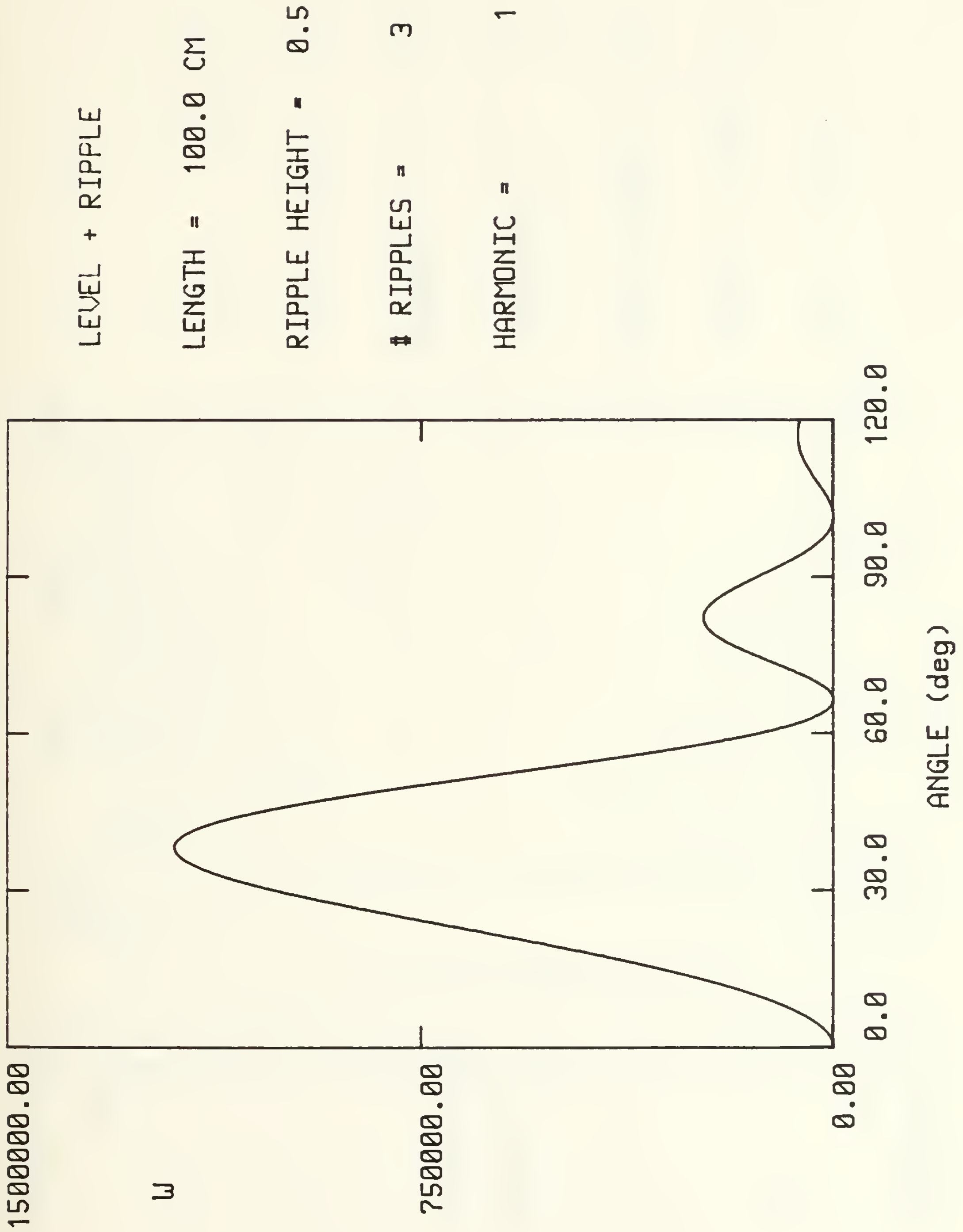

100000.00
50000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 42

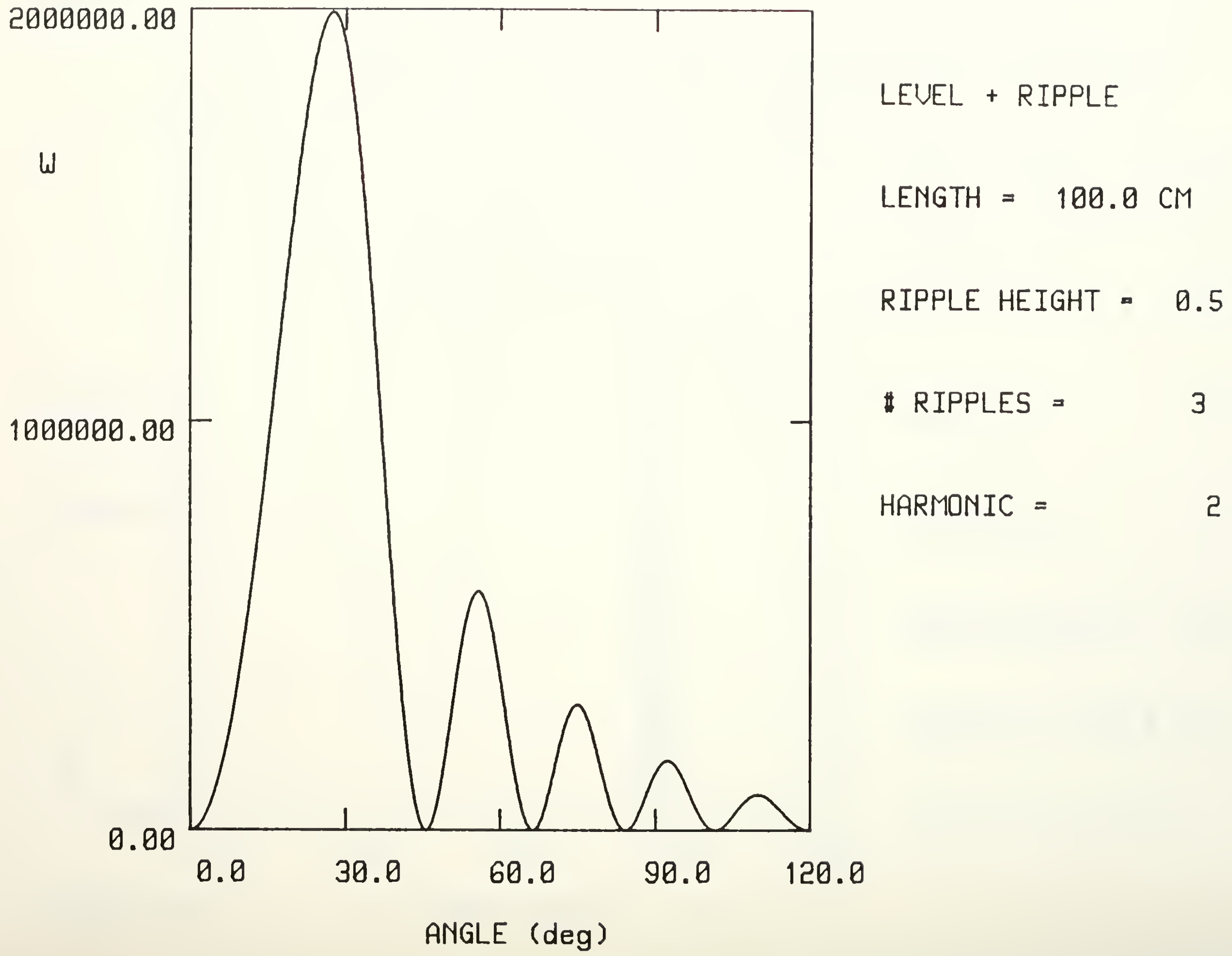

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 48

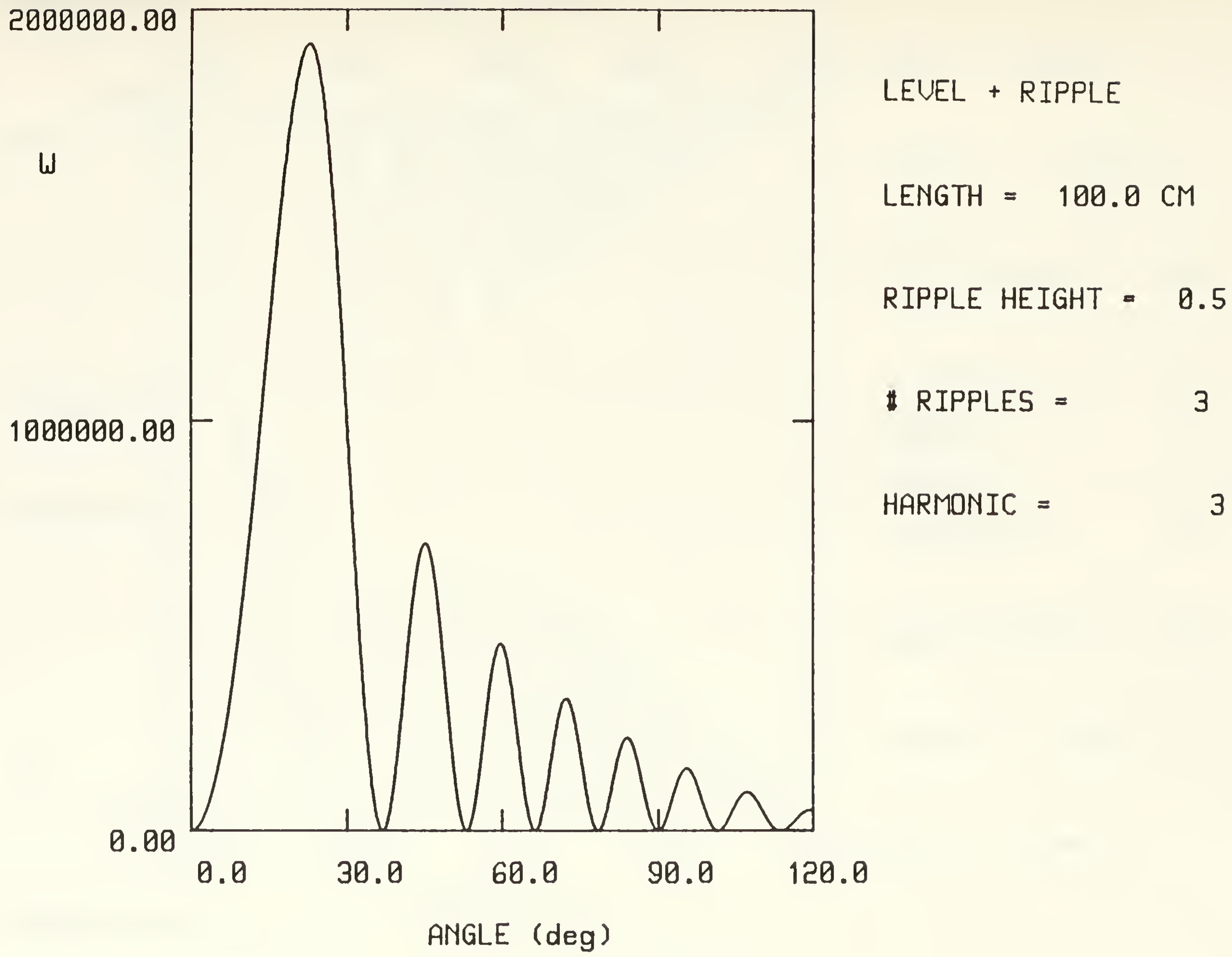

W
100000.00
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 49

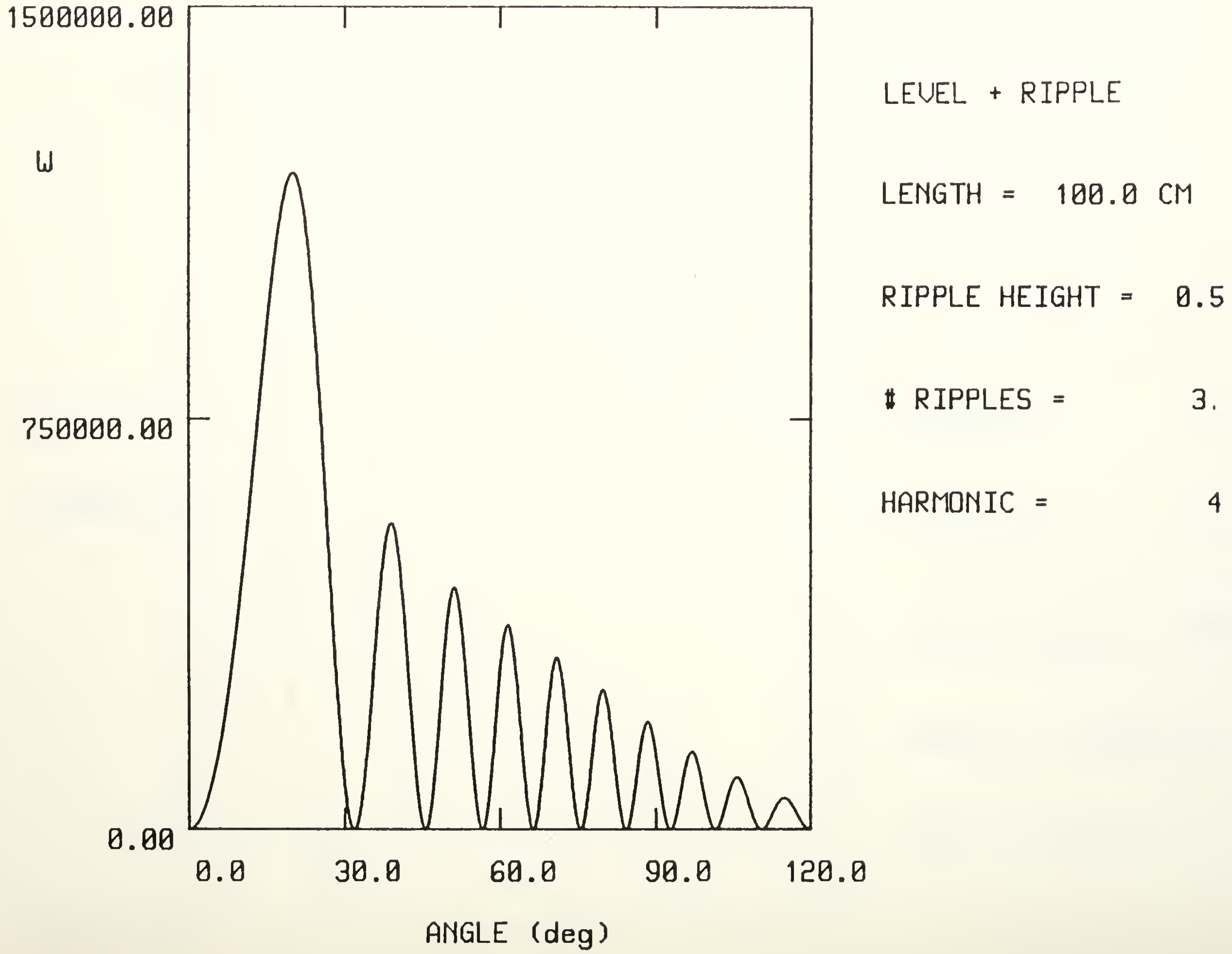

GAUSSIAN FUNCTION
BUNCH LENGTH = 100.0 CM
HARMONIC = 50
W
100000.00
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)

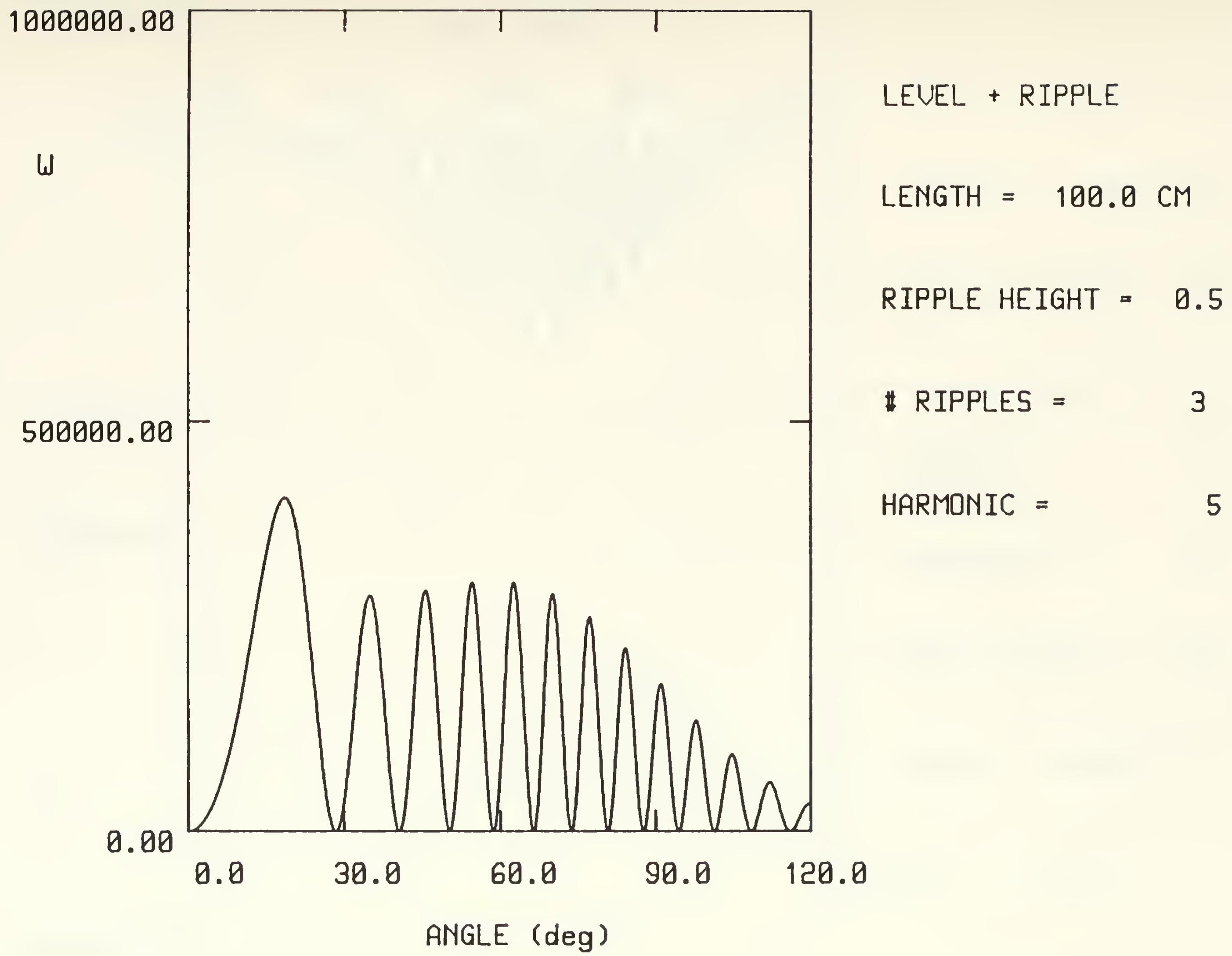
APPENDIX F: LEVEL PLUS RIPPLE COMBINATION FUNCTION

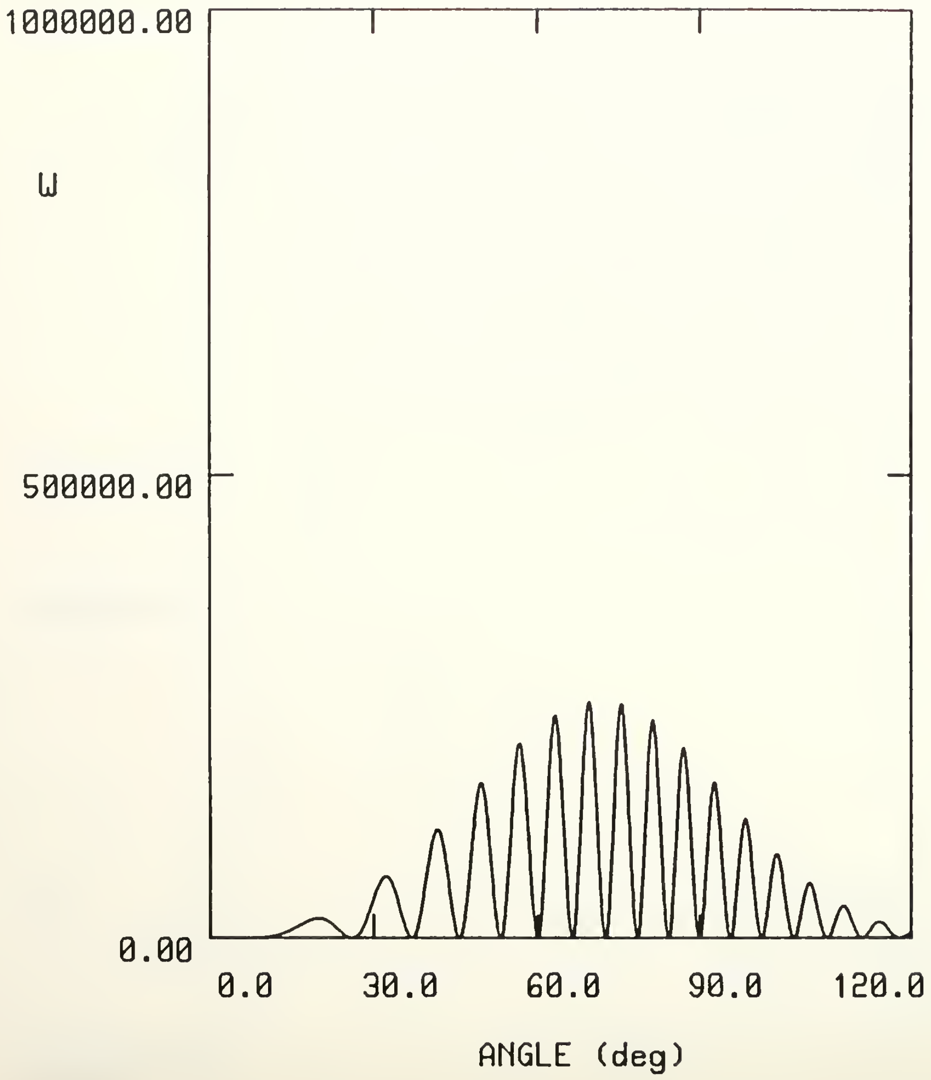



2000000.00
W
1000000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3
HARMONIC = 2


2000000.00
1000000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3
HARMONIC = 3


1500000.00
W
750000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3.
HARMONIC = 4


1000000.00
500000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3
HARMONIC = 5



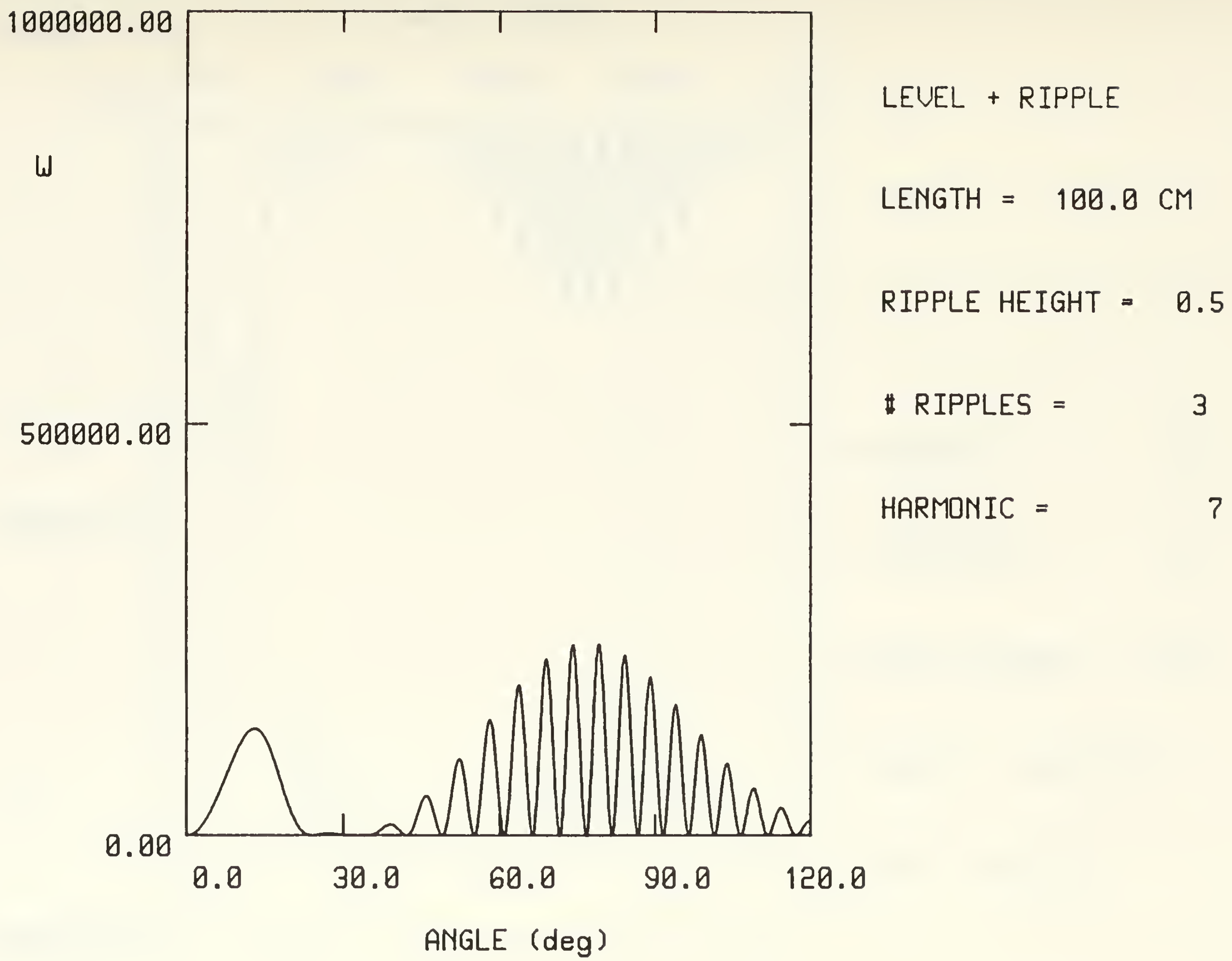
LEVEL + RIPPLE

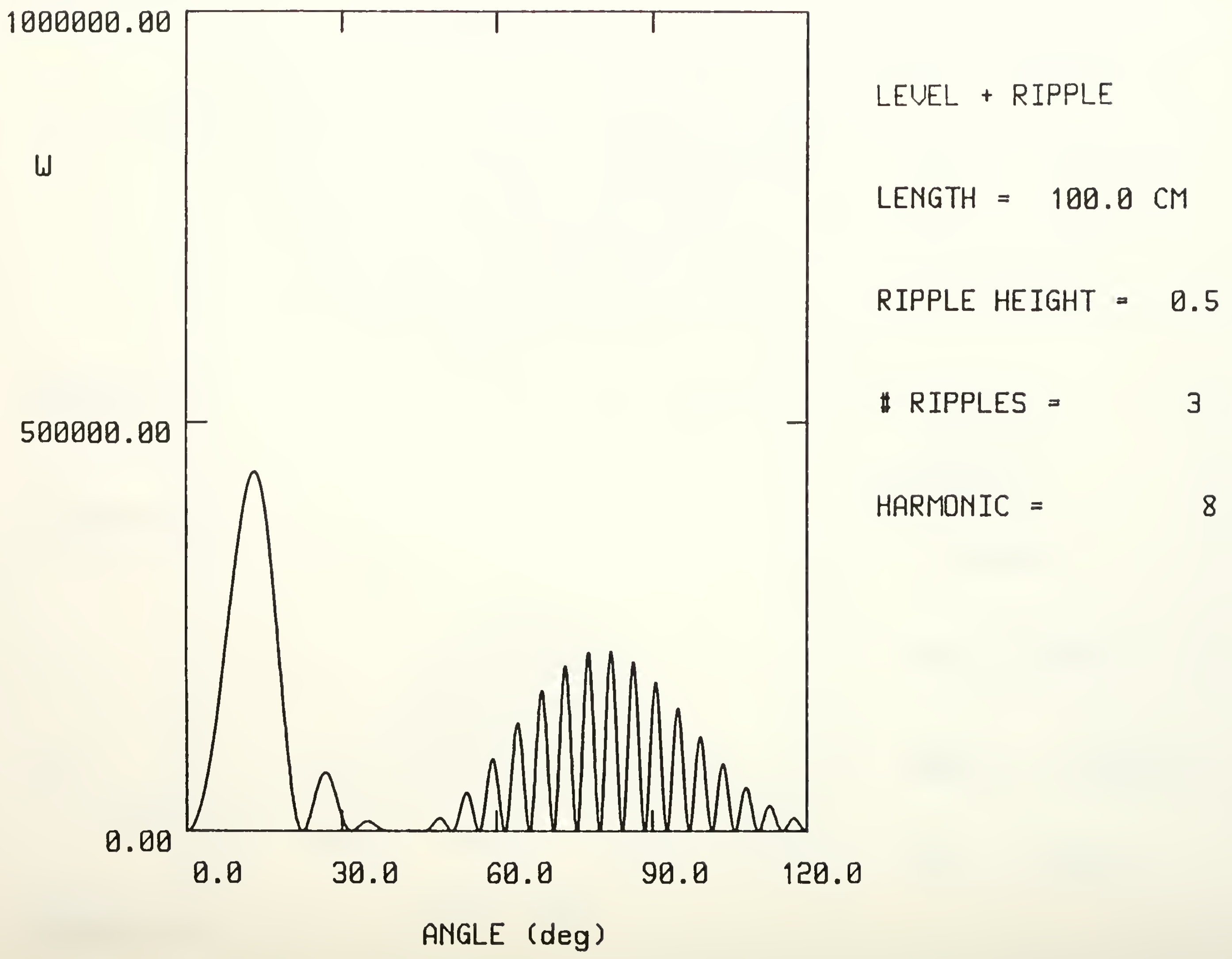
LENGTH = 100.0 CM

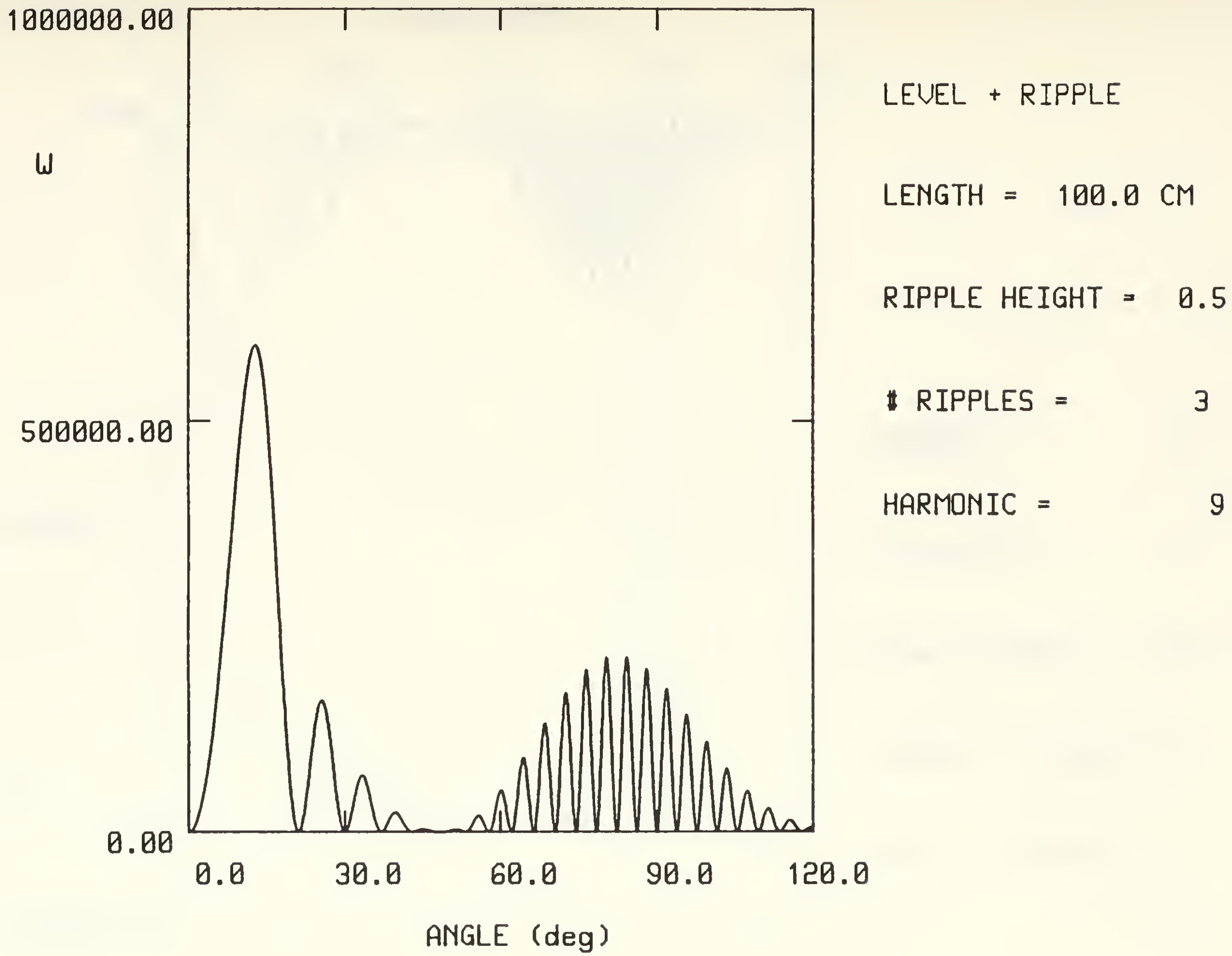
RIPPLE HEIGHT = 0.5

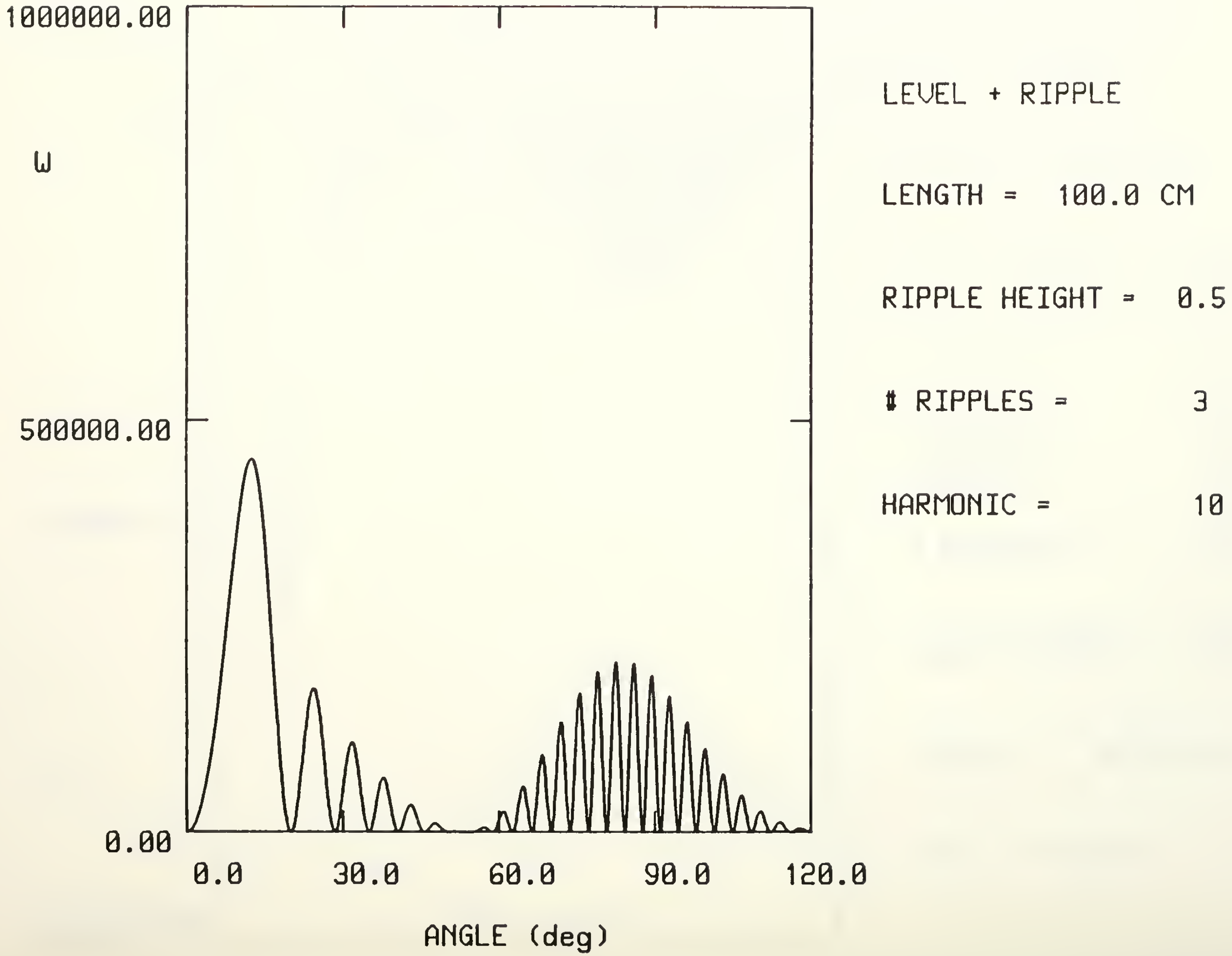
RIPPLES = 3

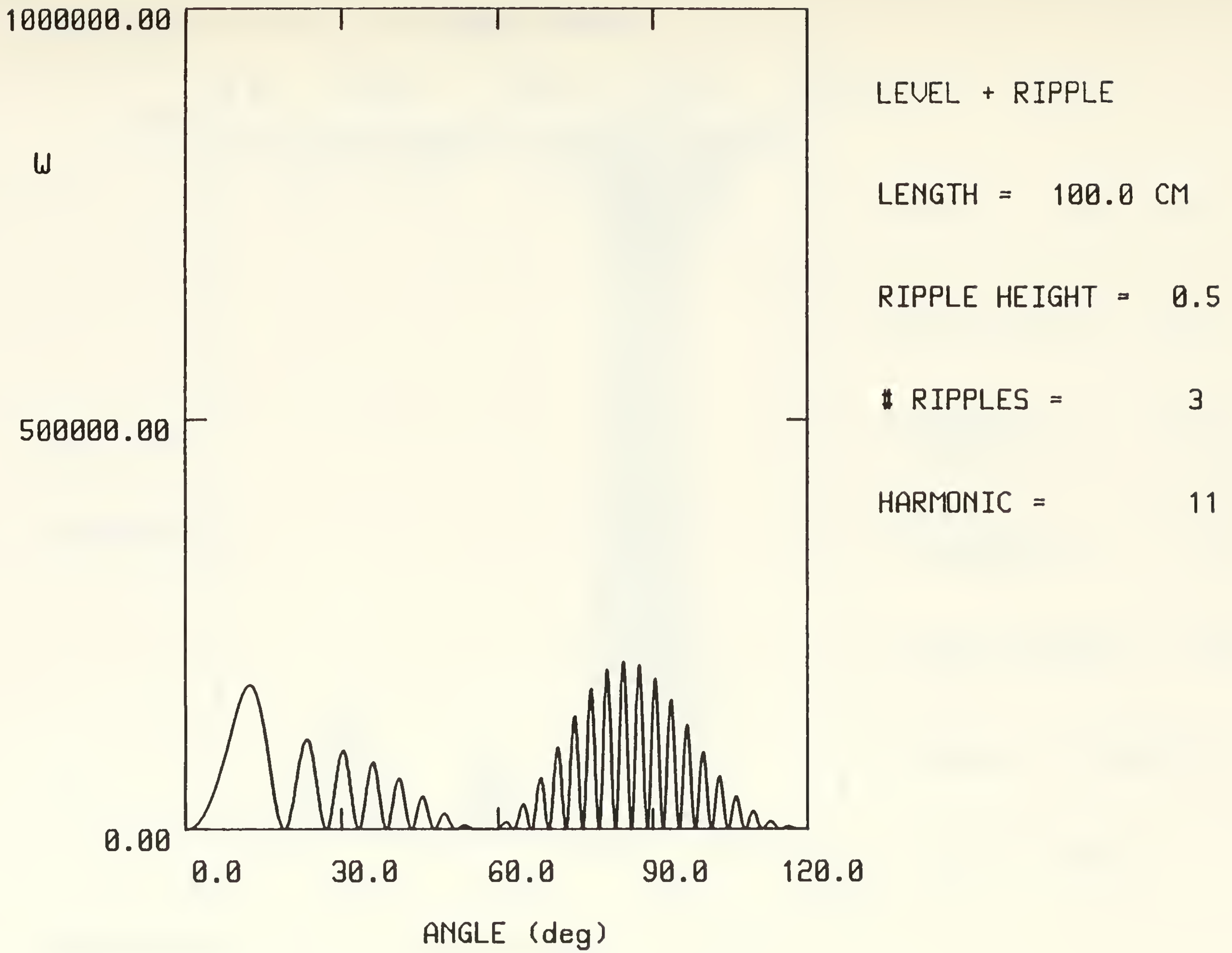
HARMONIC = 6

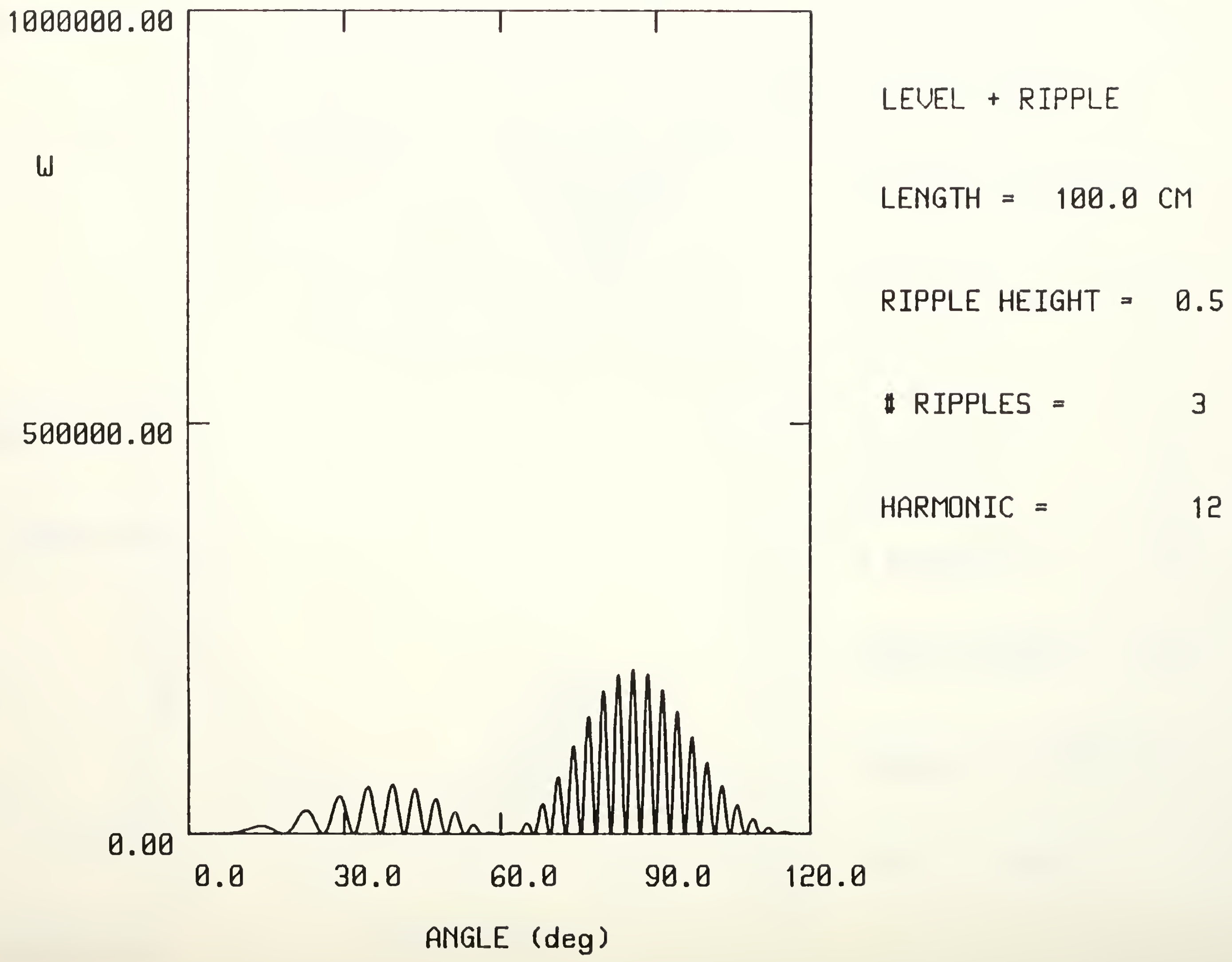

1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3
HARMONIC = 7

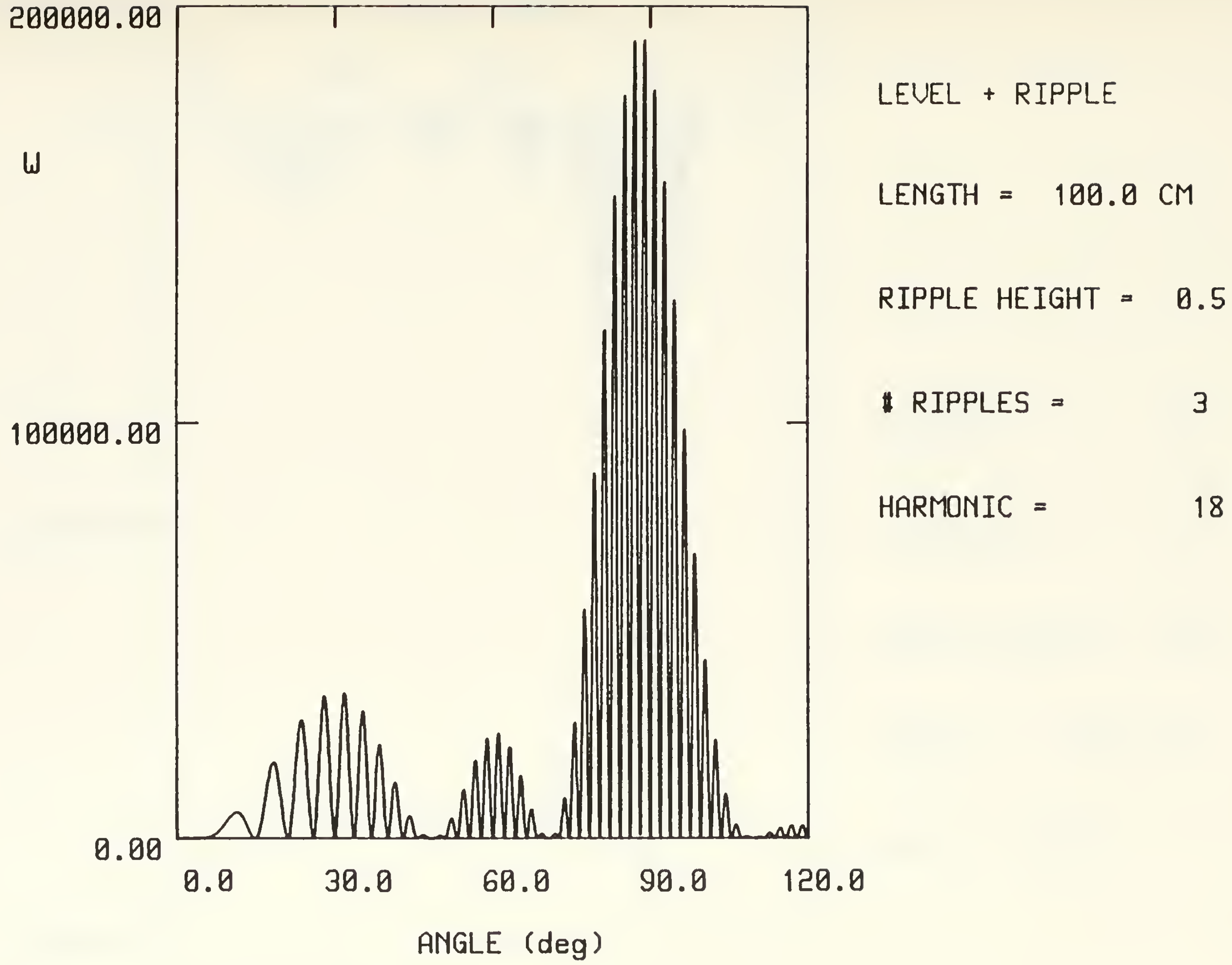

1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3
HARMONIC = 8

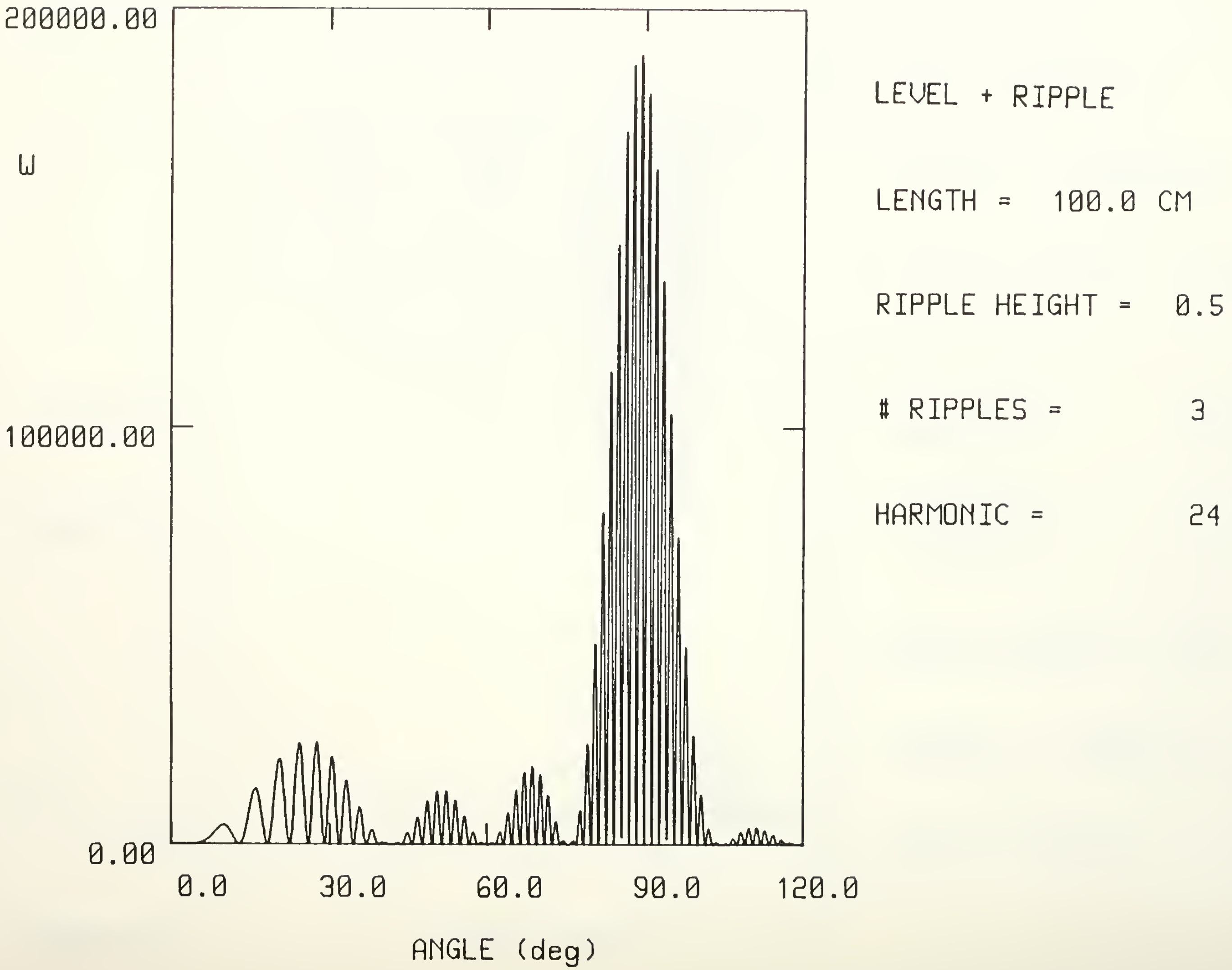

1000000.00
500000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3
HARMONIC = 9

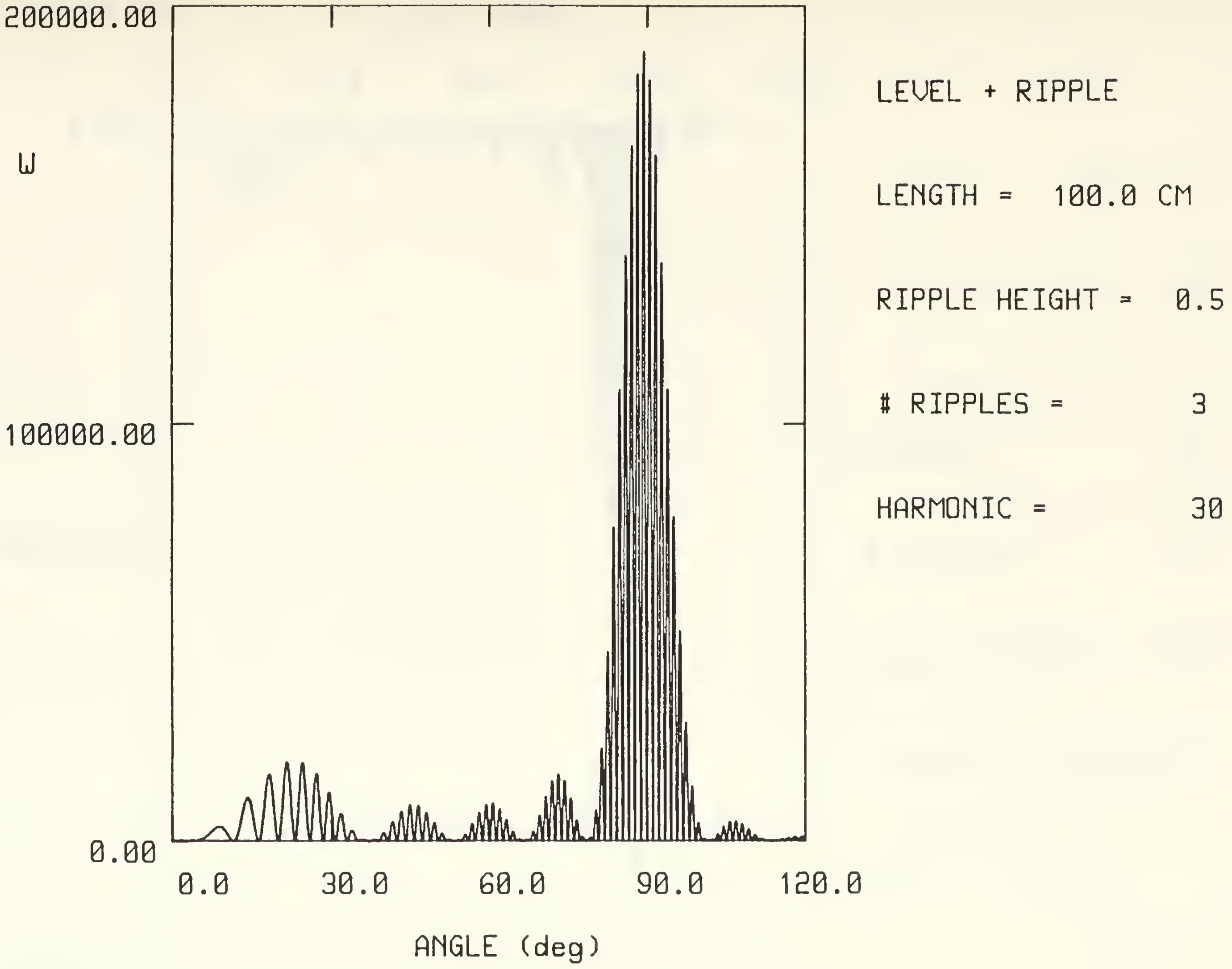

1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3
HARMONIC = 10

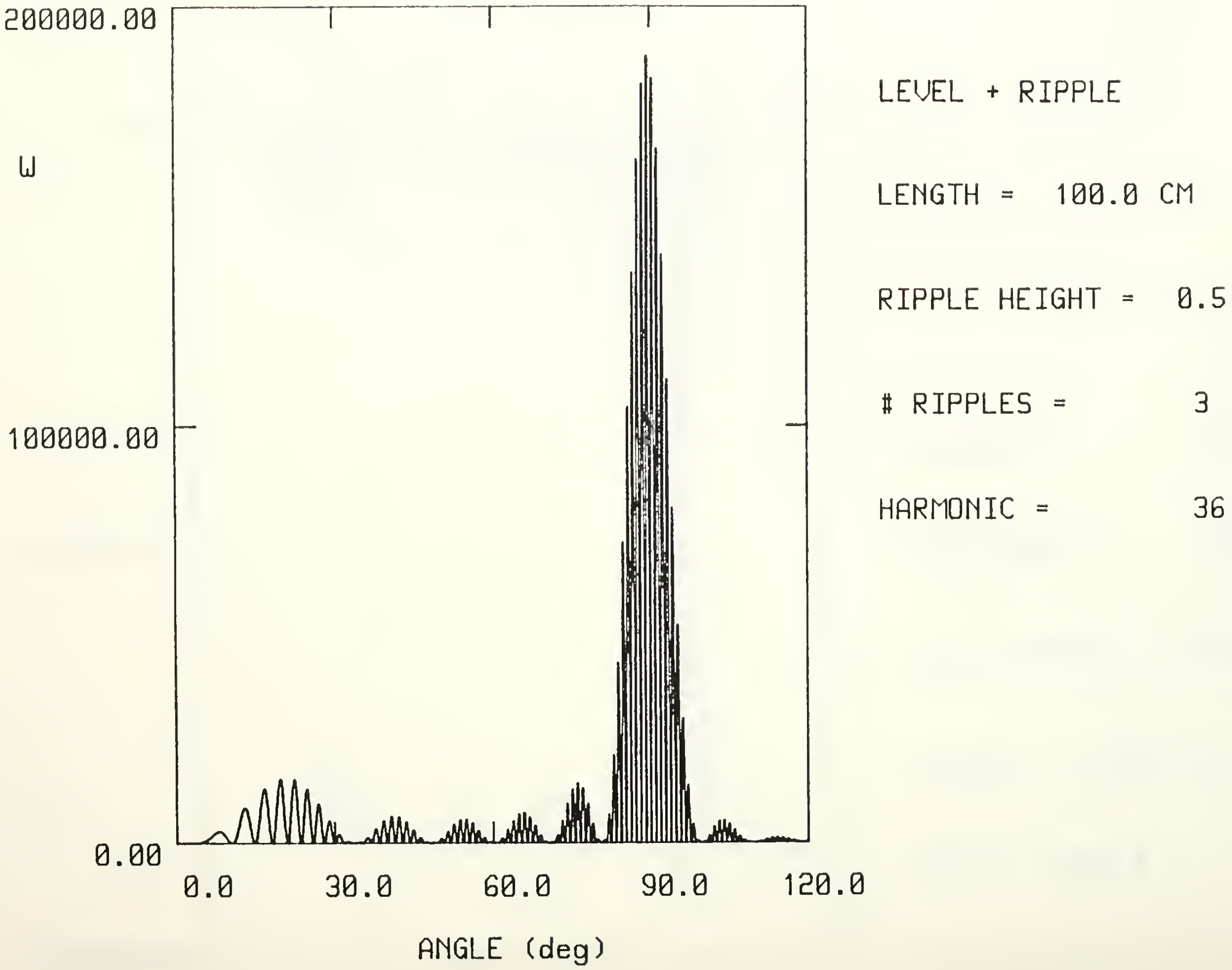

1000000.00
500000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3
HARMONIC = 11

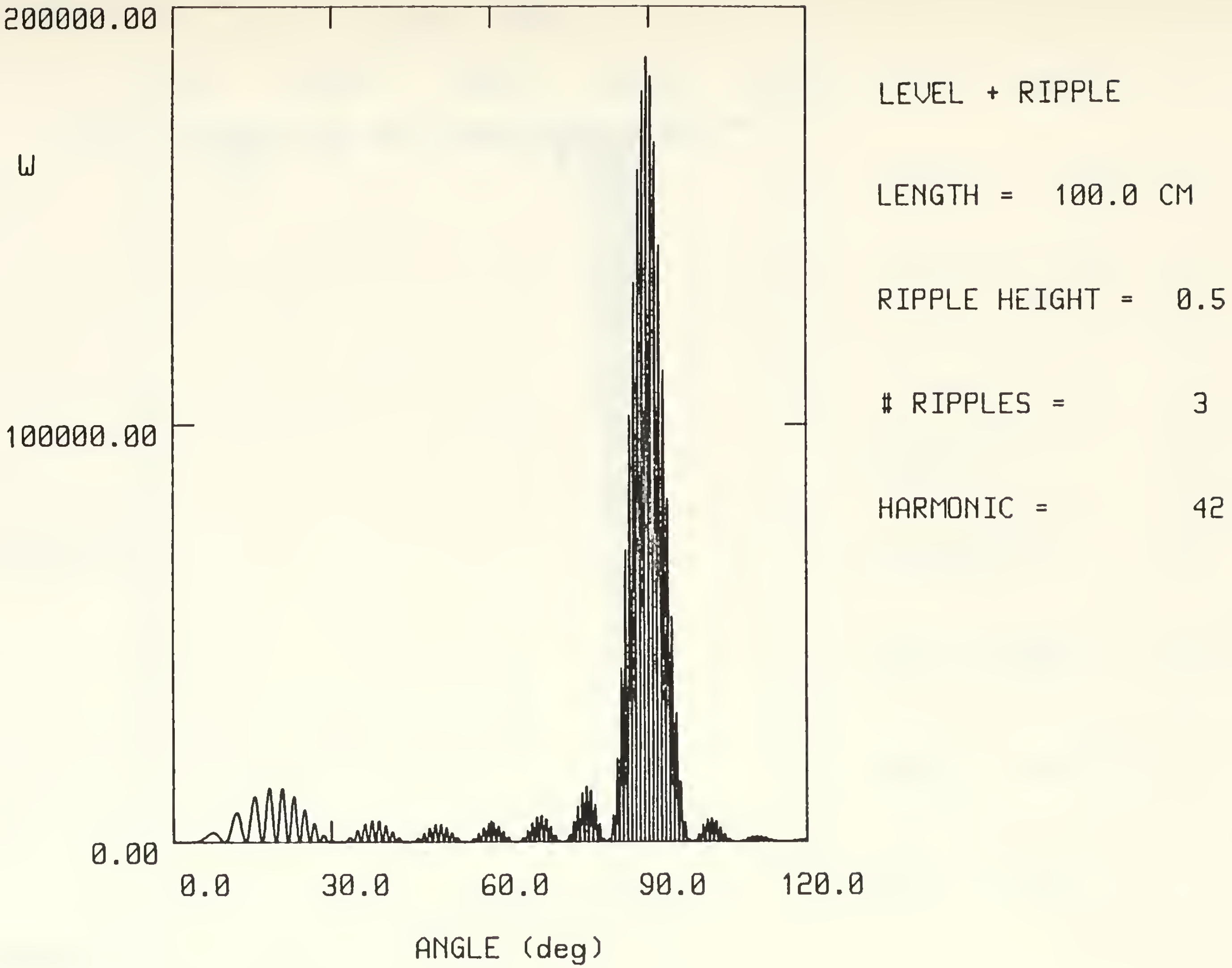

1000000.00
500000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3
HARMONIC = 12

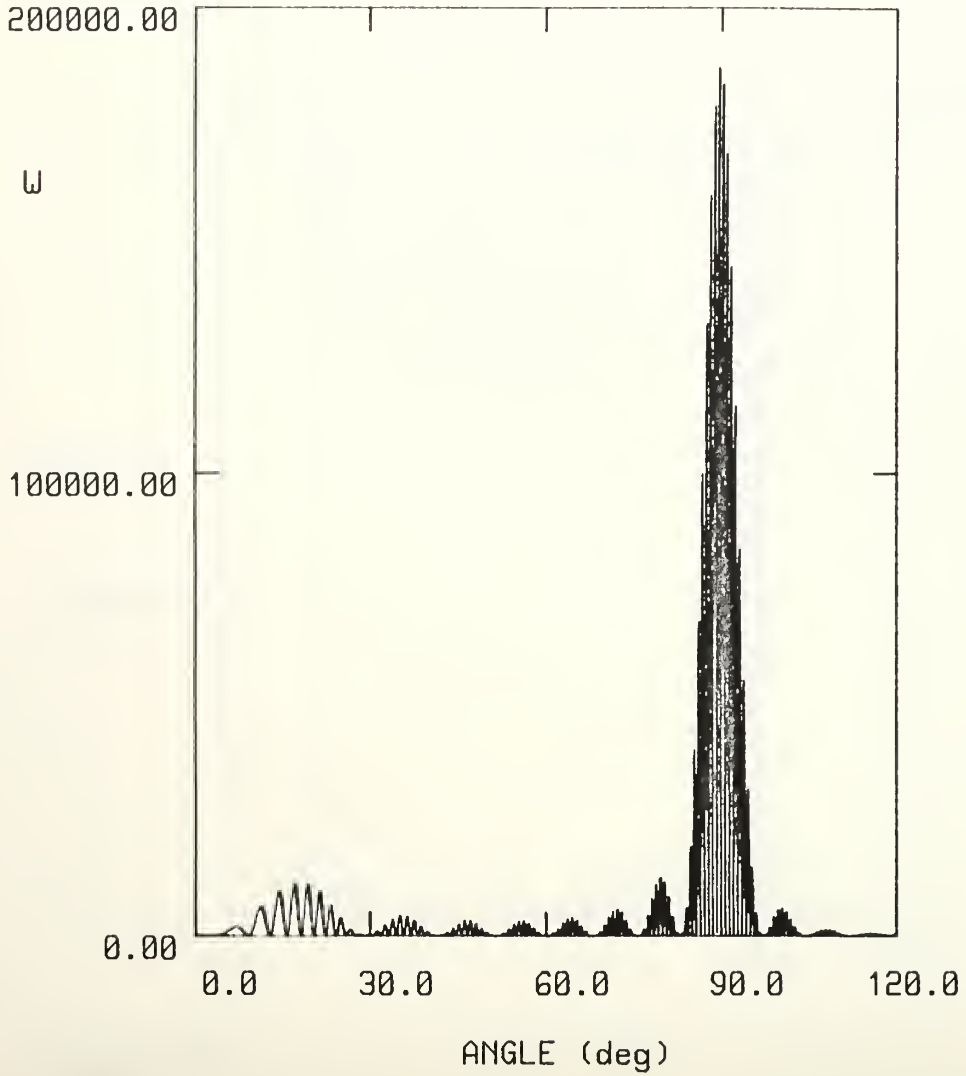

200000.00
W
100000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3
HARMONIC = 18


200000.00
W
100000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3
HARMONIC = 24


200000.00
W
100000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3
HARMONIC = 30


200000.00
W
100000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3
HARMONIC = 36


200000.00
W
100000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3
HARMONIC = 42



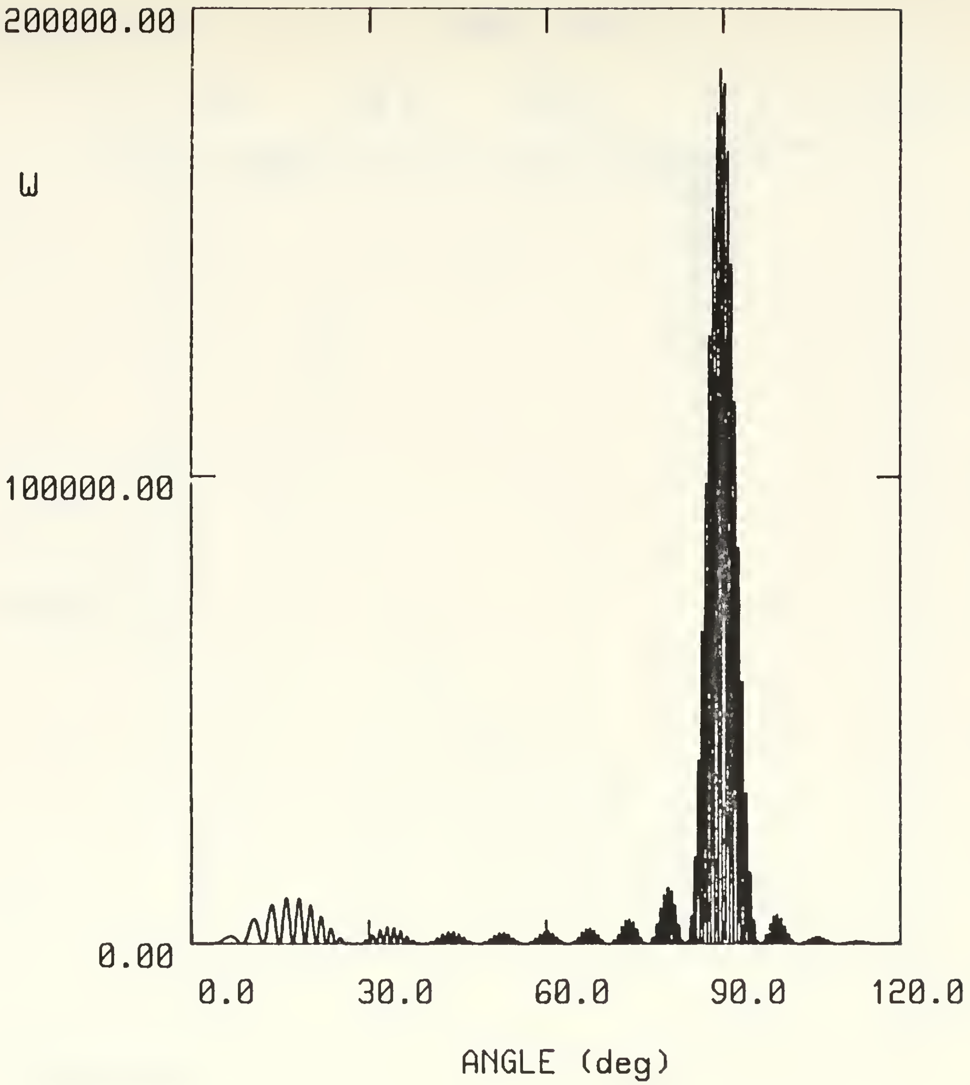
LEVEL + RIPPLE

LENGTH = 100.0 CM

RIPPLE HEIGHT = 0.5

RIPPLES = 3

HARMONIC = 48



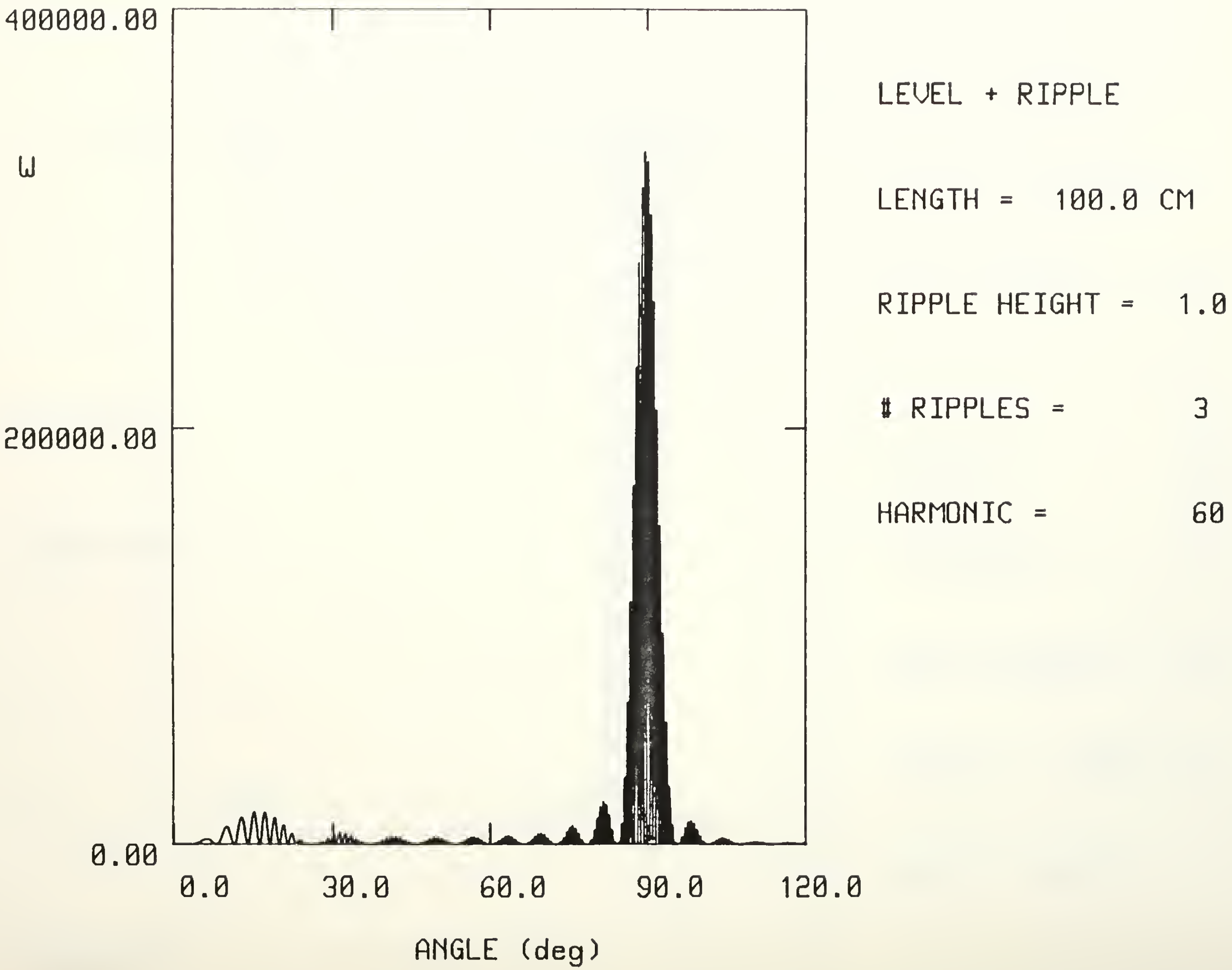
LEVEL + RIPPLE

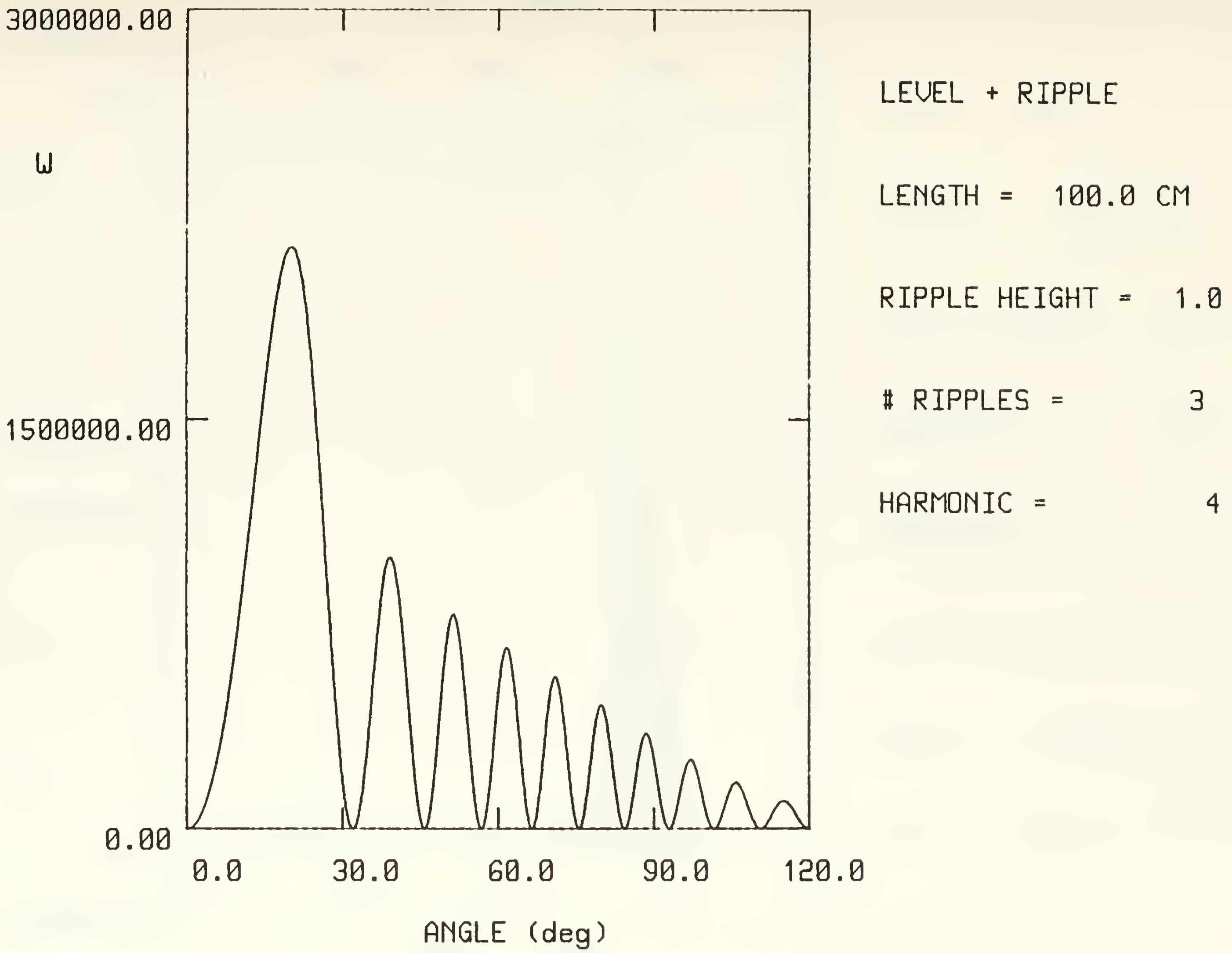
LENGTH = 100.0 CM

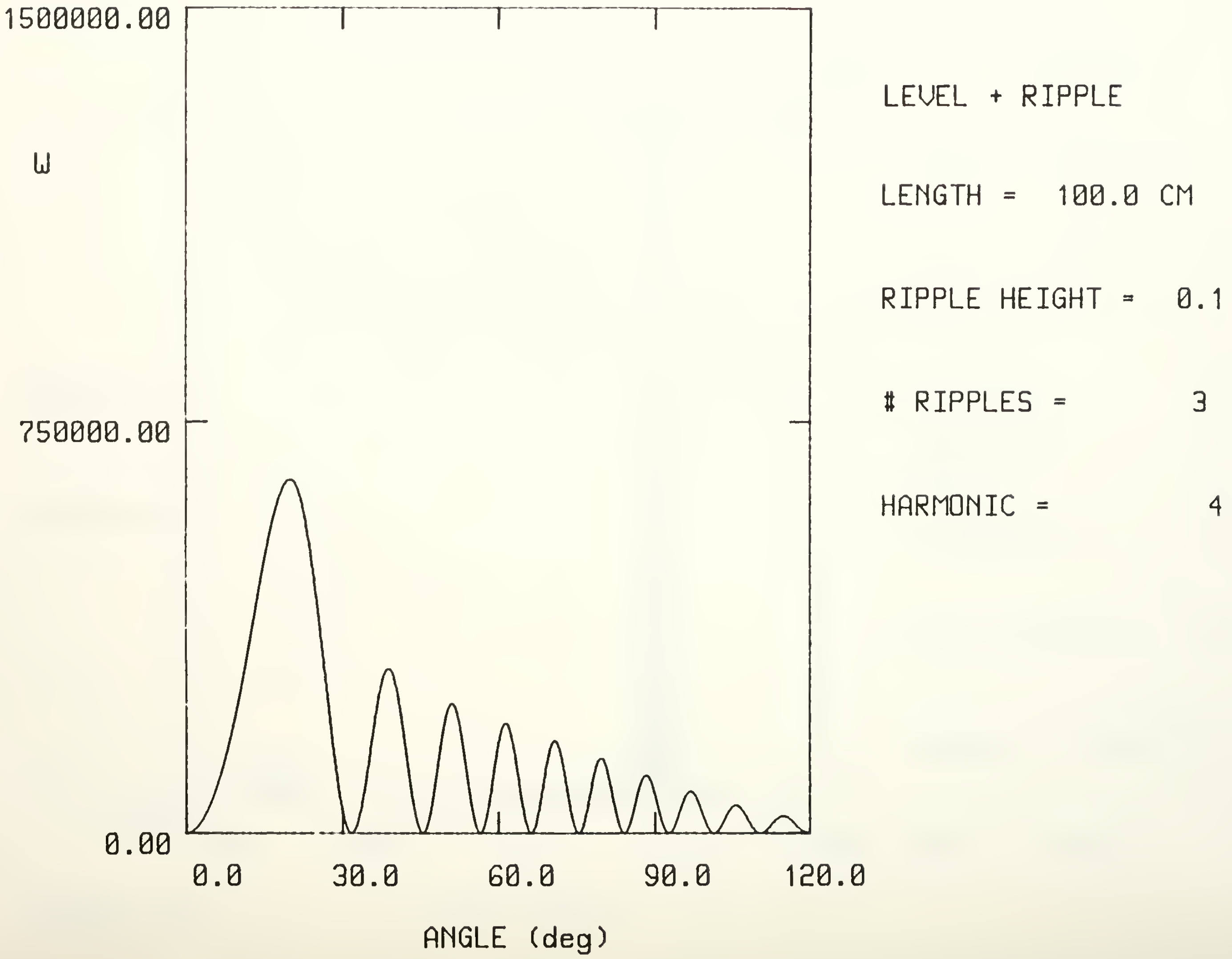
RIPPLE HEIGHT = 0.5

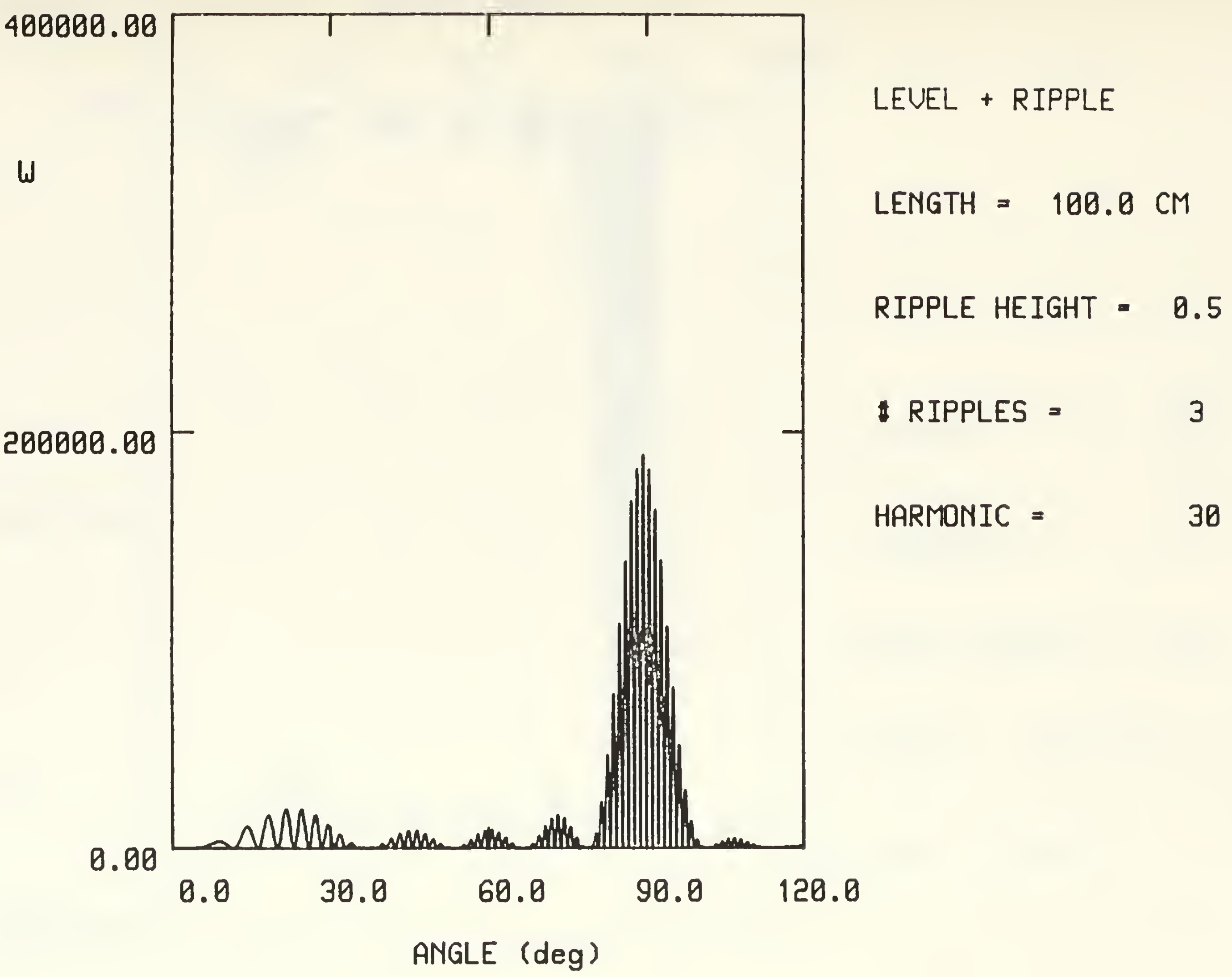
RIPPLES = 3

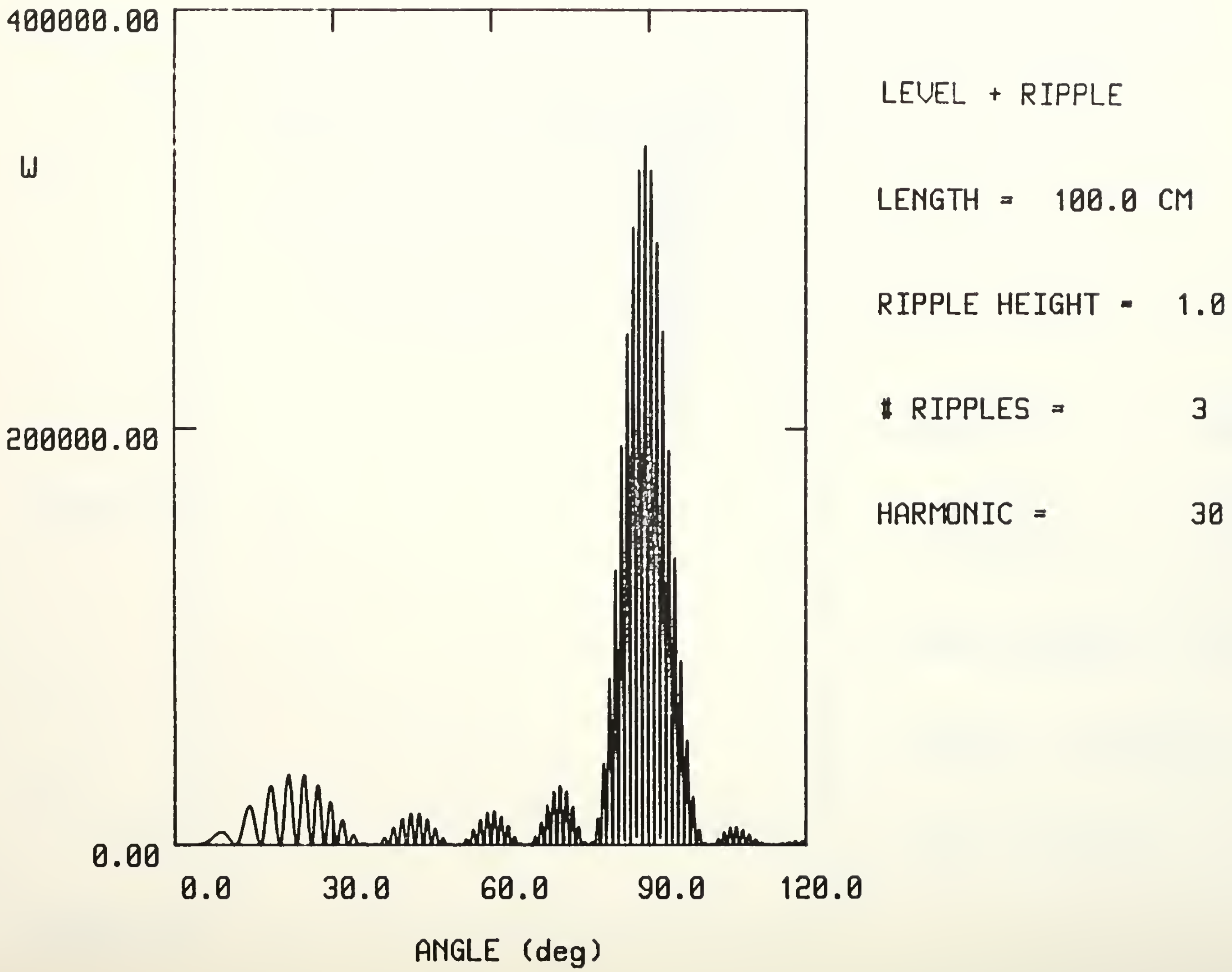
HARMONIC = 54

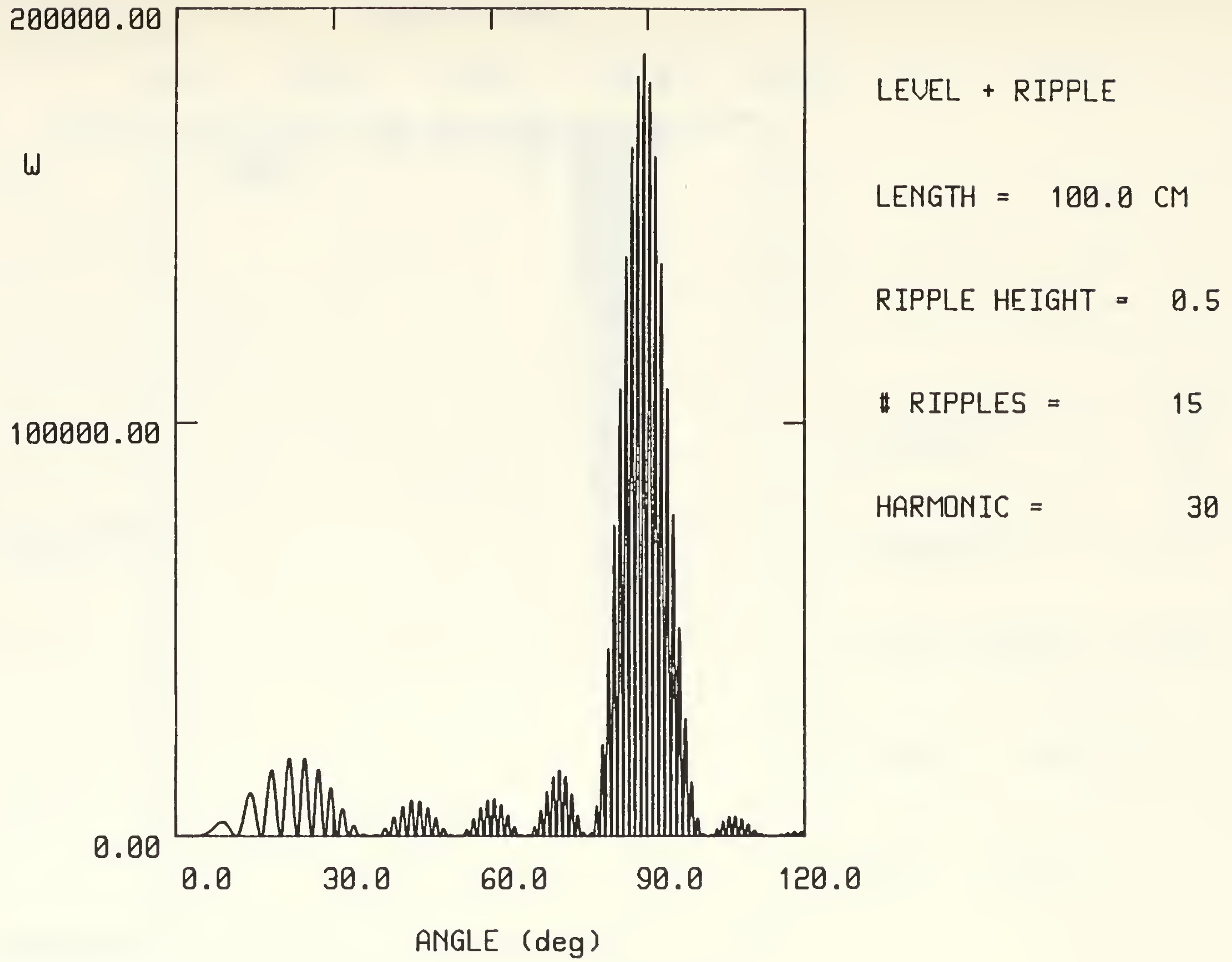

400000.00
W
200000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 1.0
# RIPPLES = 3
HARMONIC = 60

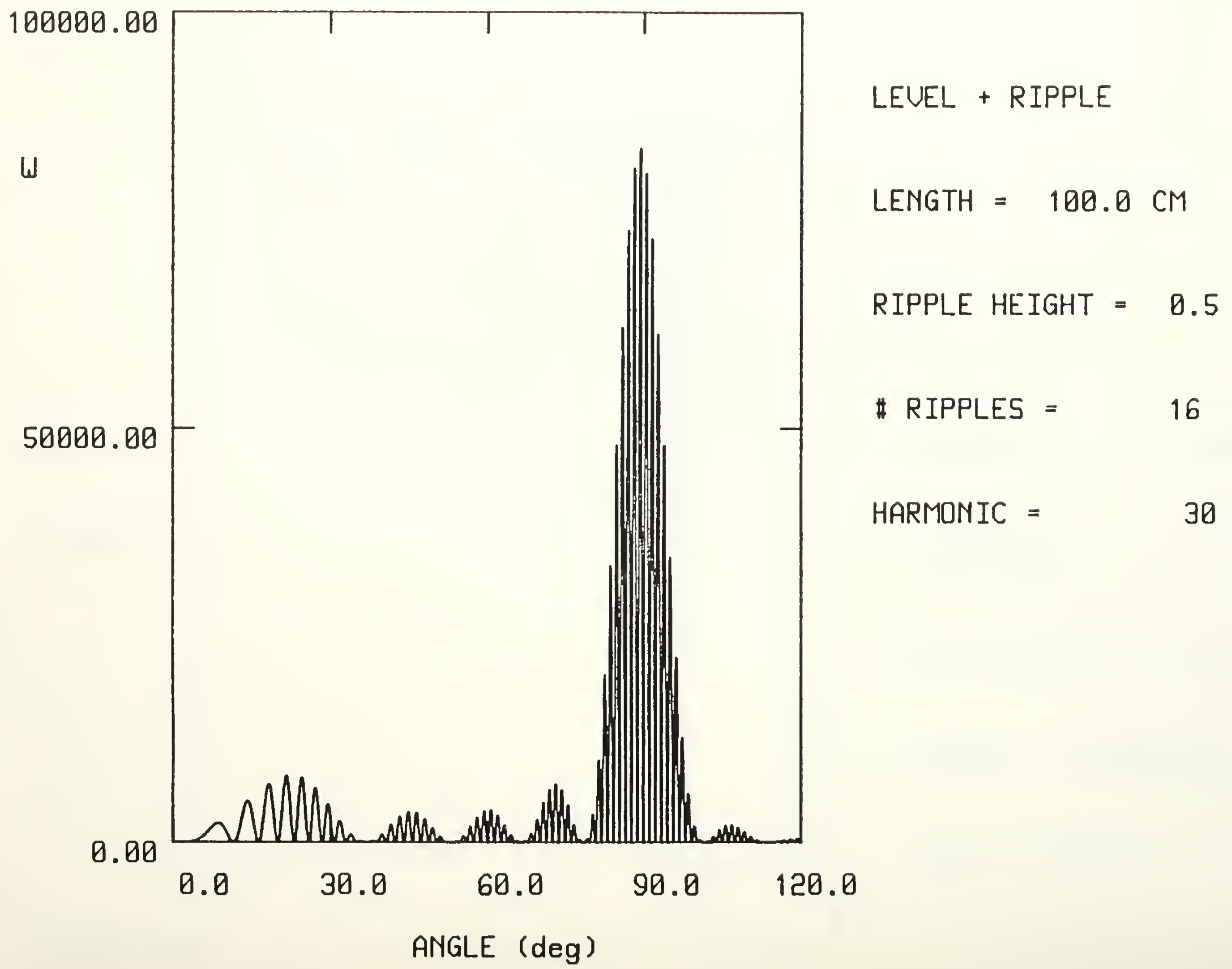

3000000.00
1500000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 1.0
# RIPPLES = 3
HARMONIC = 4

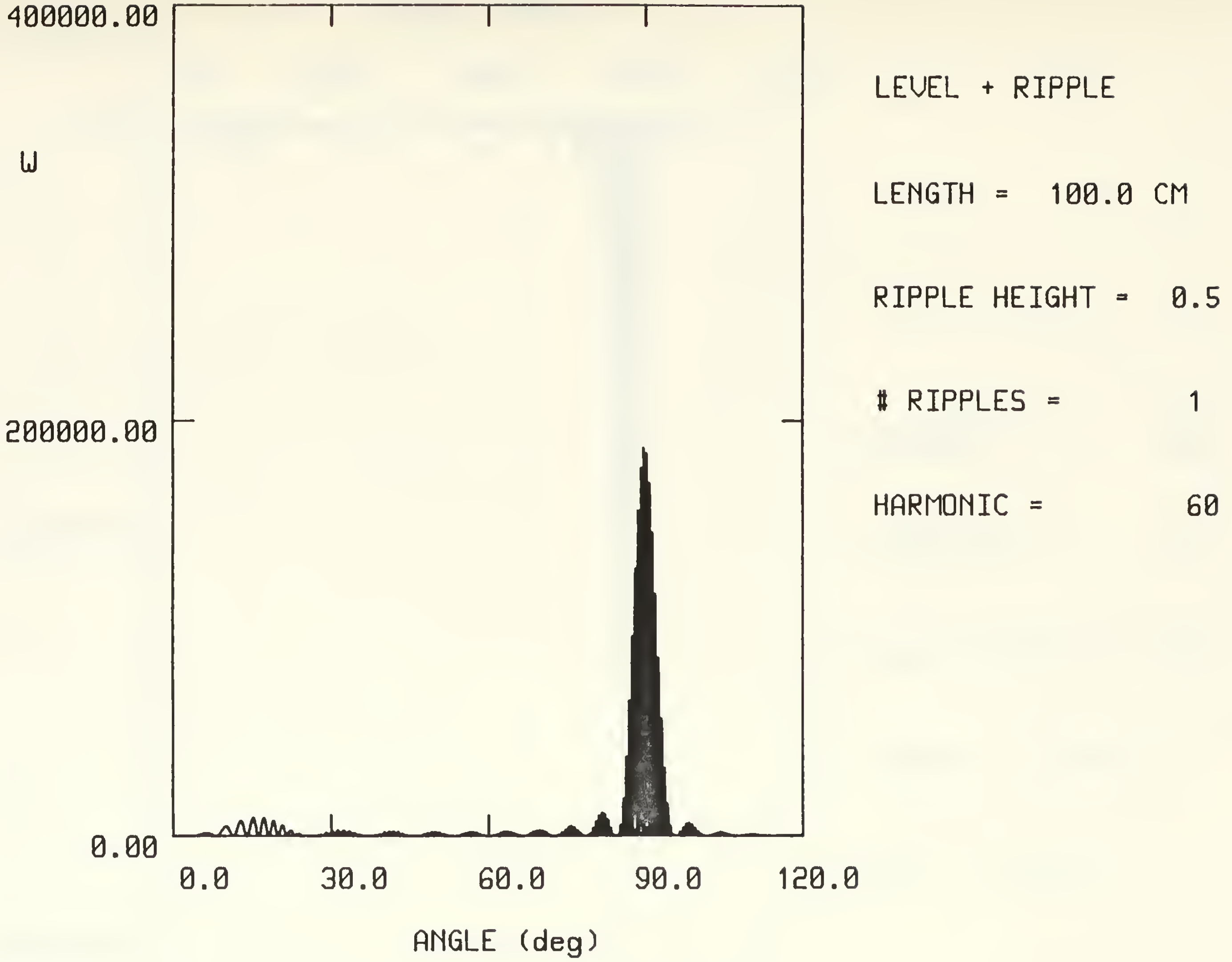

1500000.00
W
750000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.1
# RIPPLES = 3
HARMONIC = 4

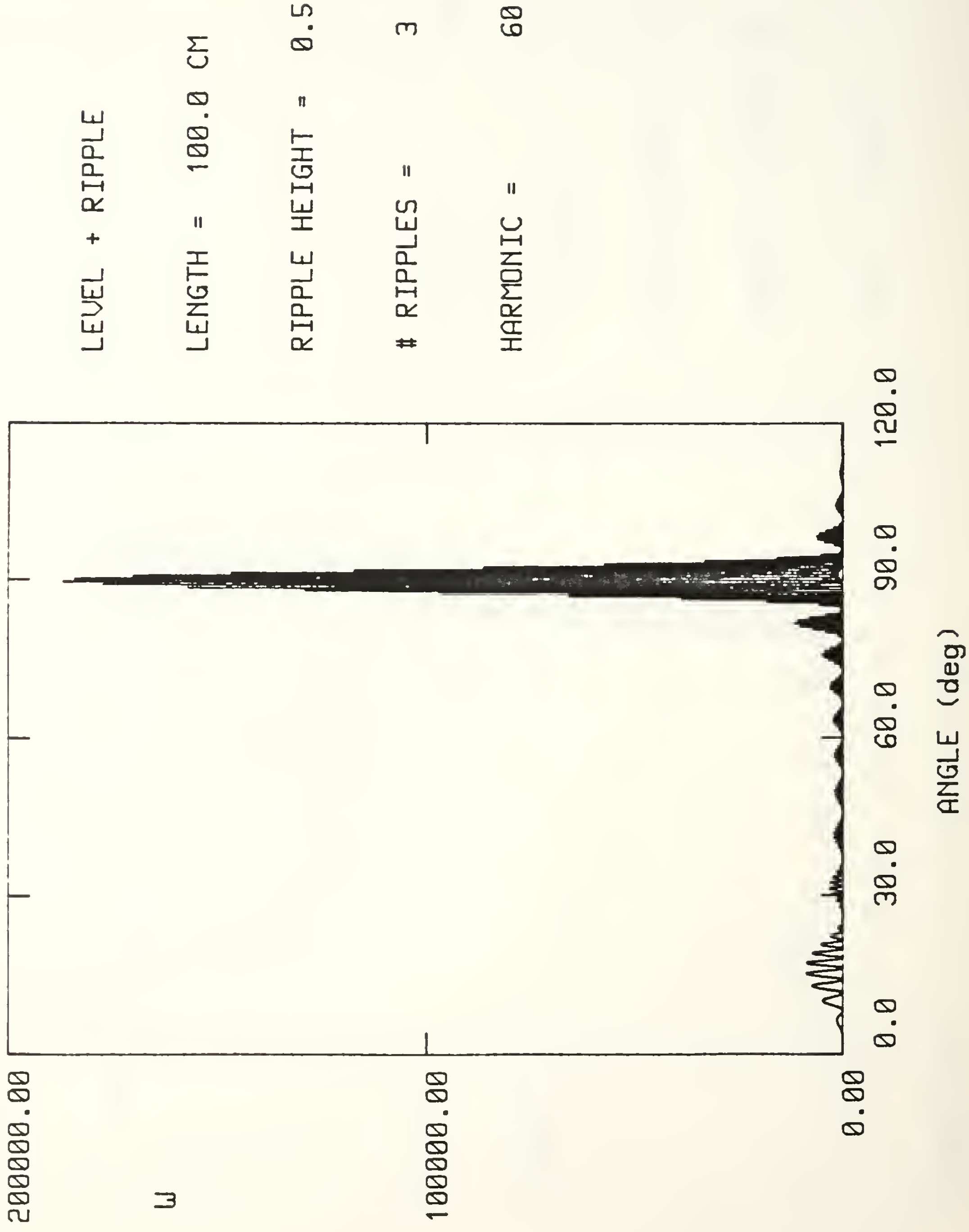

400000.00
W
200000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3
HARMONIC = 30

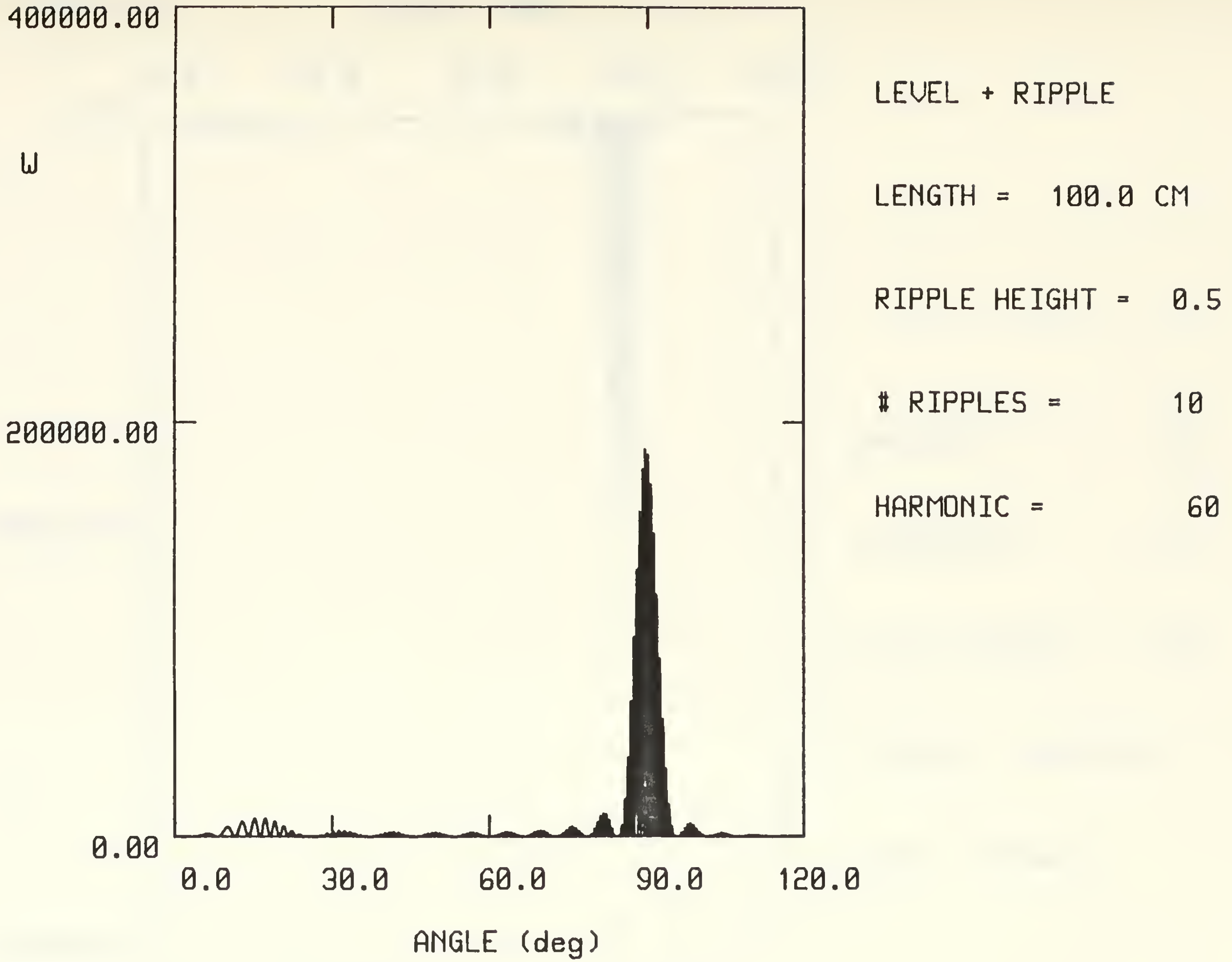

400000.00
W
200000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 1.0
# RIPPLES = 3
HARMONIC = 30

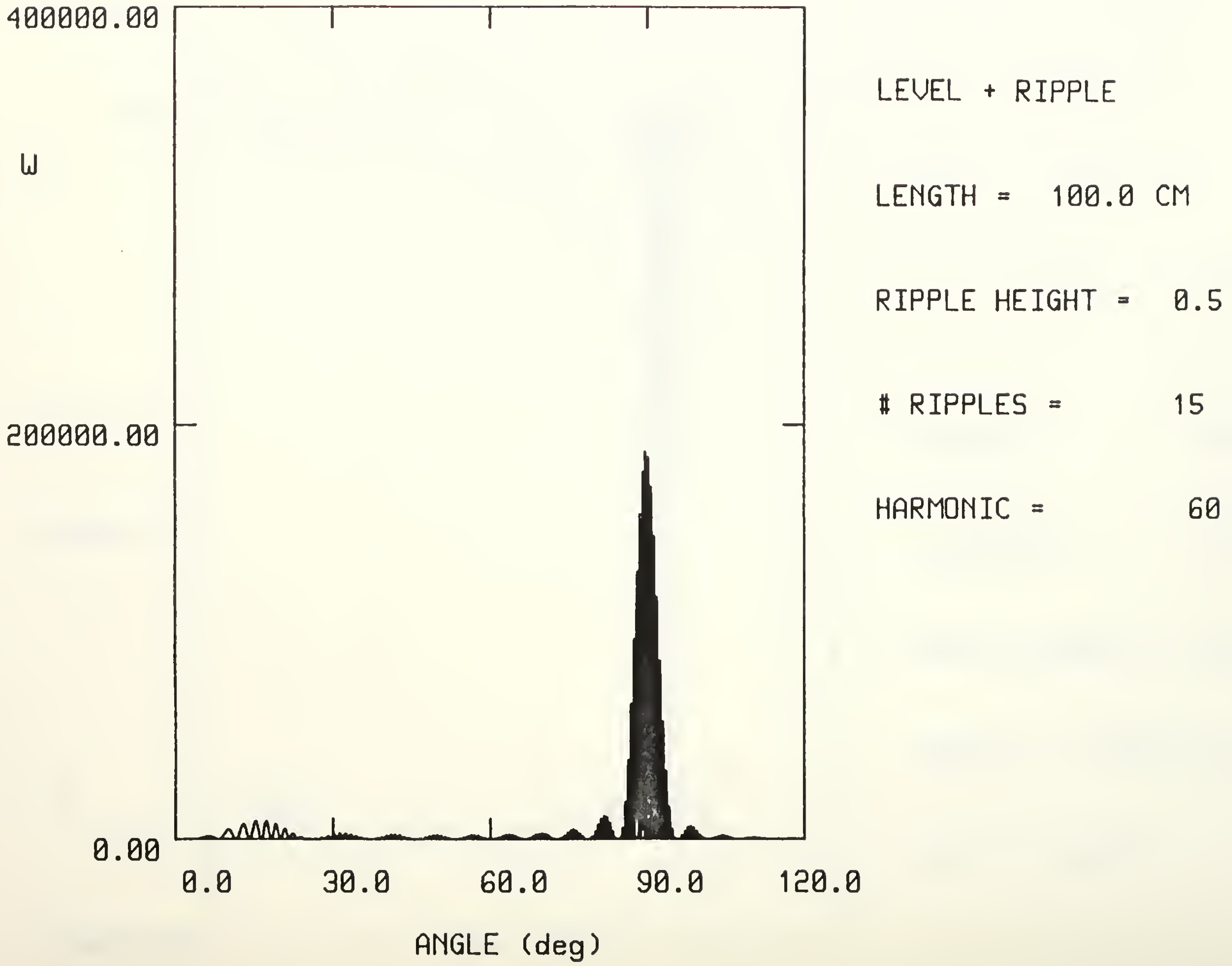

200000.00
W
100000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 15
HARMONIC = 30

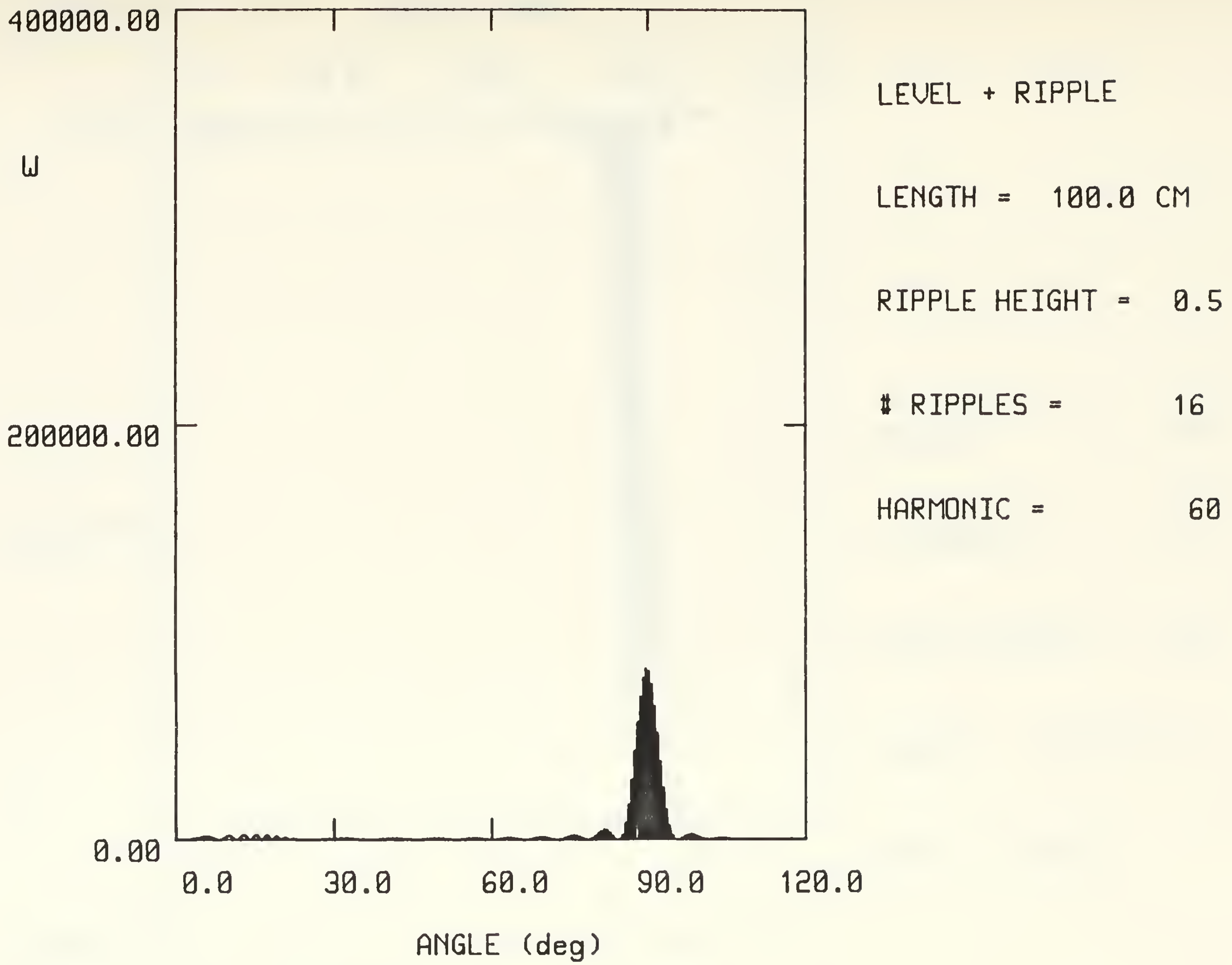

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 16
HARMONIC = 30

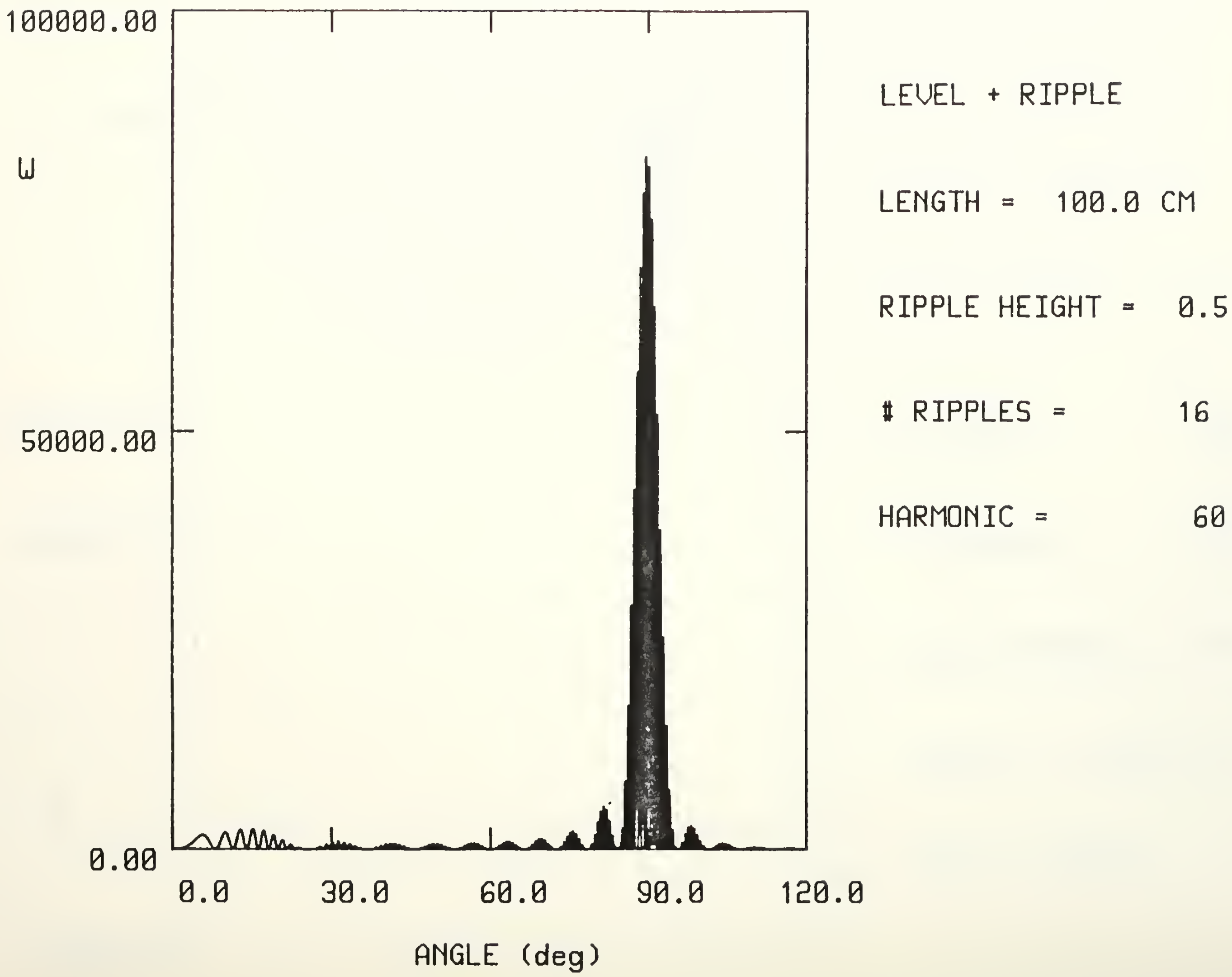

400000.00
W
200000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 1
HARMONIC = 60

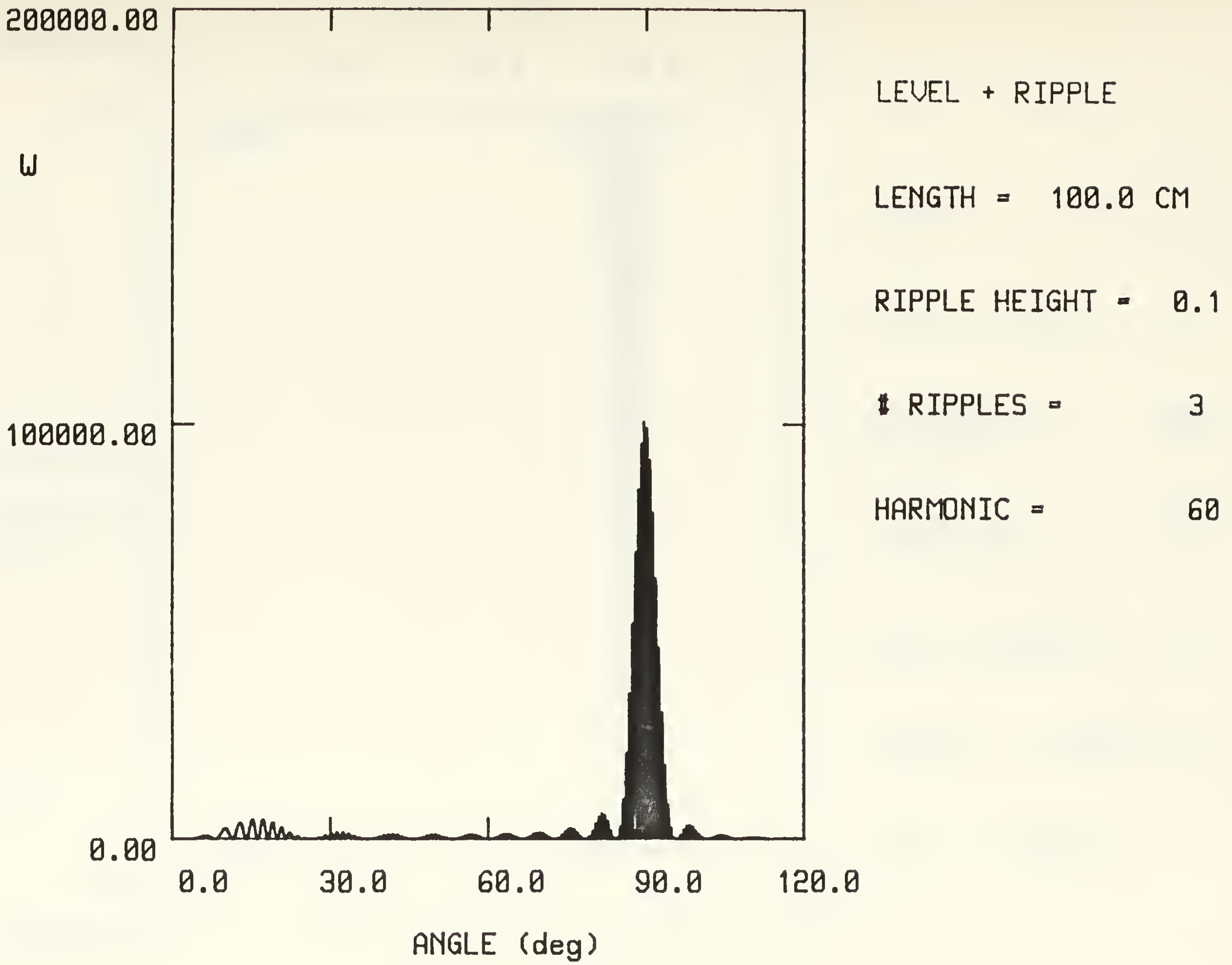

LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 3
HARMONIC = 60
E
200000.00
100000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)

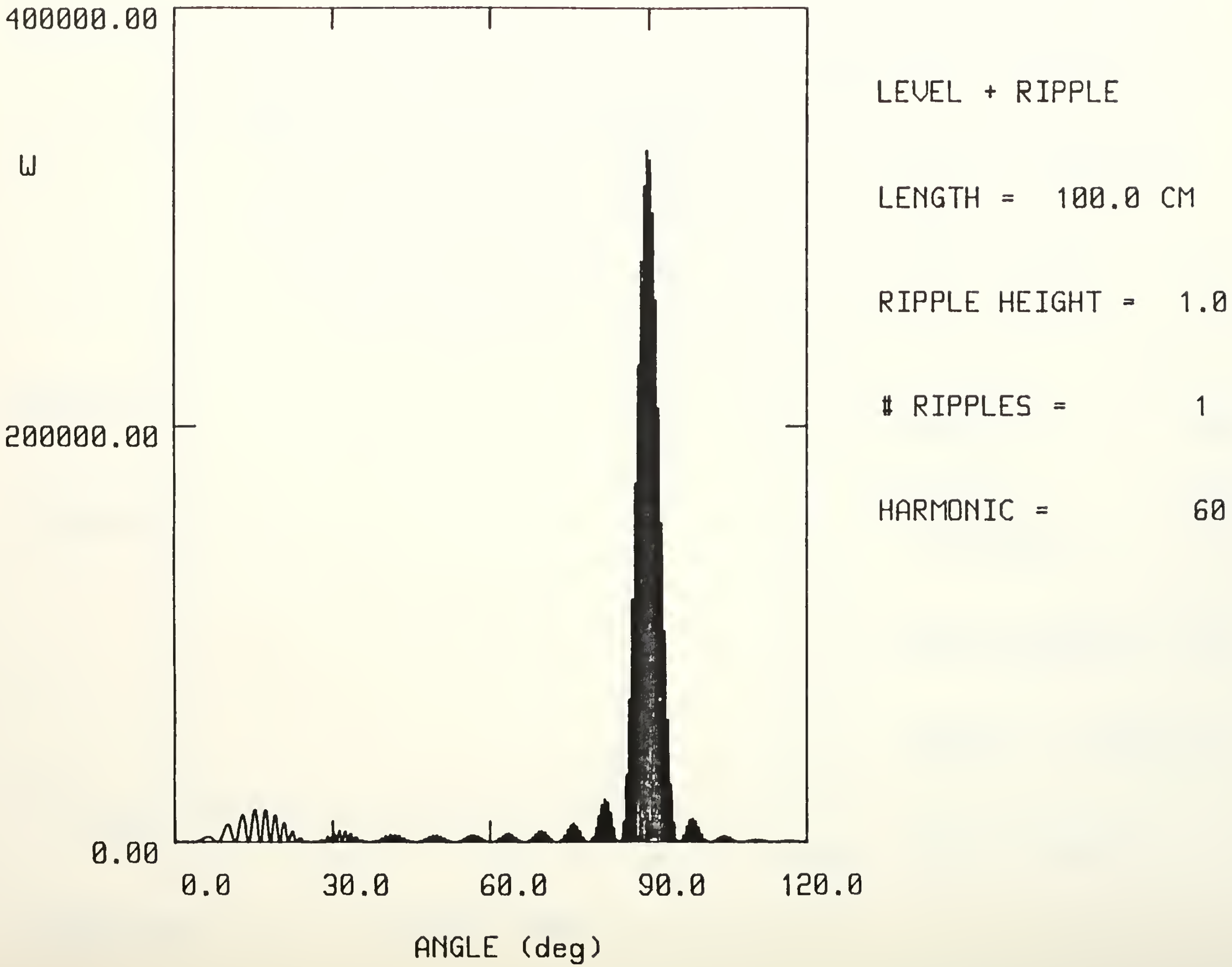

W
400000.00
200000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 10
HARMONIC = 60

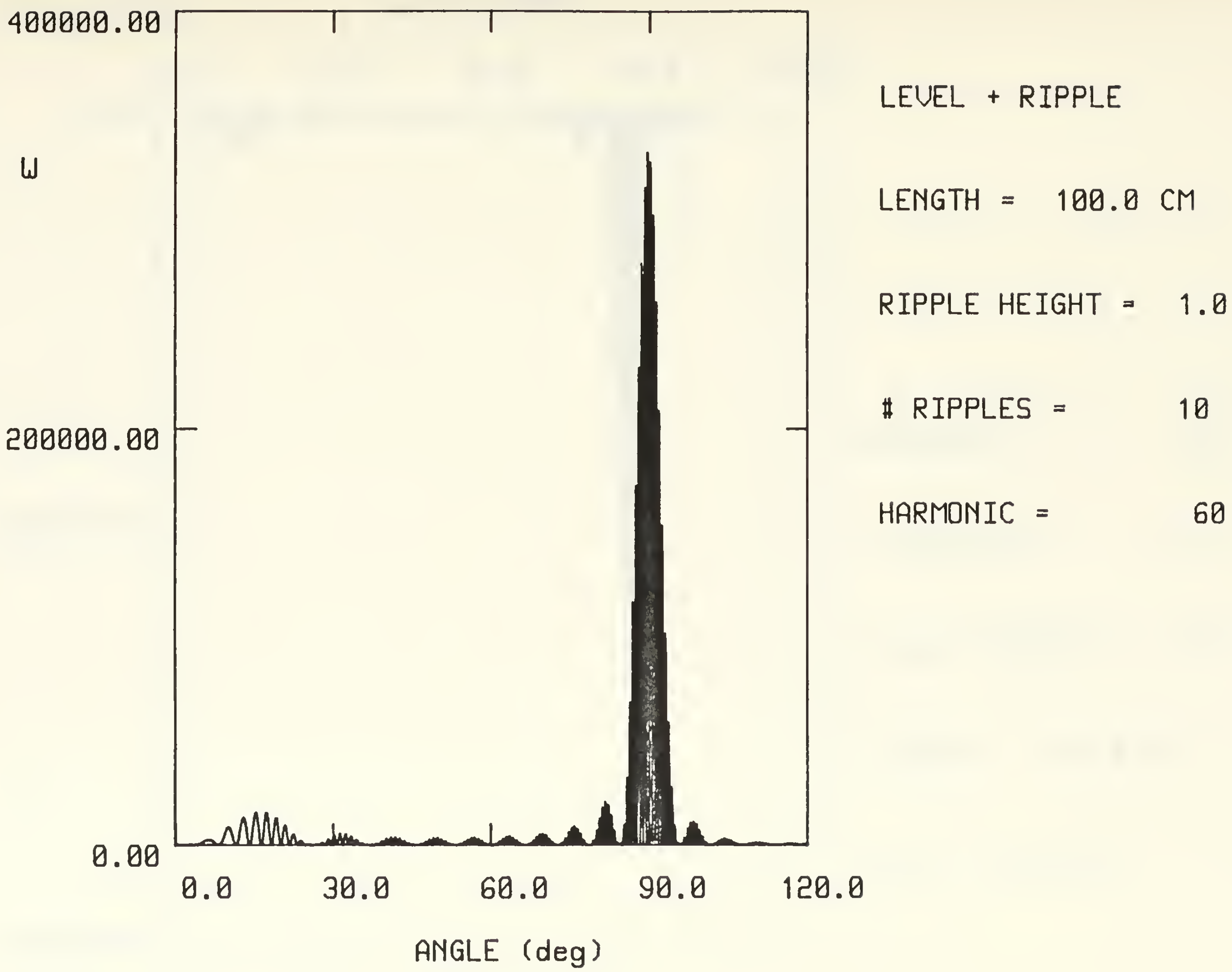

400000.00
W
200000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 15
HARMONIC = 60

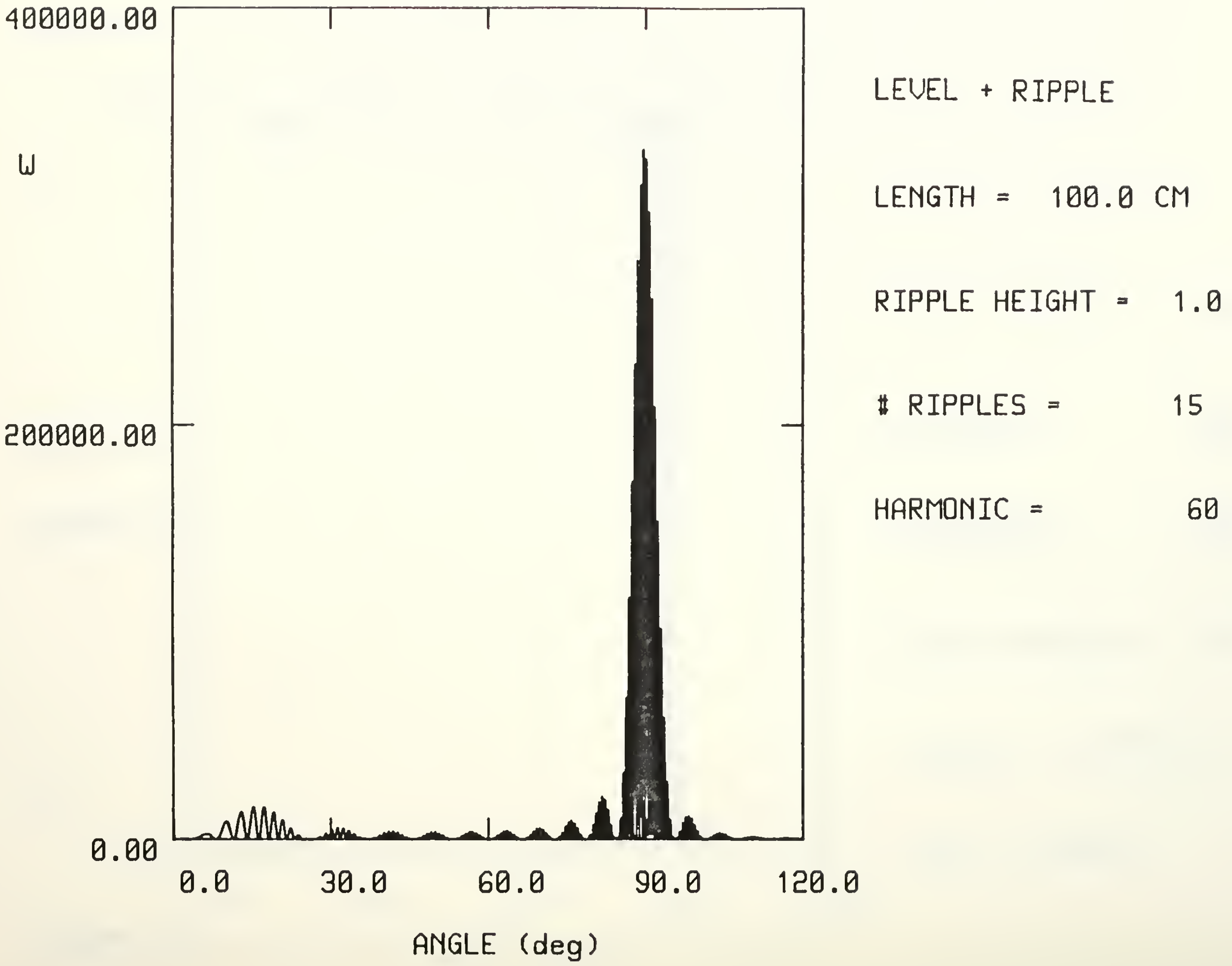

400000.00
W
200000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 16
HARMONIC = 60

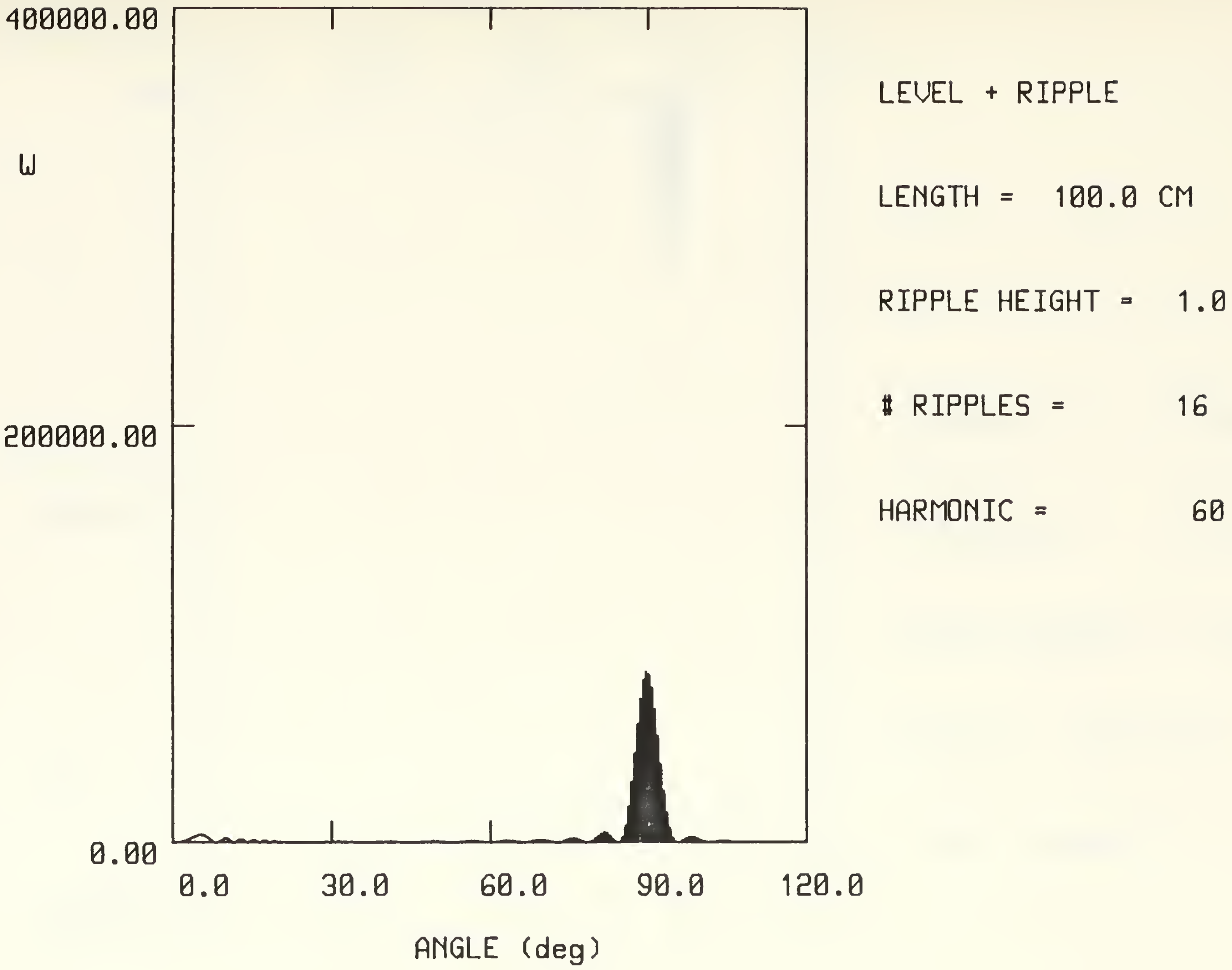

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.5
# RIPPLES = 16
HARMONIC = 60

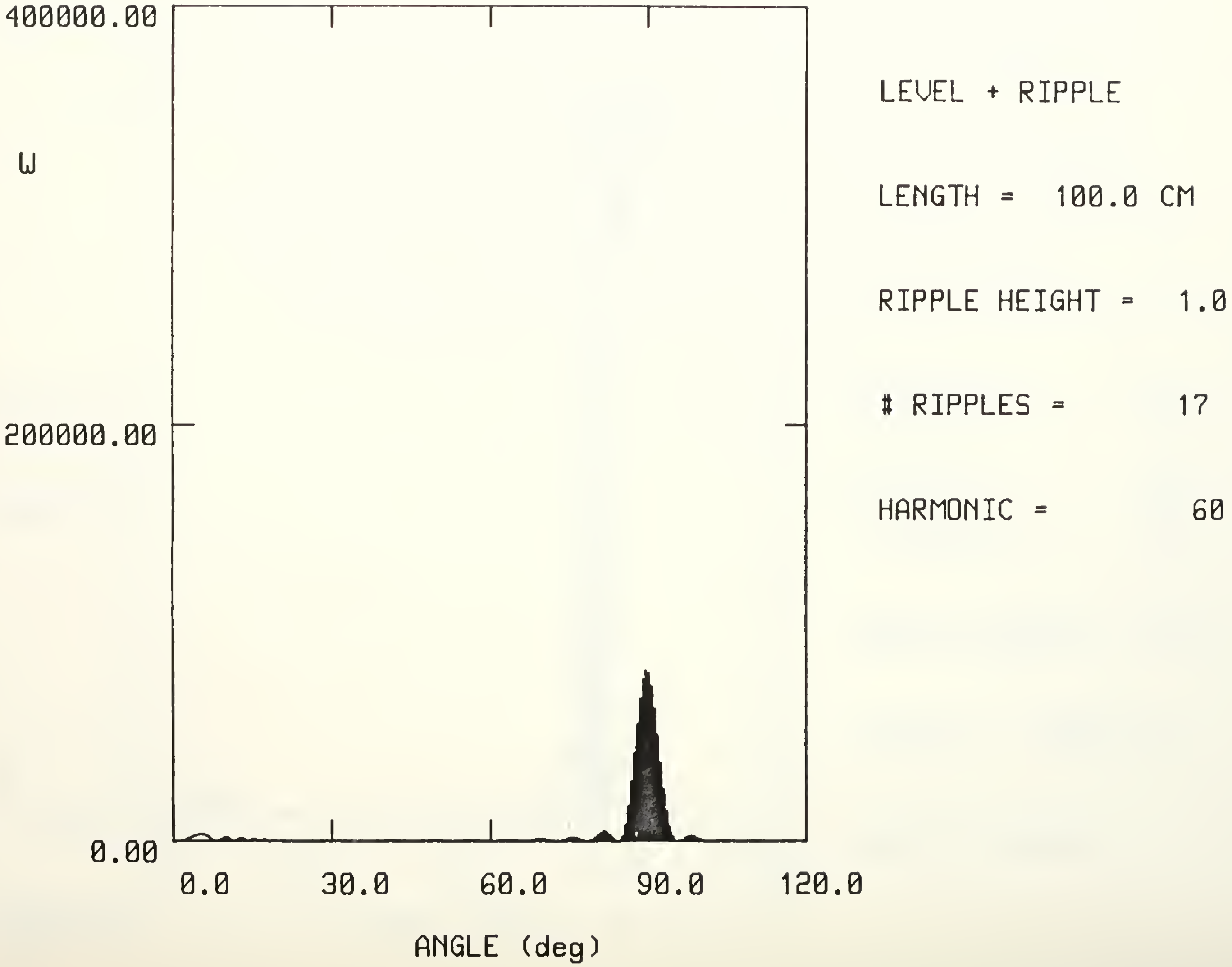

200000.00
W
100000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 0.1
# RIPPLES = 3
HARMONIC = 60

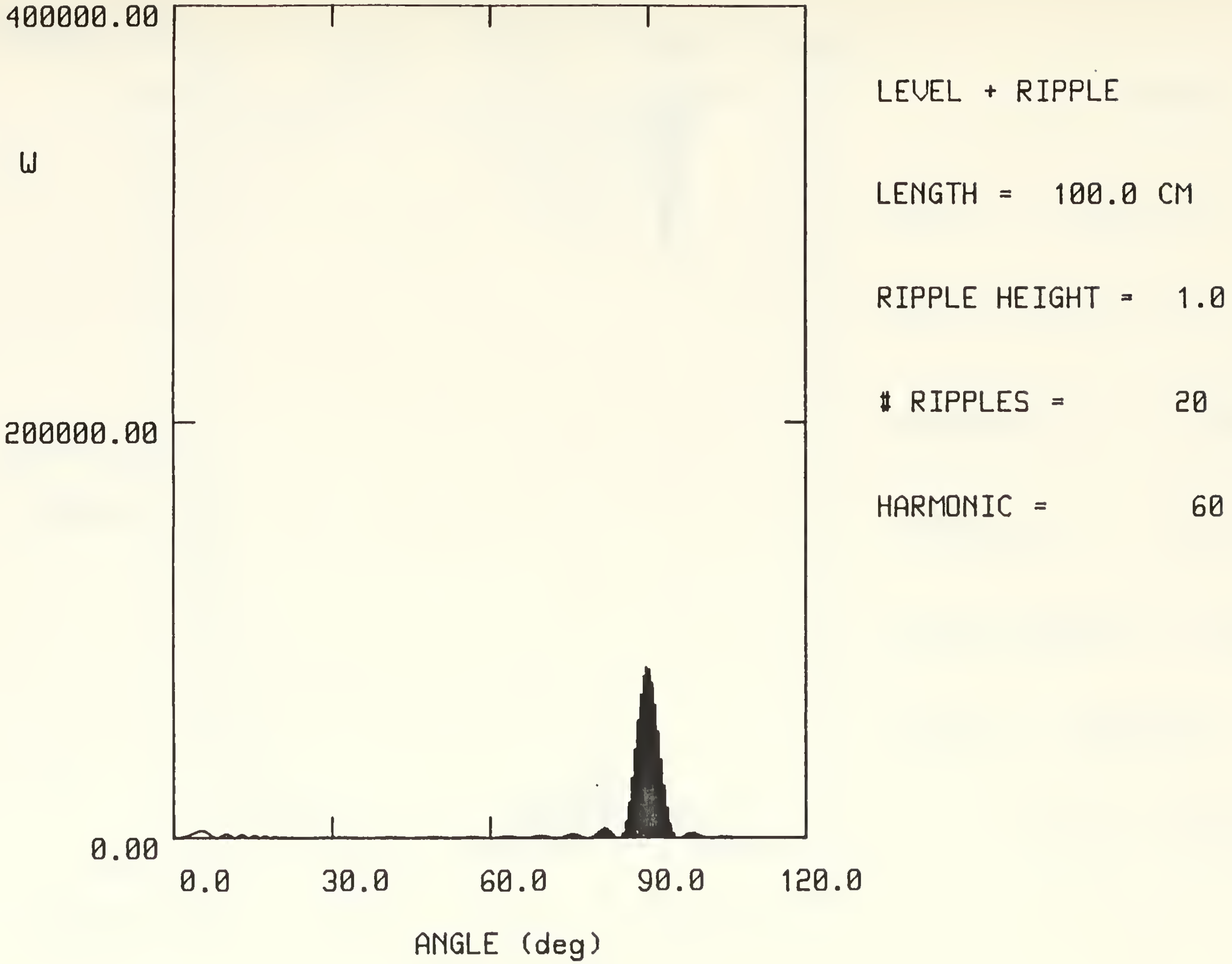

400000.00
W
200000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 1.0
# RIPPLES = 1
HARMONIC = 60

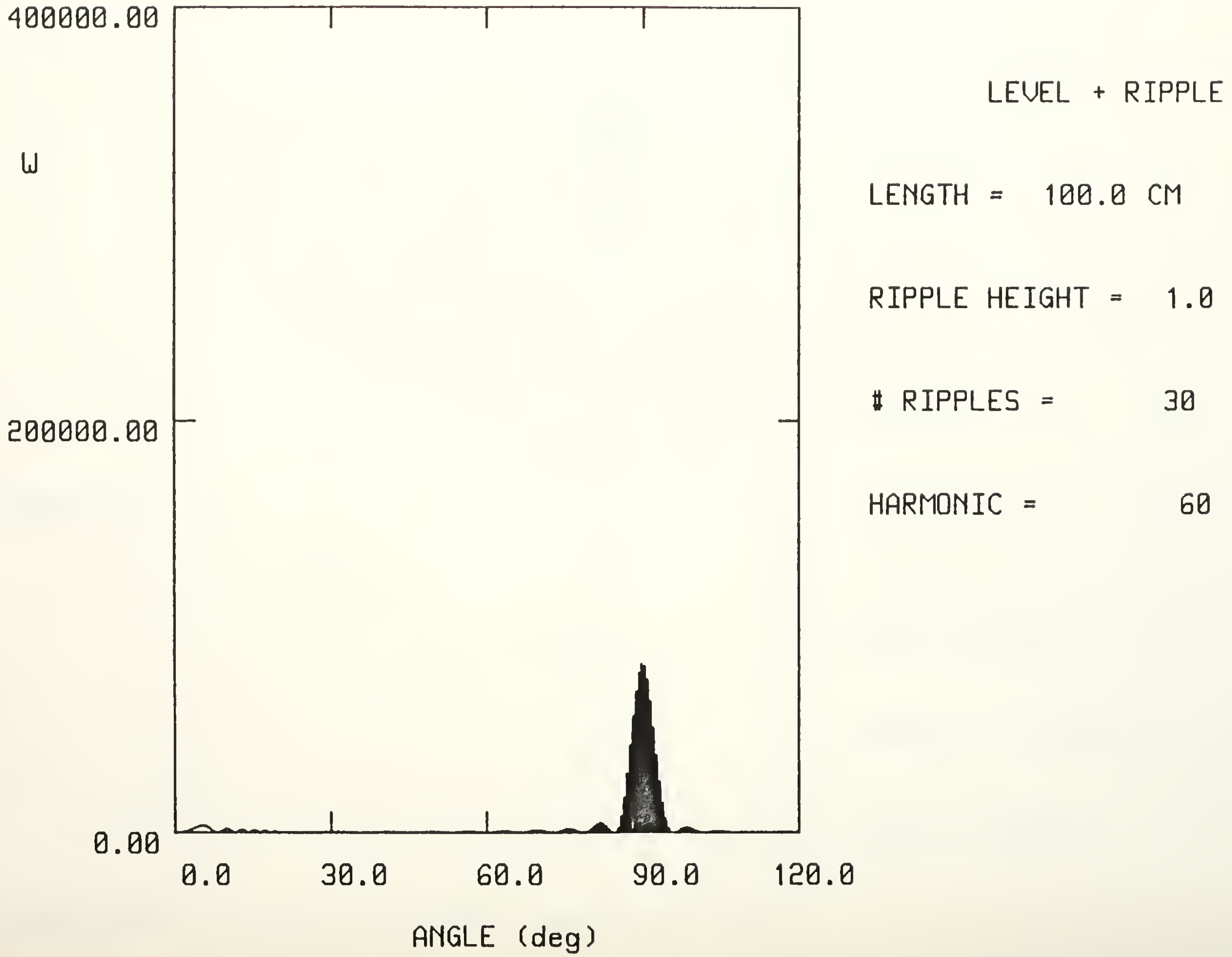

400000.00
W
200000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 1.0
# RIPPLES = 10
HARMONIC = 60

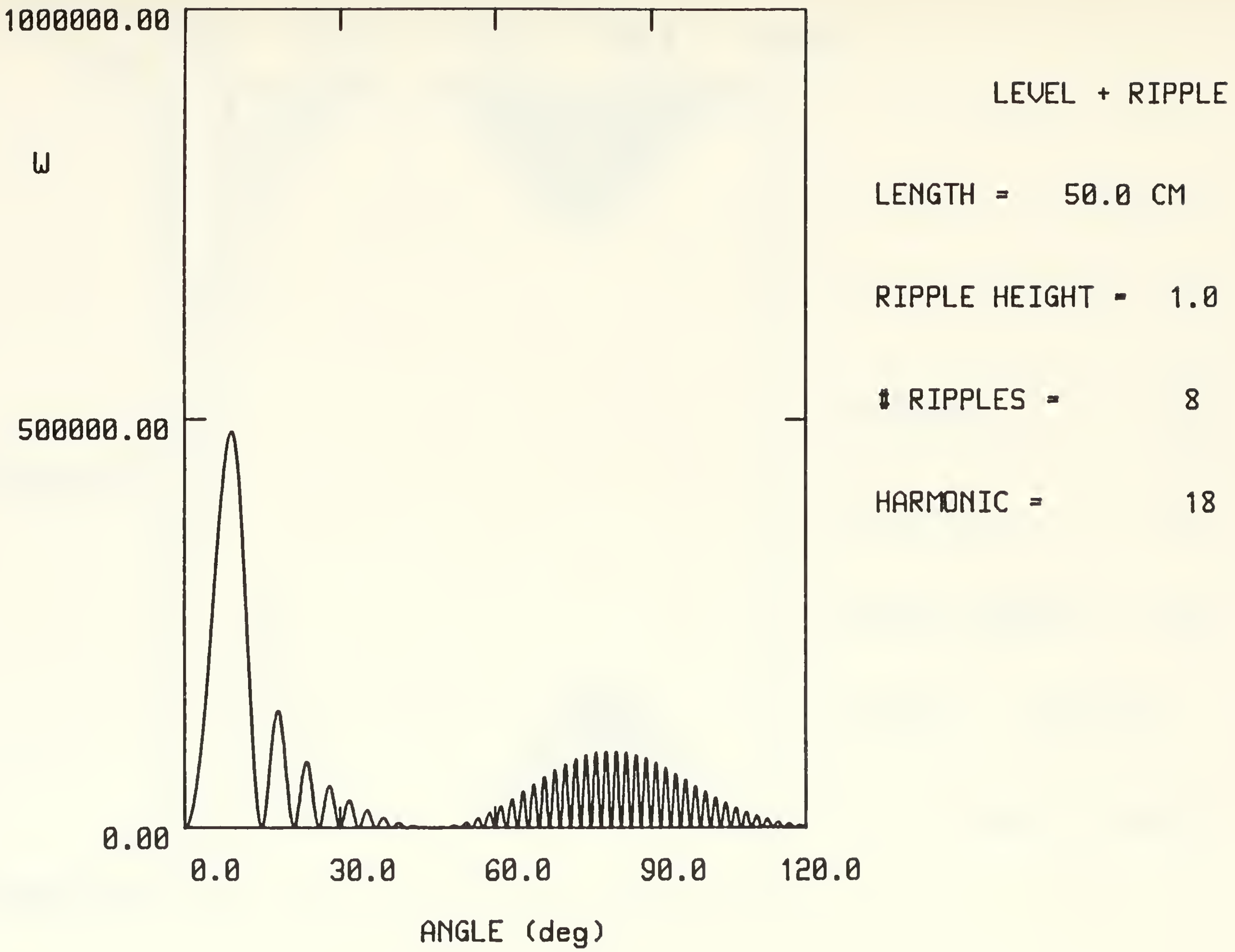

400000.00
W
200000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 1.0
# RIPPLES = 15
HARMONIC = 60

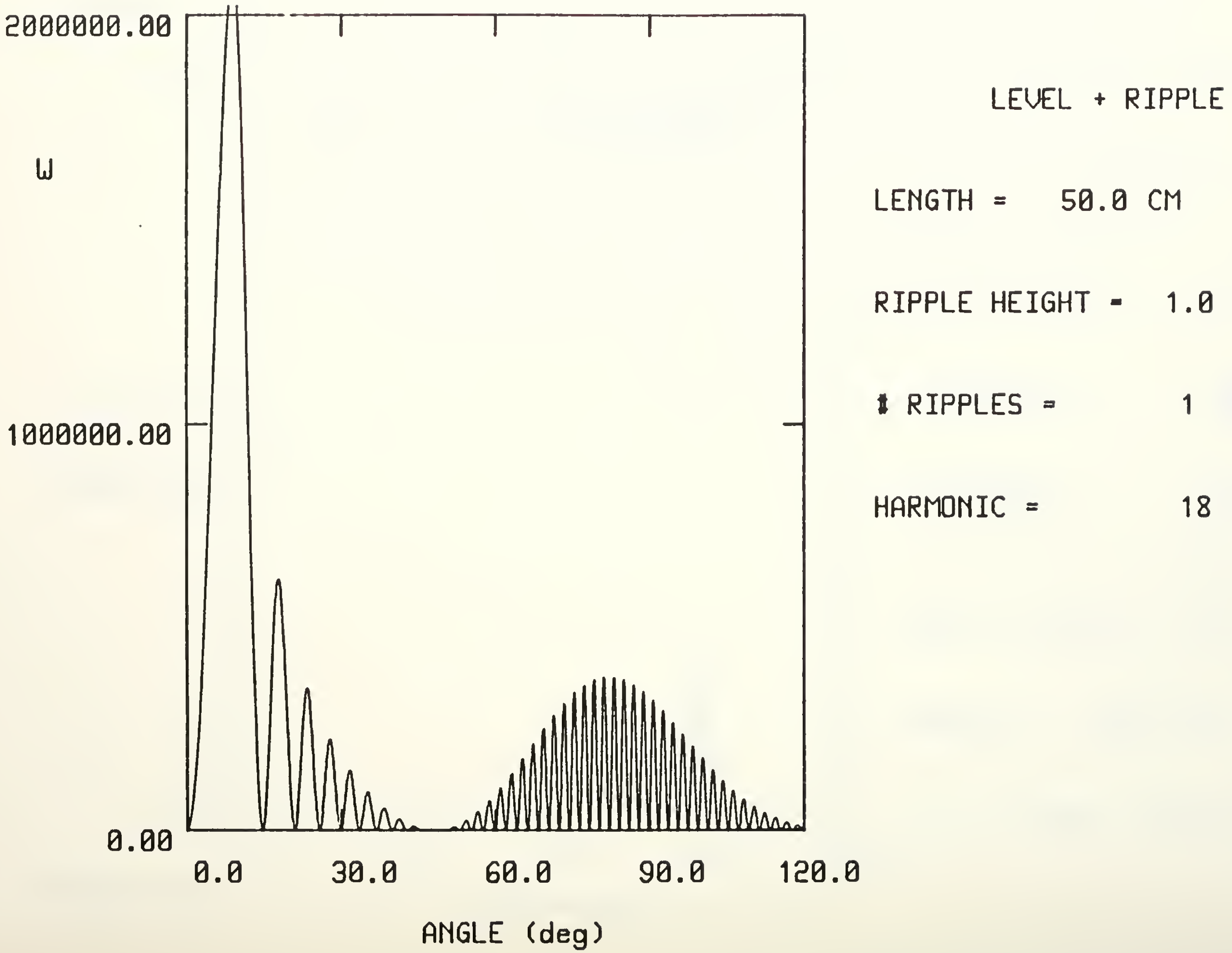

400000.00
W
200000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 1.0
# RIPPLES = 16
HARMONIC = 60

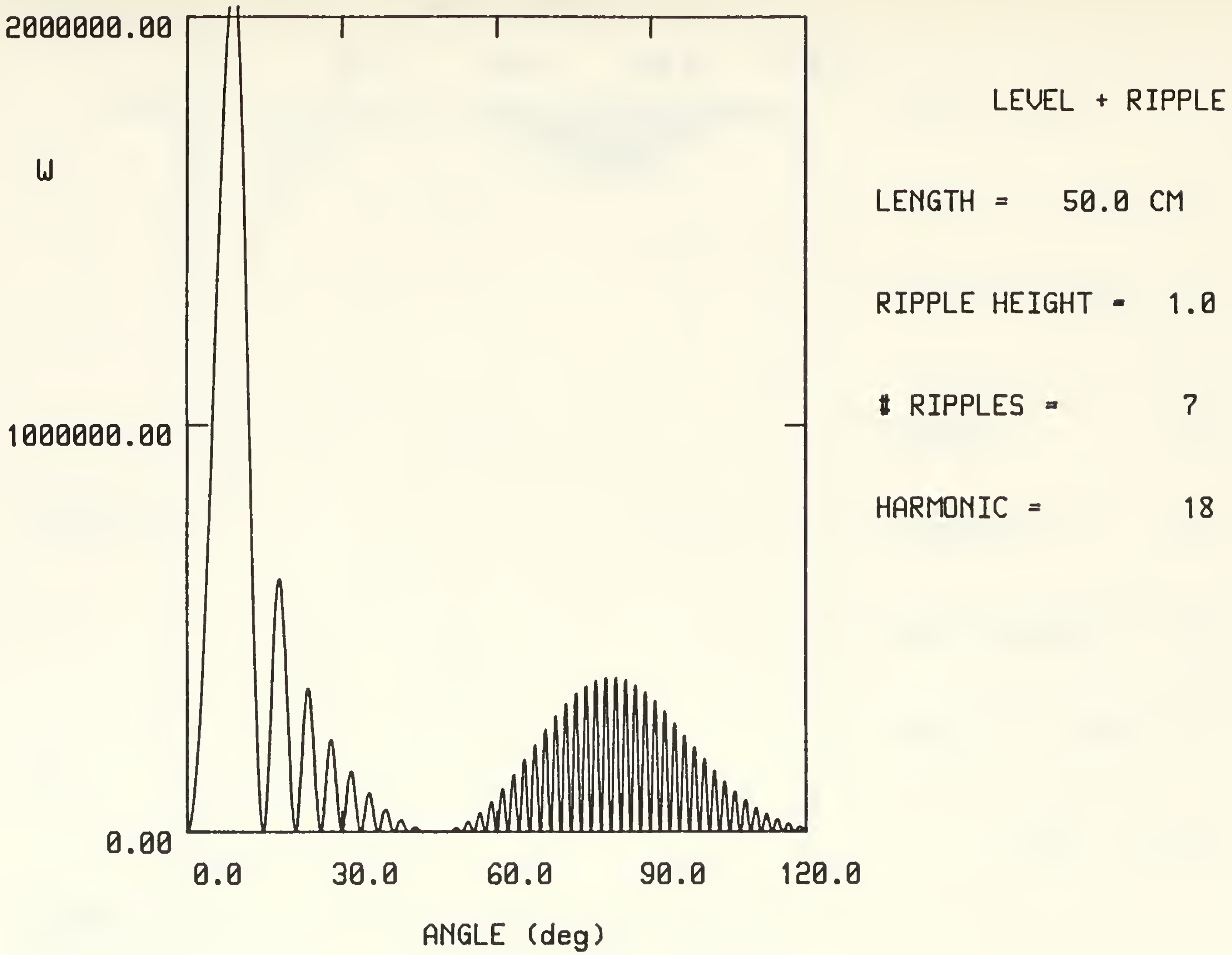

400000.00
W
200000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 1.0
# RIPPLES = 17
HARMONIC = 60

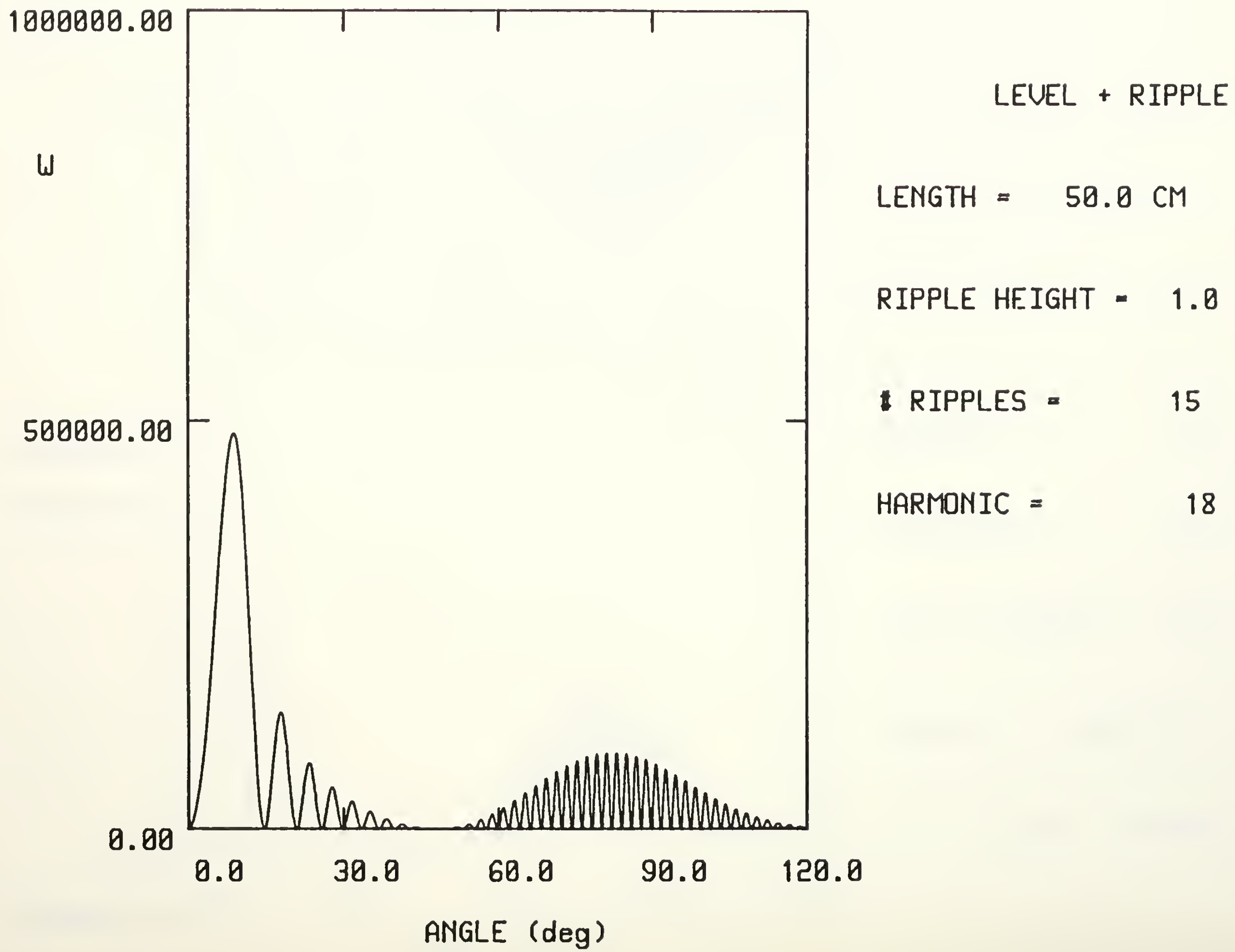

400000.00
W
200000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 1.0
# RIPPLES = 20
HARMONIC = 60


400000.00
200000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 100.0 CM
RIPPLE HEIGHT = 1.0
# RIPPLES = 30
HARMONIC = 60

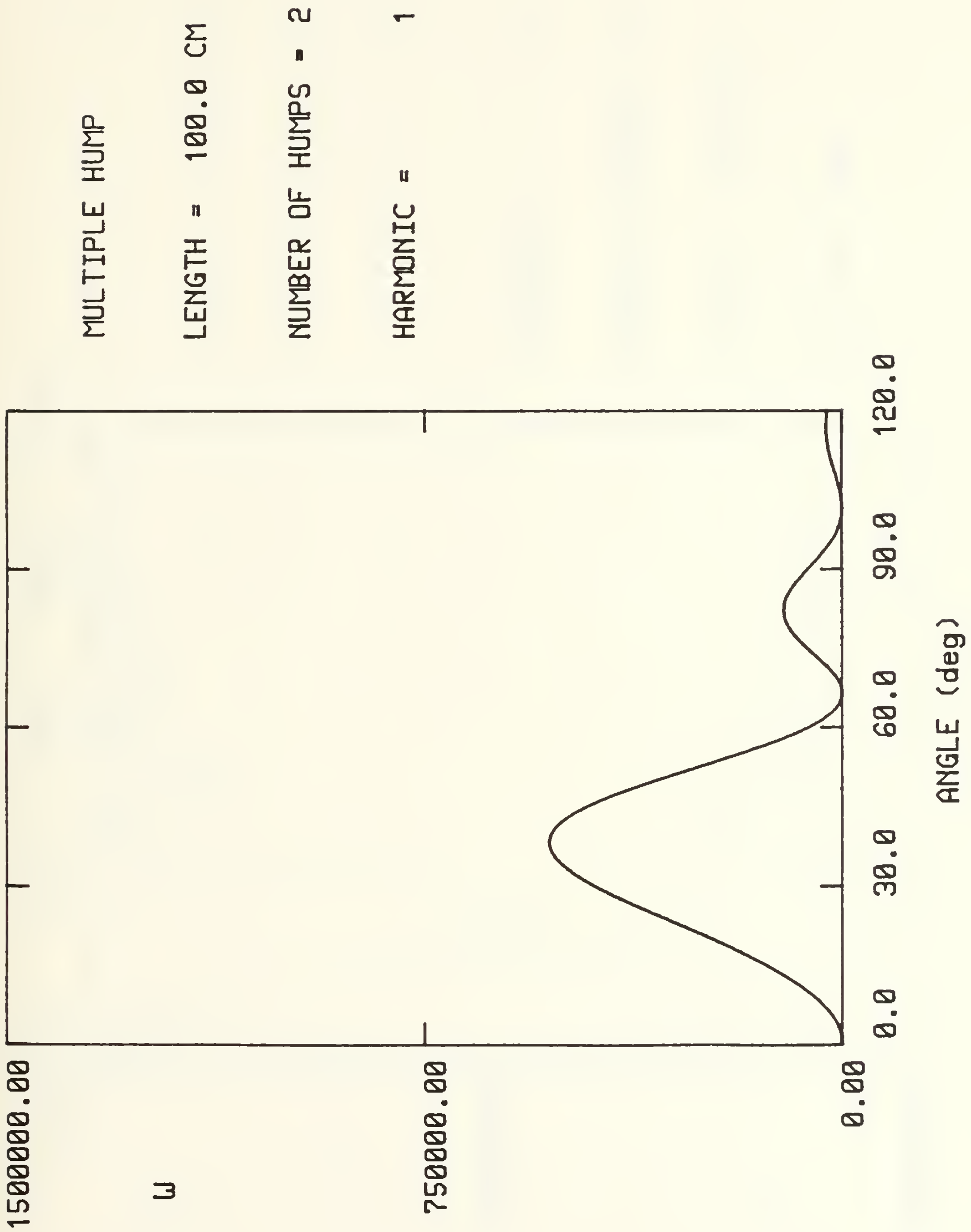

1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 50.0 CM
RIPPLE HEIGHT = 1.0
# RIPPLES = 8
HARMONIC = 18

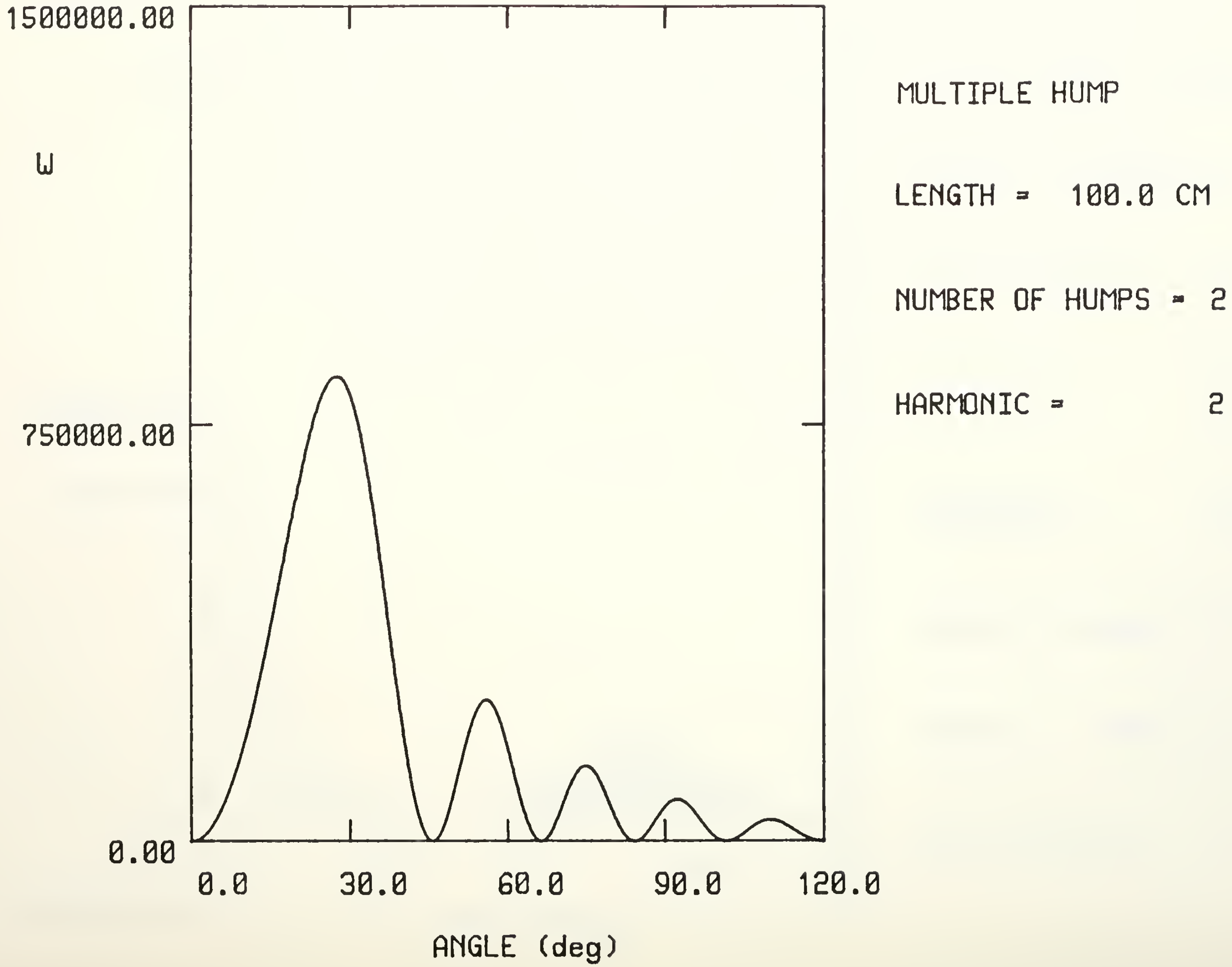

2000000.00
W
1000000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 50.0 CM
RIPPLE HEIGHT - 1.0
# RIPPLES = 1
HARMONIC = 18

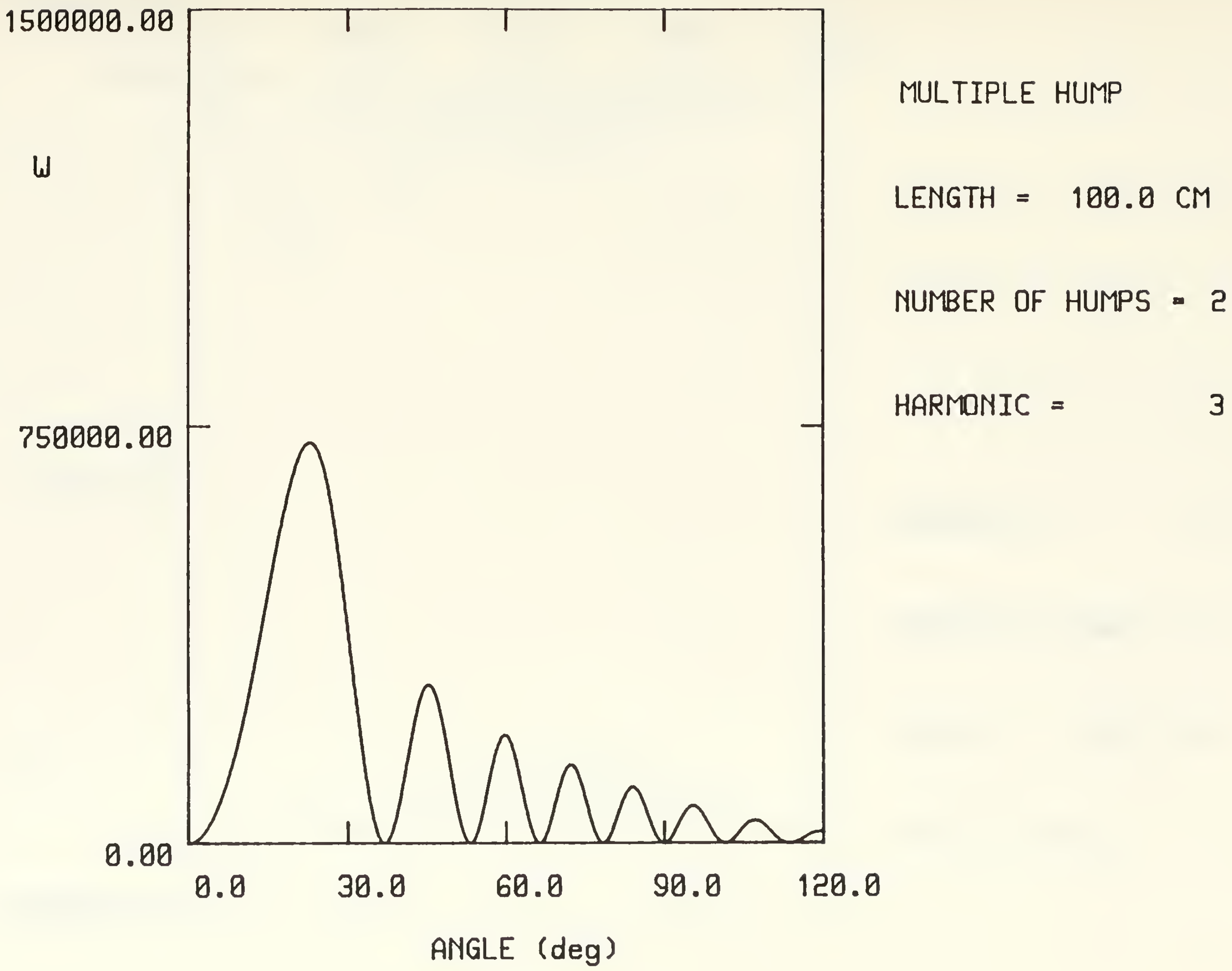

2000000.00
1000000.00
0.00
W
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 50.0 CM
RIPPLE HEIGHT = 1.0
# RIPPLES = 7
HARMONIC = 18

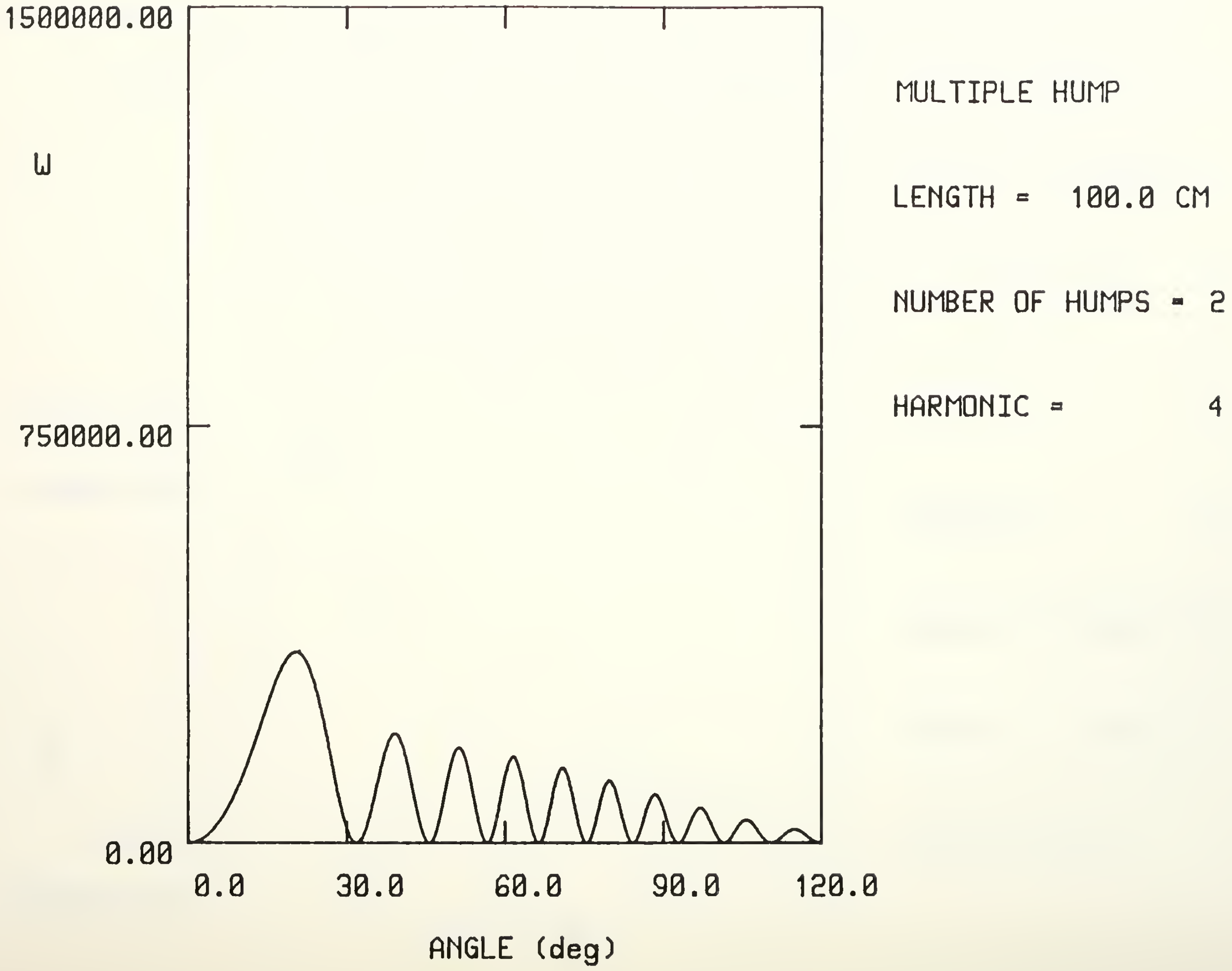

1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
LEVEL + RIPPLE
LENGTH = 50.0 CM
RIPPLE HEIGHT = 1.0
# RIPPLES = 15
HARMONIC = 18

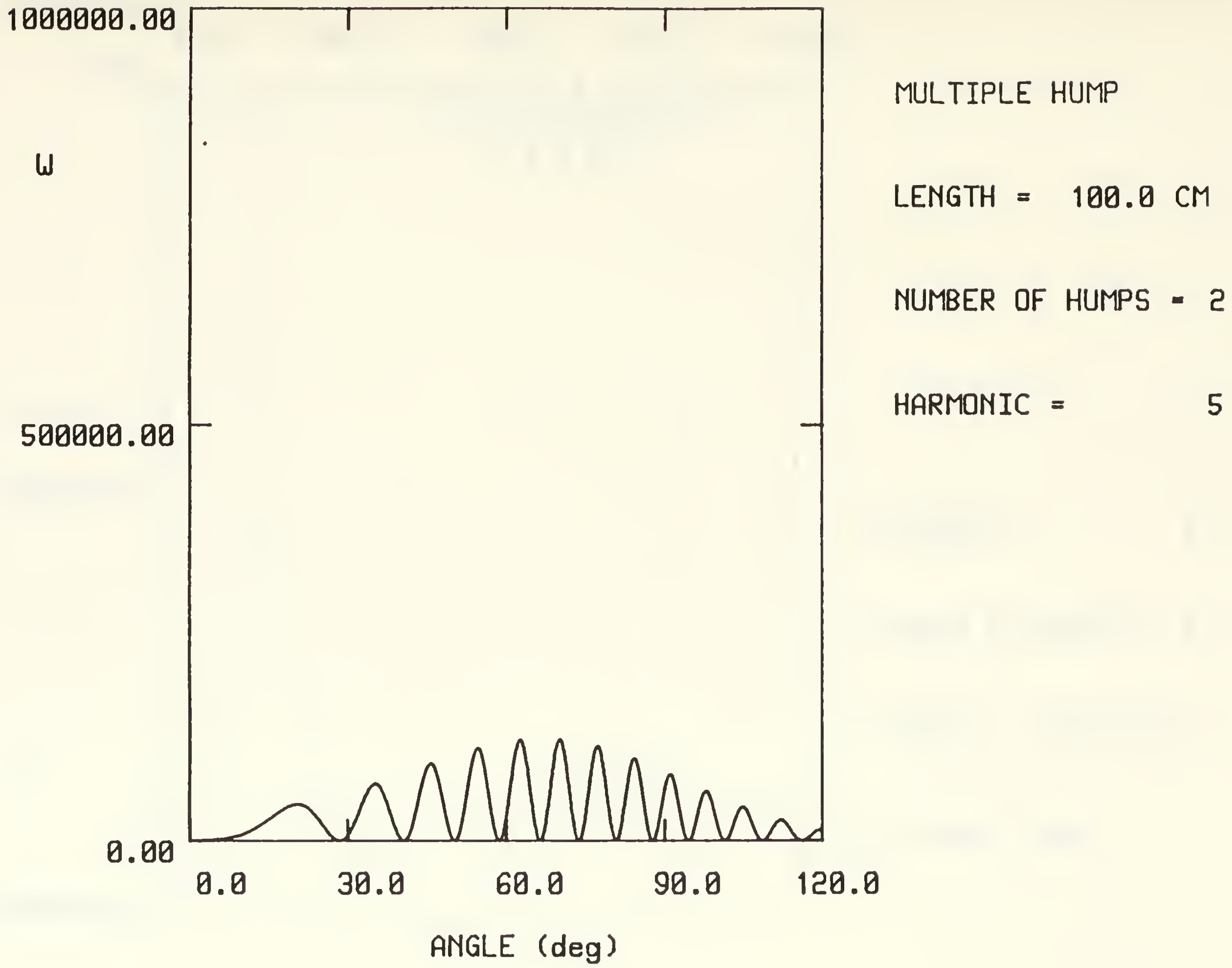
APPENDIX G: MULTIPLE HUMP FUNCTION

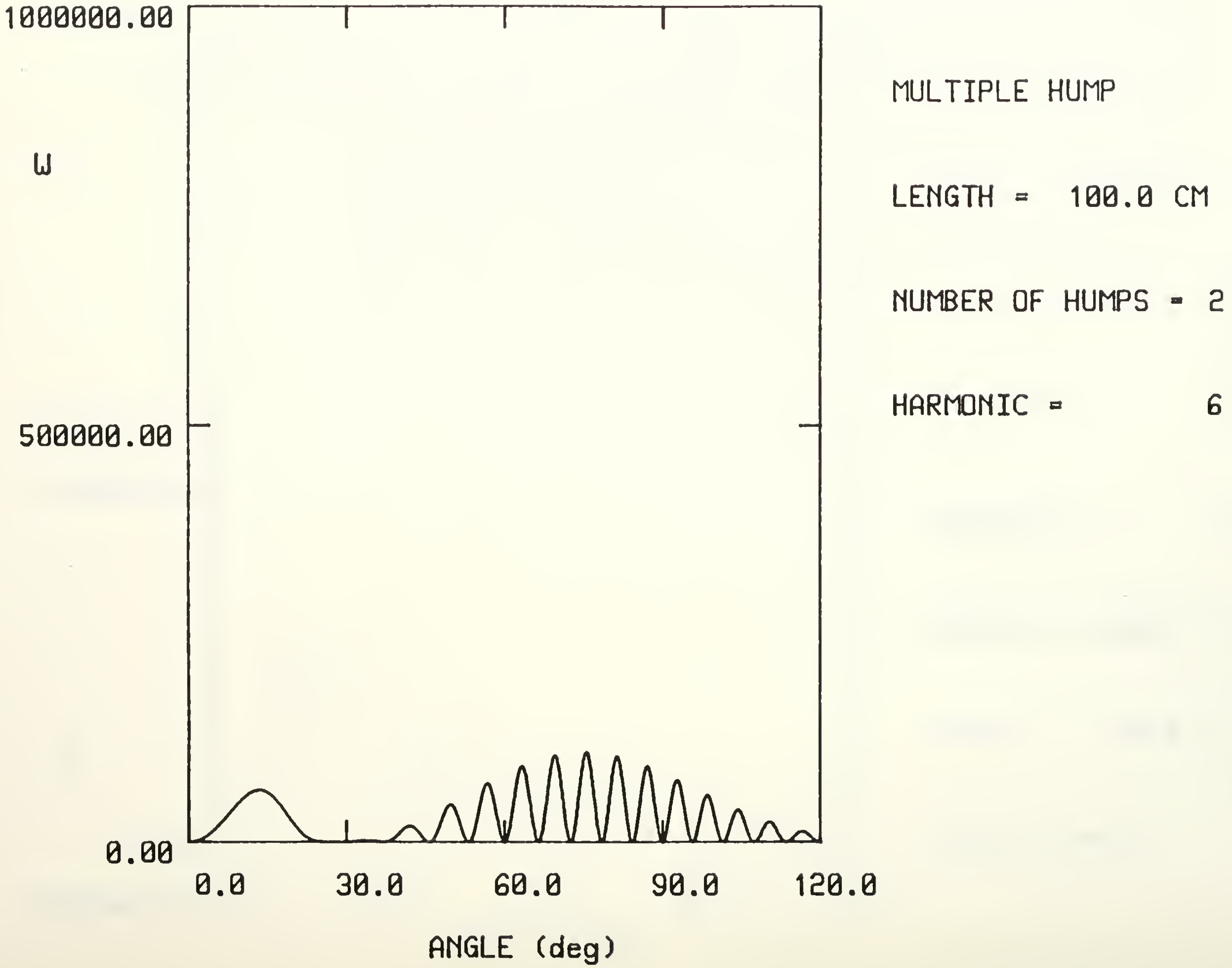


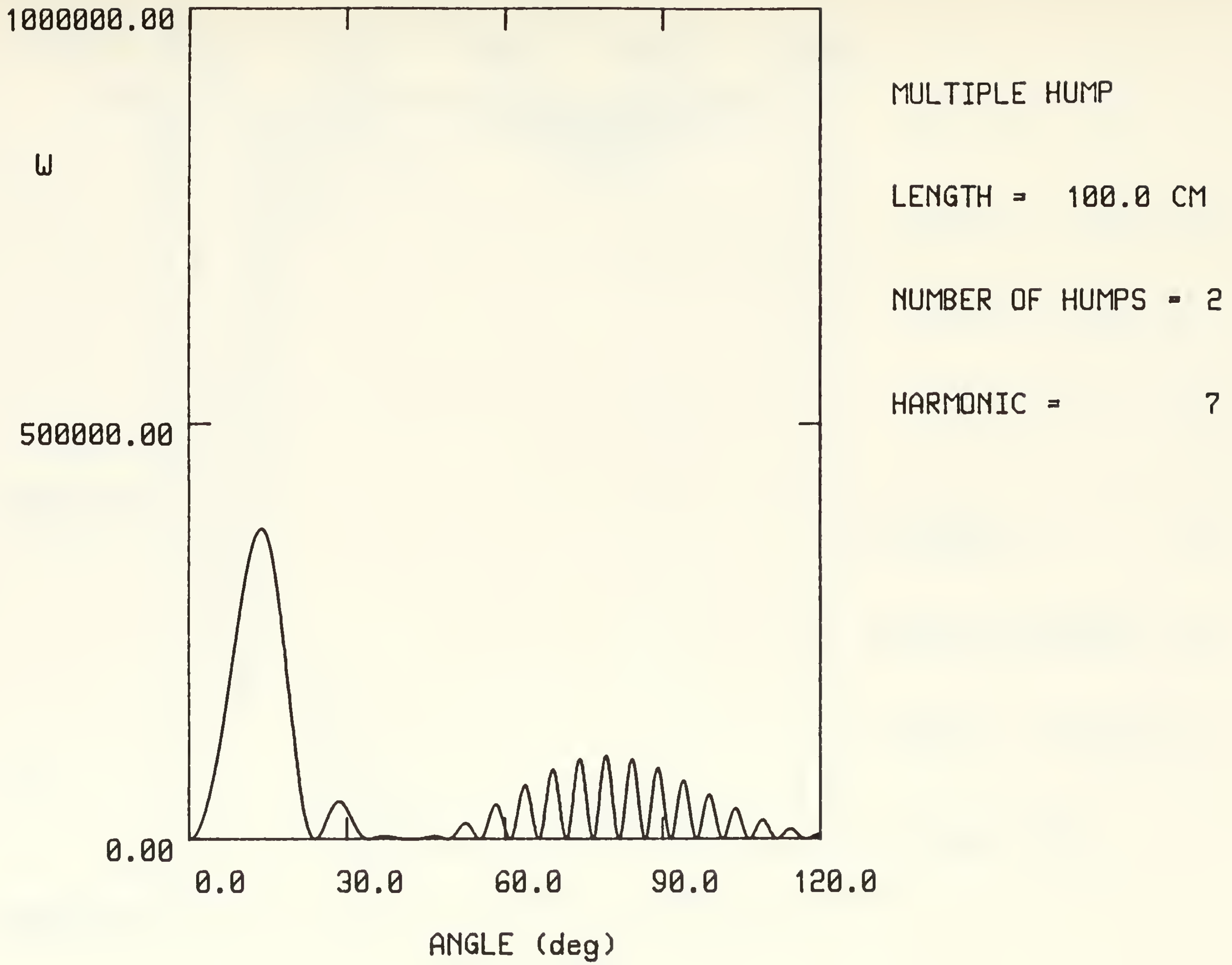

1500000.00
W
750000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
MULTIPLE HUMP
LENGTH = 100.0 CM
NUMBER OF HUMPS = 2
HARMONIC = 2

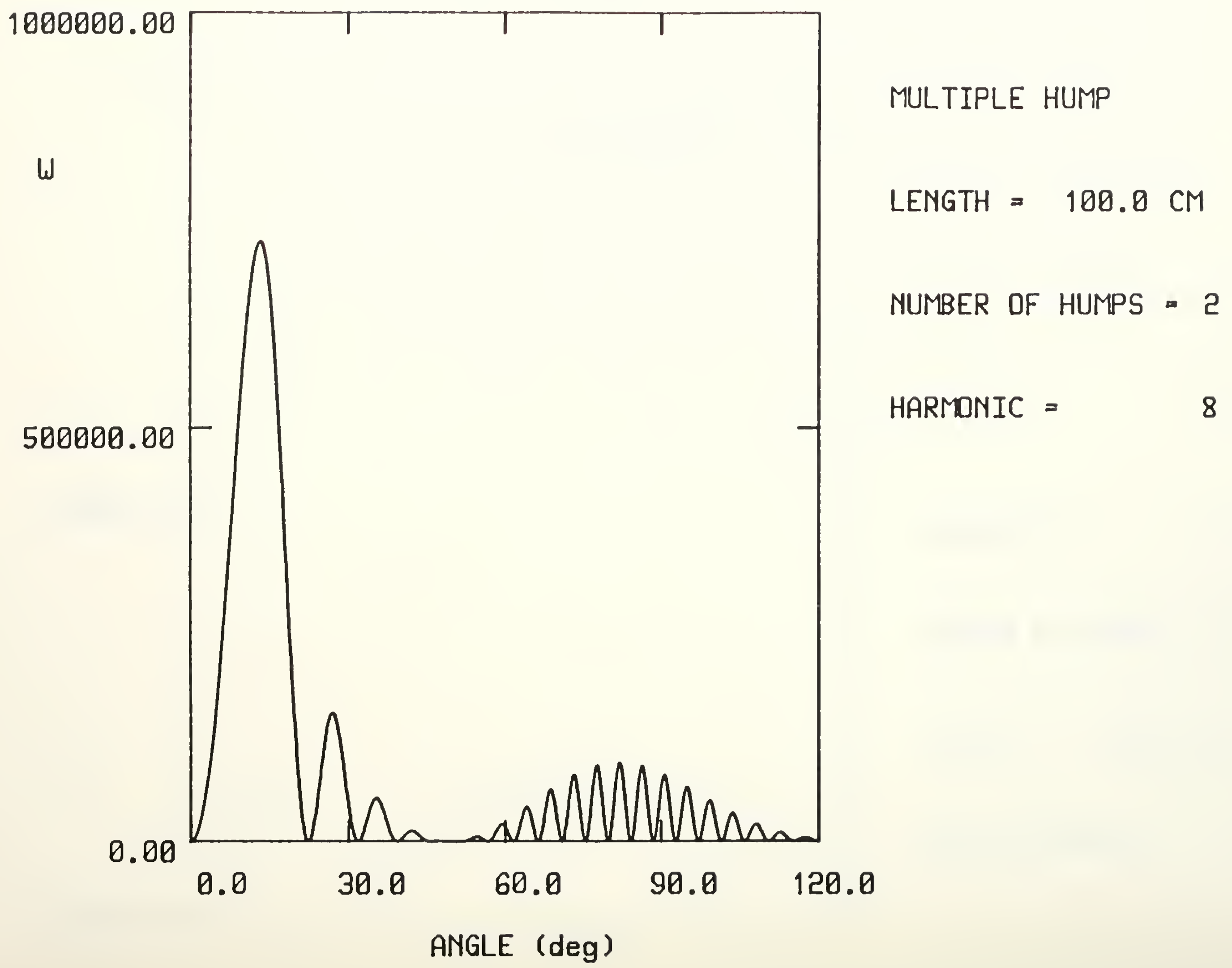

1500000.00
W
750000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
MULTIPLE HUMP
LENGTH = 100.0 CM
NUMBER OF HUMPS = 2
HARMONIC = 3

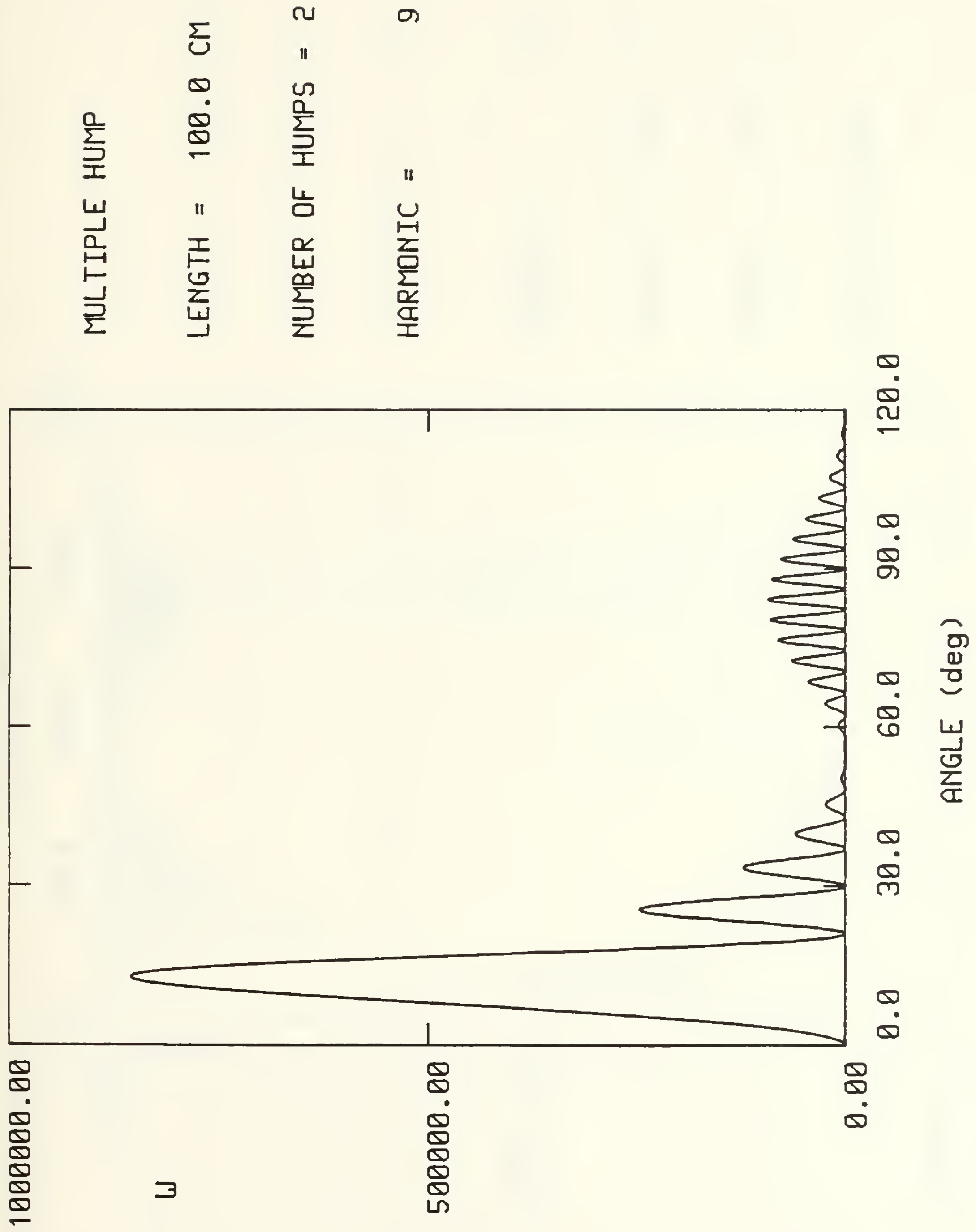

1500000.00
W
750000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
MULTIPLE HUMP
LENGTH = 100.0 CM
NUMBER OF HUMPS = 2
HARMONIC = 4

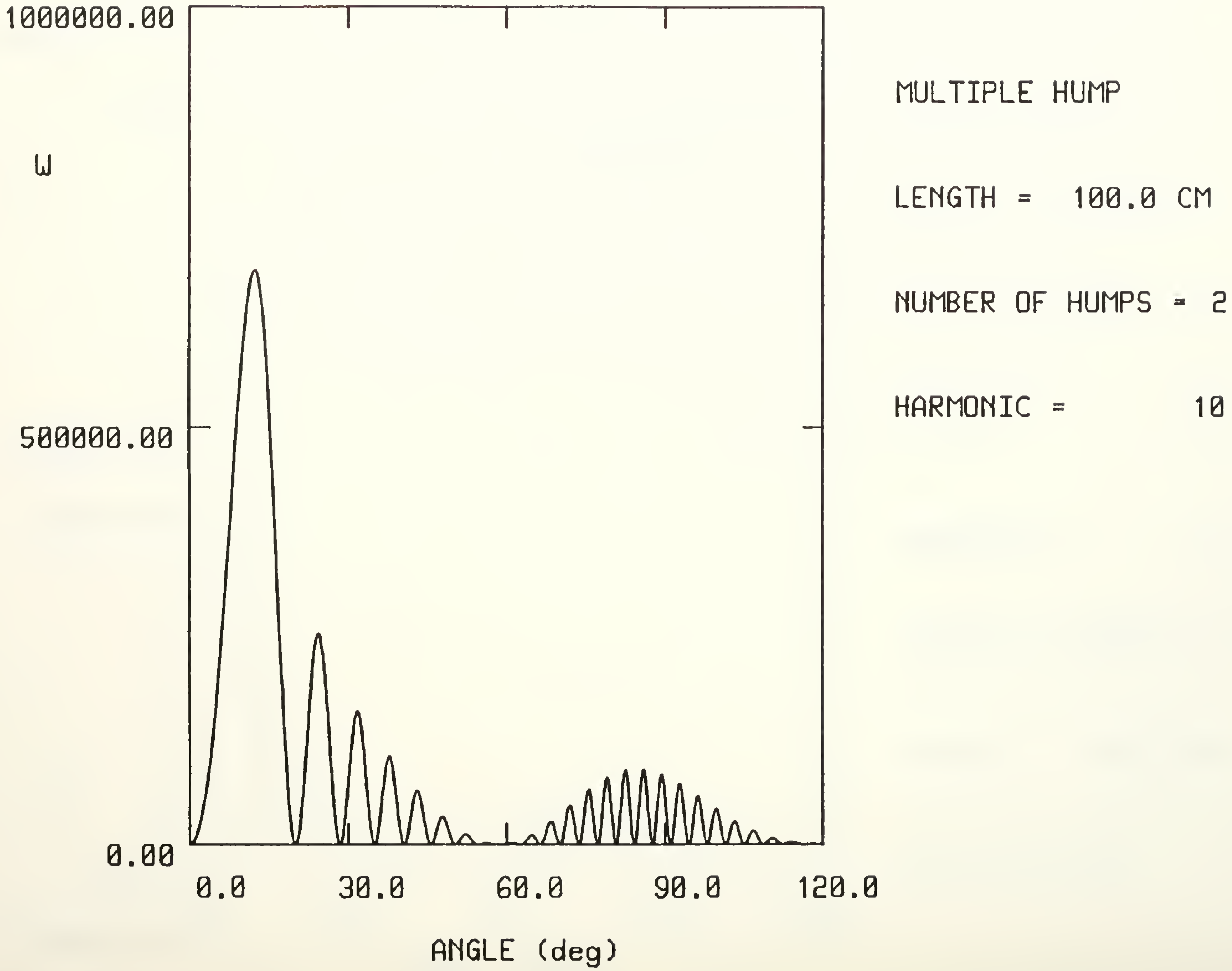

1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
MULTIPLE HUMP
LENGTH = 100.0 CM
NUMBER OF HUMPS = 2
HARMONIC = 5

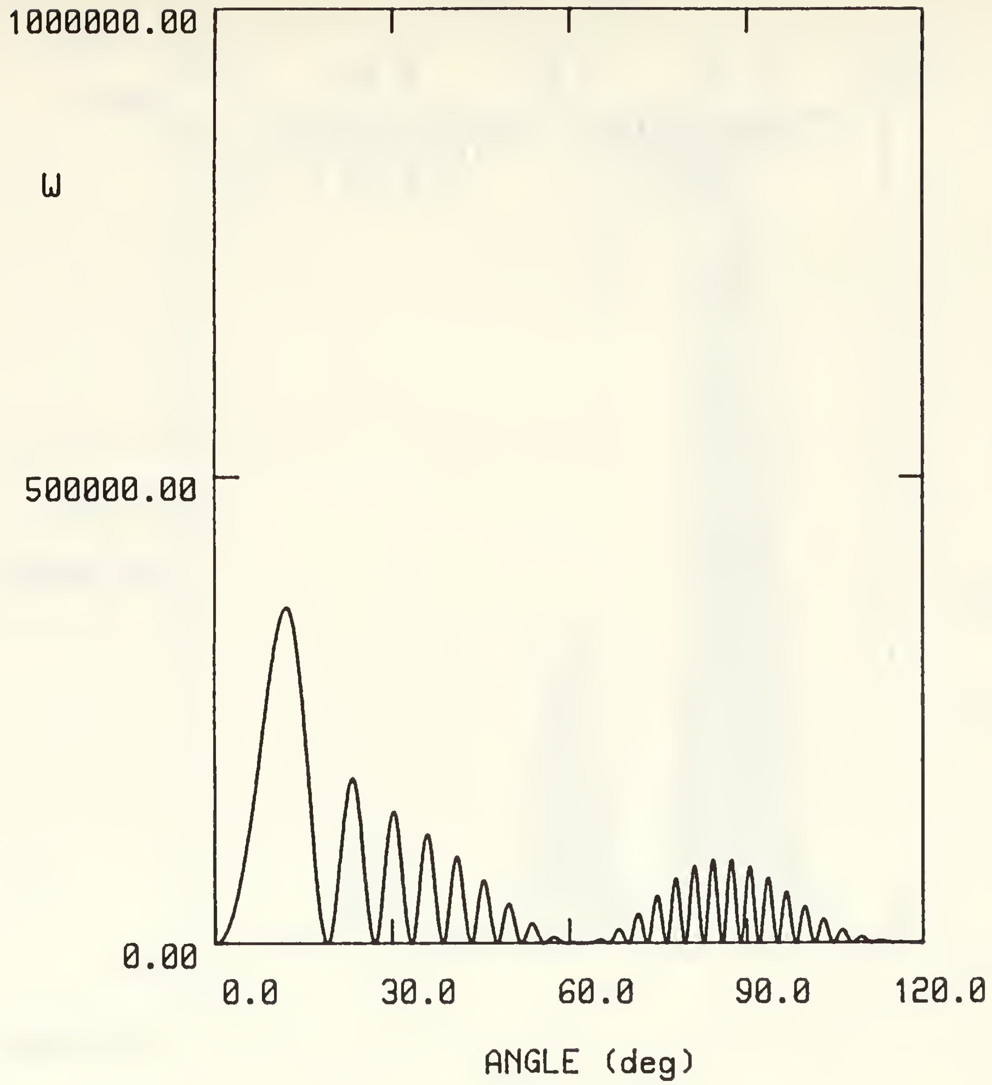

1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
MULTIPLE HUMP
LENGTH = 100.0 CM
NUMBER OF HUMPS = 2
HARMONIC = 6


1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
MULTIPLE HUMP
LENGTH = 100.0 CM
NUMBER OF HUMPS = 2
HARMONIC = 7


1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
MULTIPLE HUMP
LENGTH = 100.0 CM
NUMBER OF HUMPS = 2
HARMONIC = 8


MULTIPLE HUMP
LENGTH = 100.0 CM
NUMBER OF HUMPS = 2
HARMONIC = 9
W
1000000.00
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)


1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
MULTIPLE HUMP
LENGTH = 100.0 CM
NUMBER OF HUMPS = 2
HARMONIC = 10

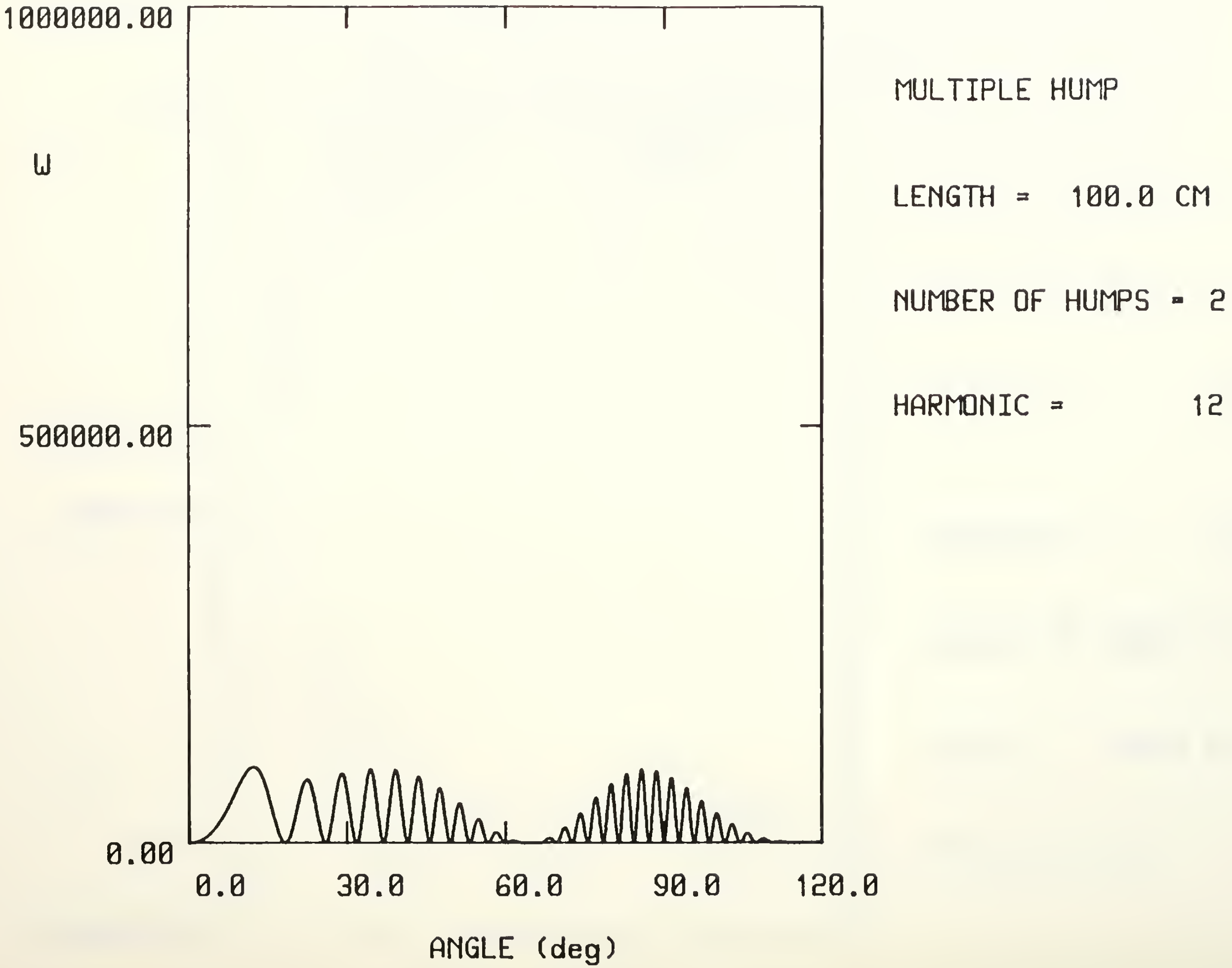


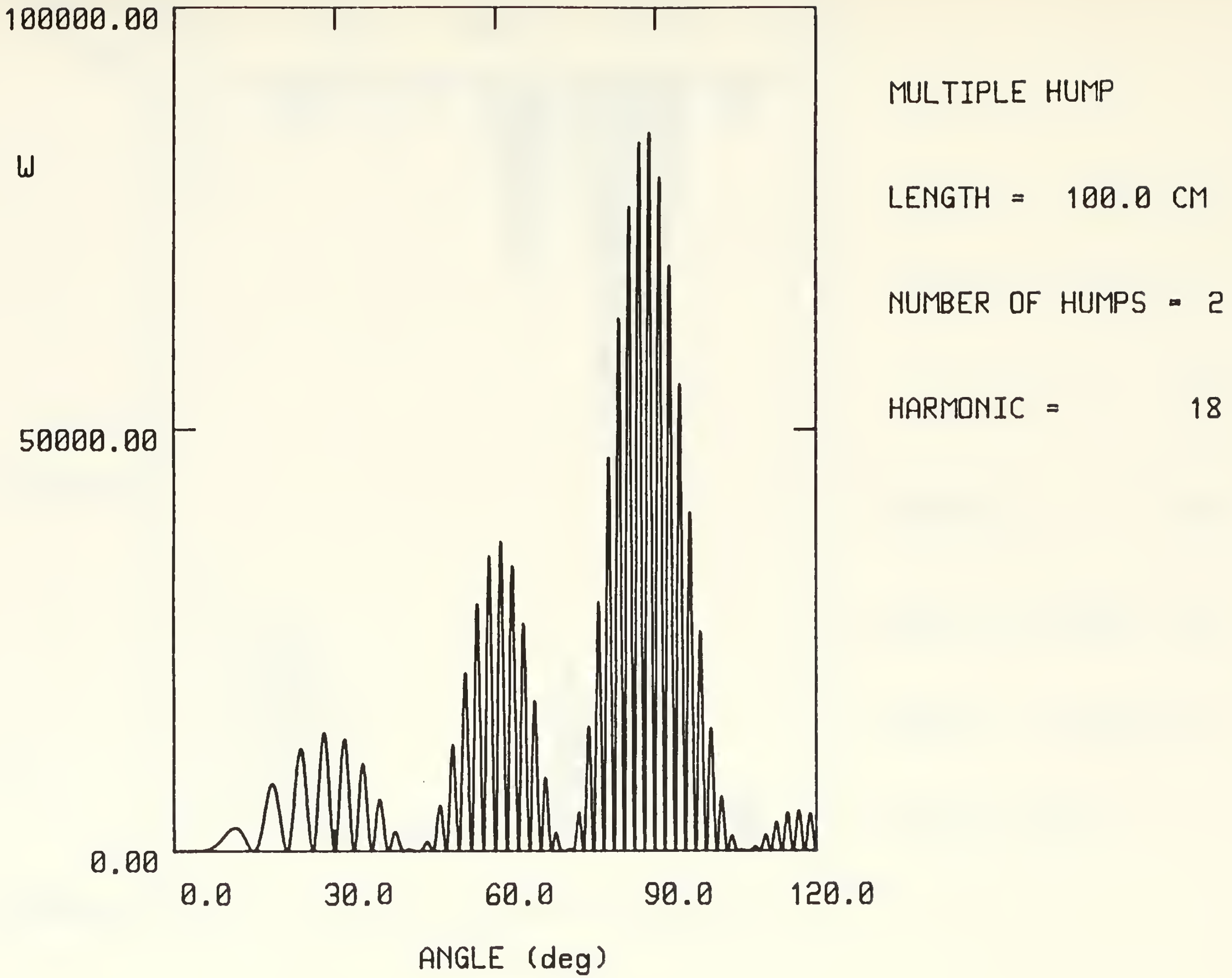
MULTIPLE HUMP

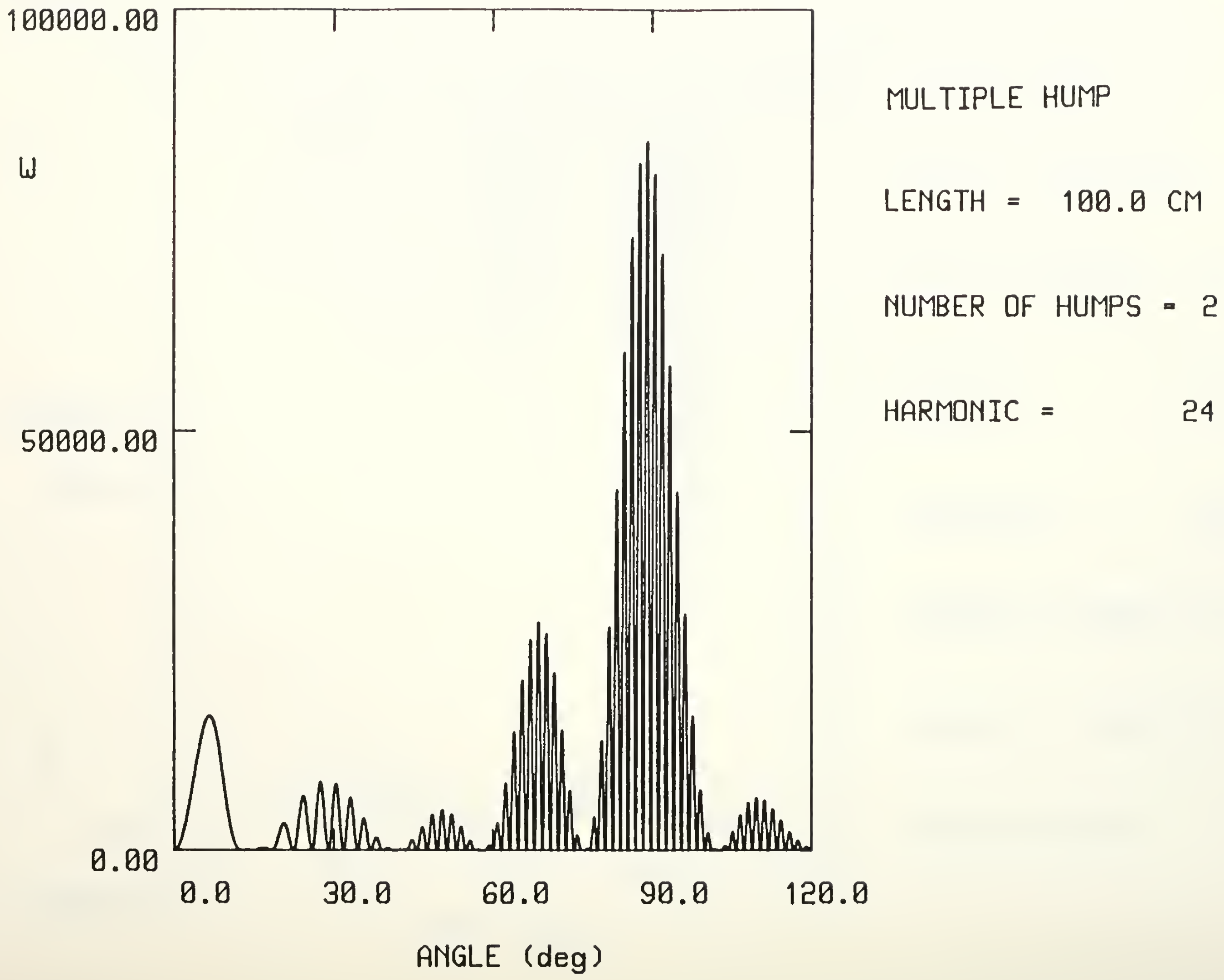
LENGTH = 100.0 CM

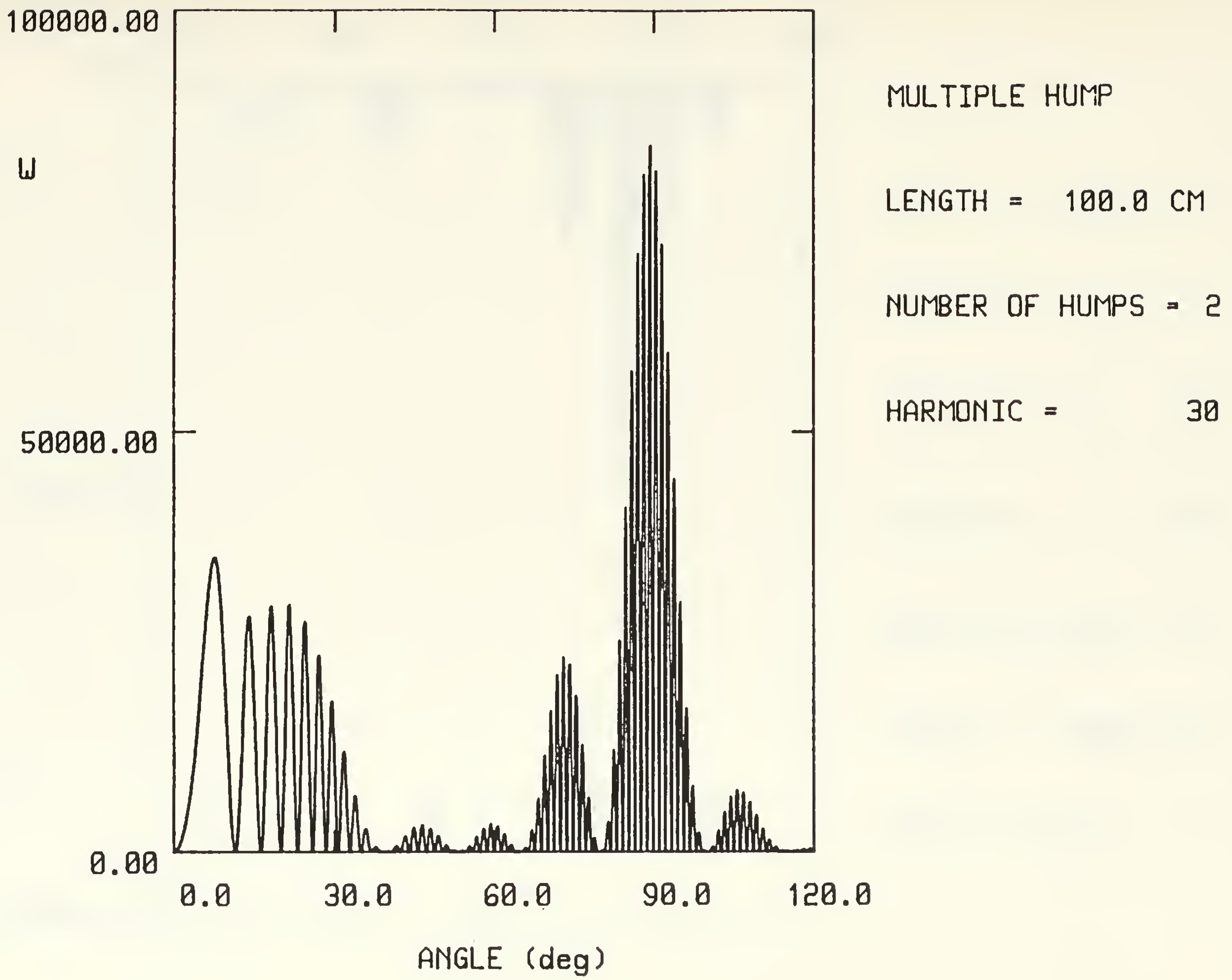
NUMBER OF HUMPS = 2

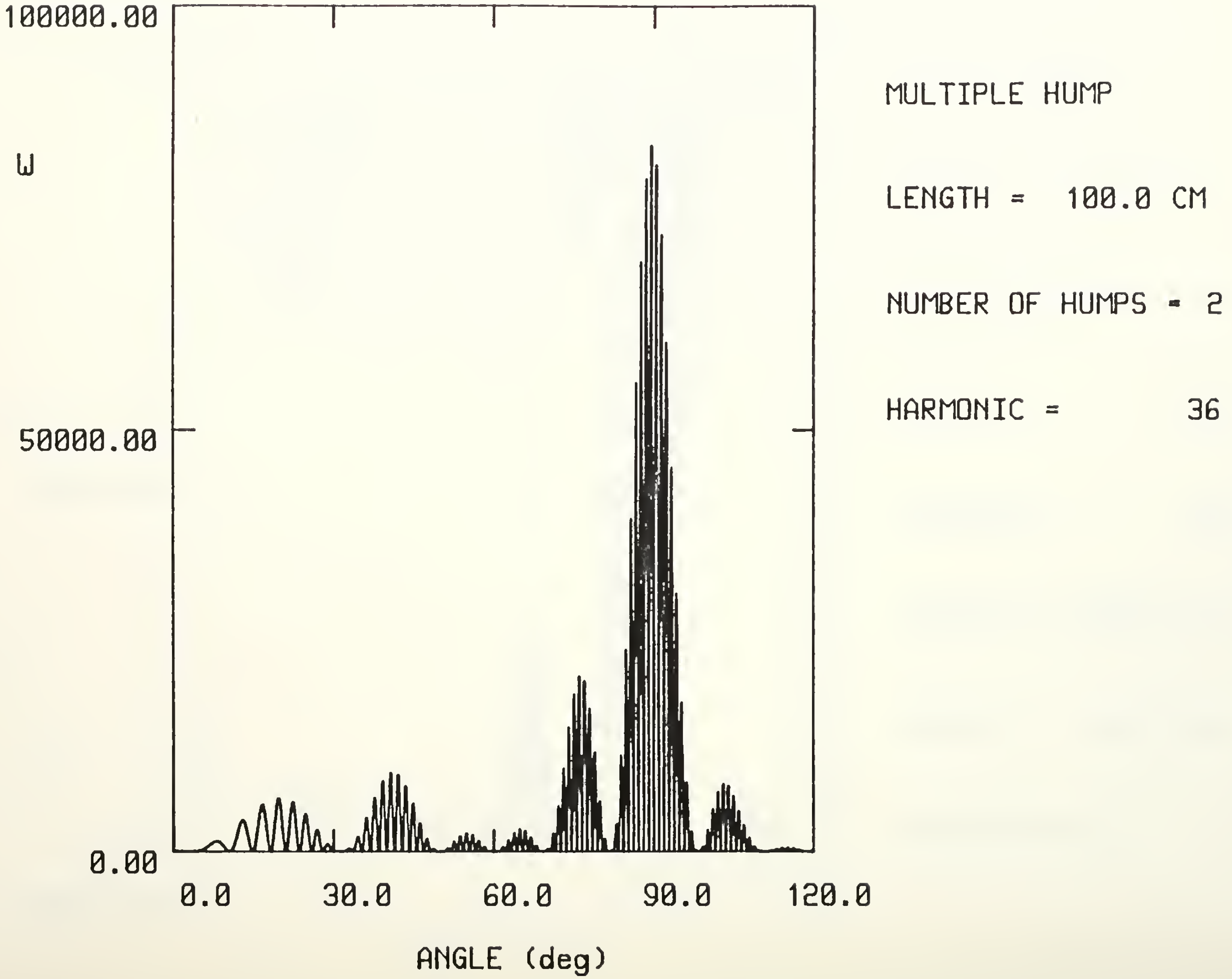
HARMONIC = 11

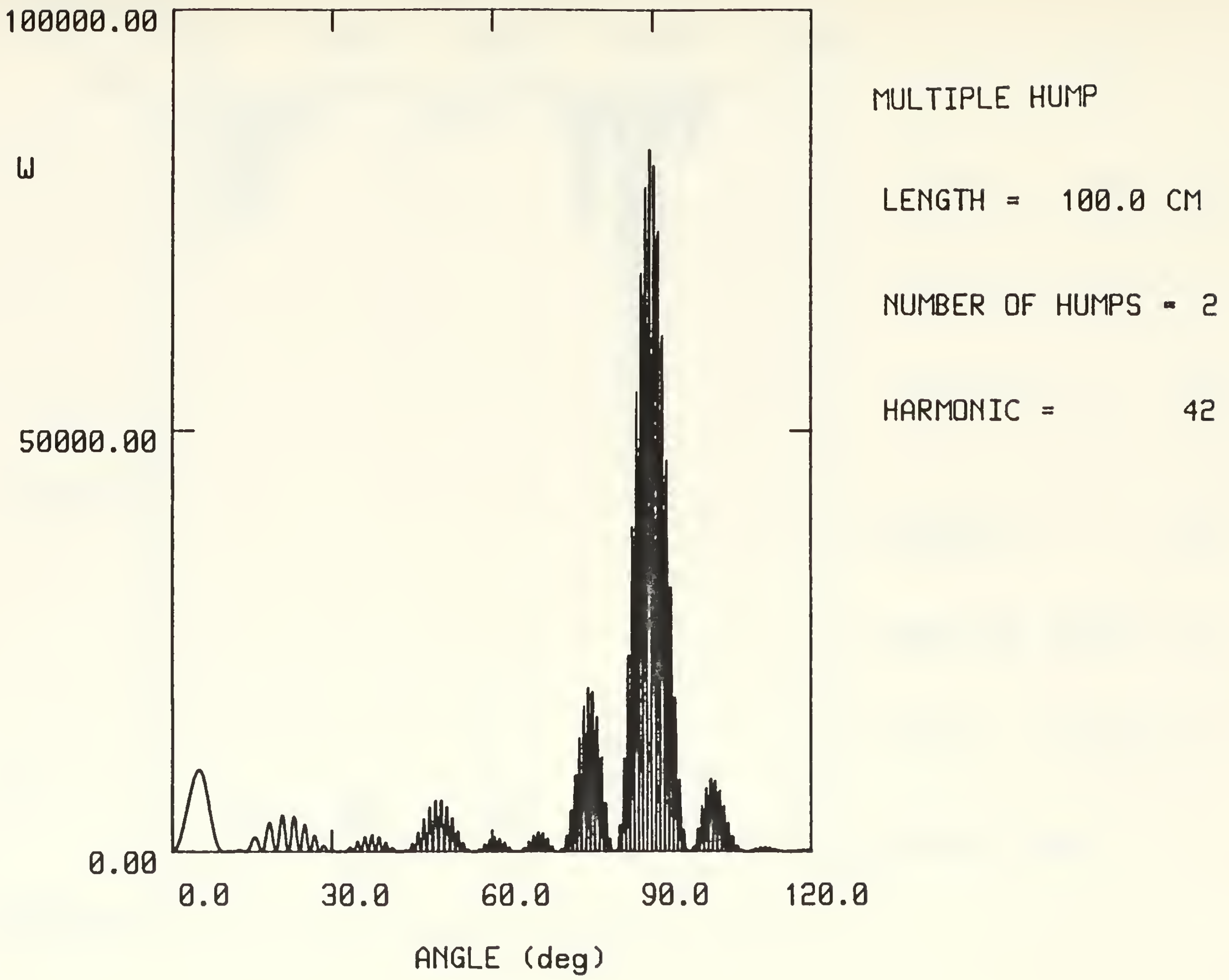

1000000.00
W
500000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
MULTIPLE HUMP
LENGTH = 100.0 CM
NUMBER OF HUMPS = 2
HARMONIC = 12

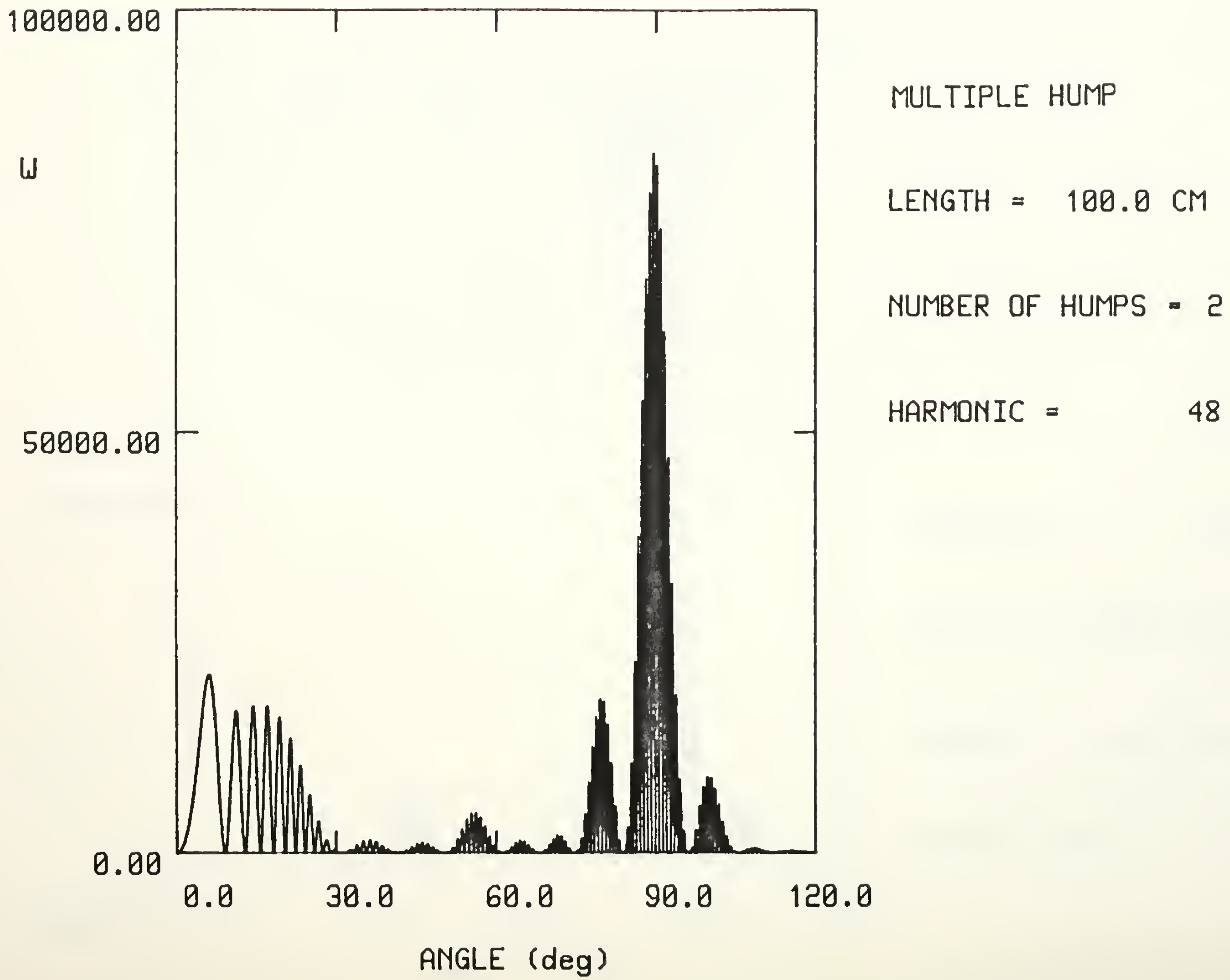

100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
MULTIPLE HUMP
LENGTH = 100.0 CM
NUMBER OF HUMPS = 2
HARMONIC = 18


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
MULTIPLE HUMP
LENGTH = 100.0 CM
NUMBER OF HUMPS = 2
HARMONIC = 24


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
MULTIPLE HUMP
LENGTH = 100.0 CM
NUMBER OF HUMPS = 2
HARMONIC = 30


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
MULTIPLE HUMP
LENGTH = 100.0 CM
NUMBER OF HUMPS = 2
HARMONIC = 36


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
MULTIPLE HUMP
LENGTH = 100.0 CM
NUMBER OF HUMPS = 2
HARMONIC = 42


100000.00
W
50000.00
0.00
0.0
30.0
60.0
90.0
120.0
ANGLE (deg)
MULTIPLE HUMP
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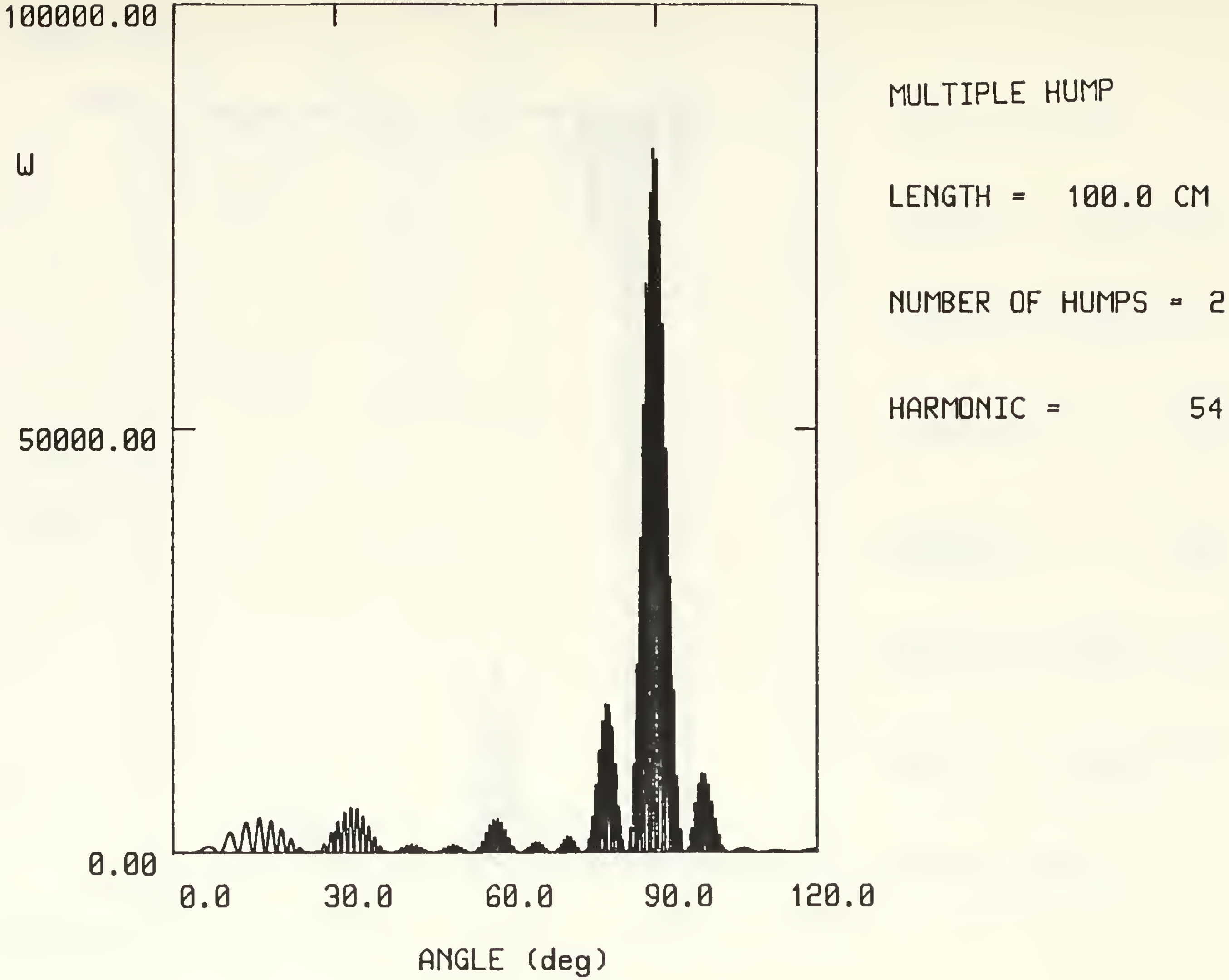
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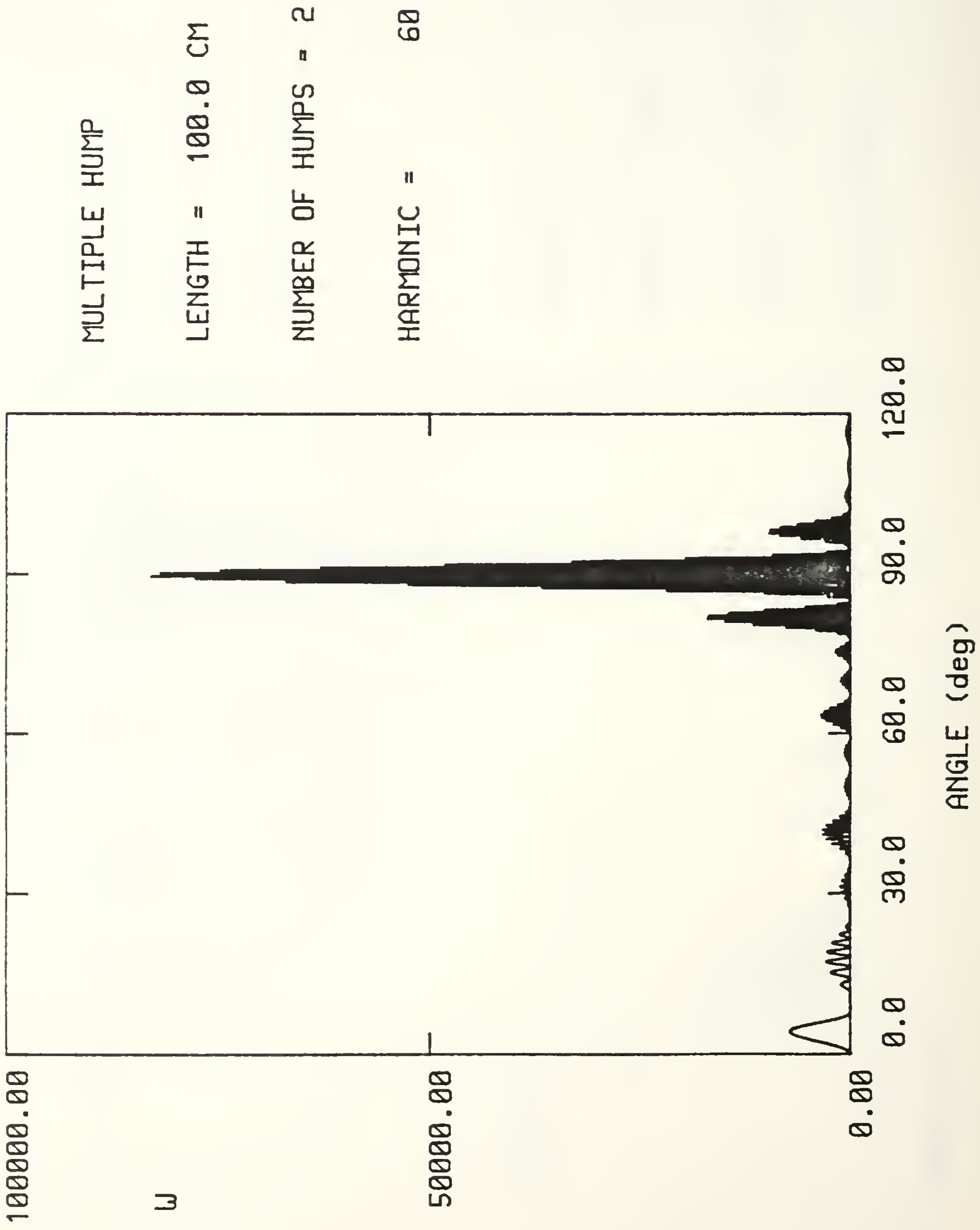
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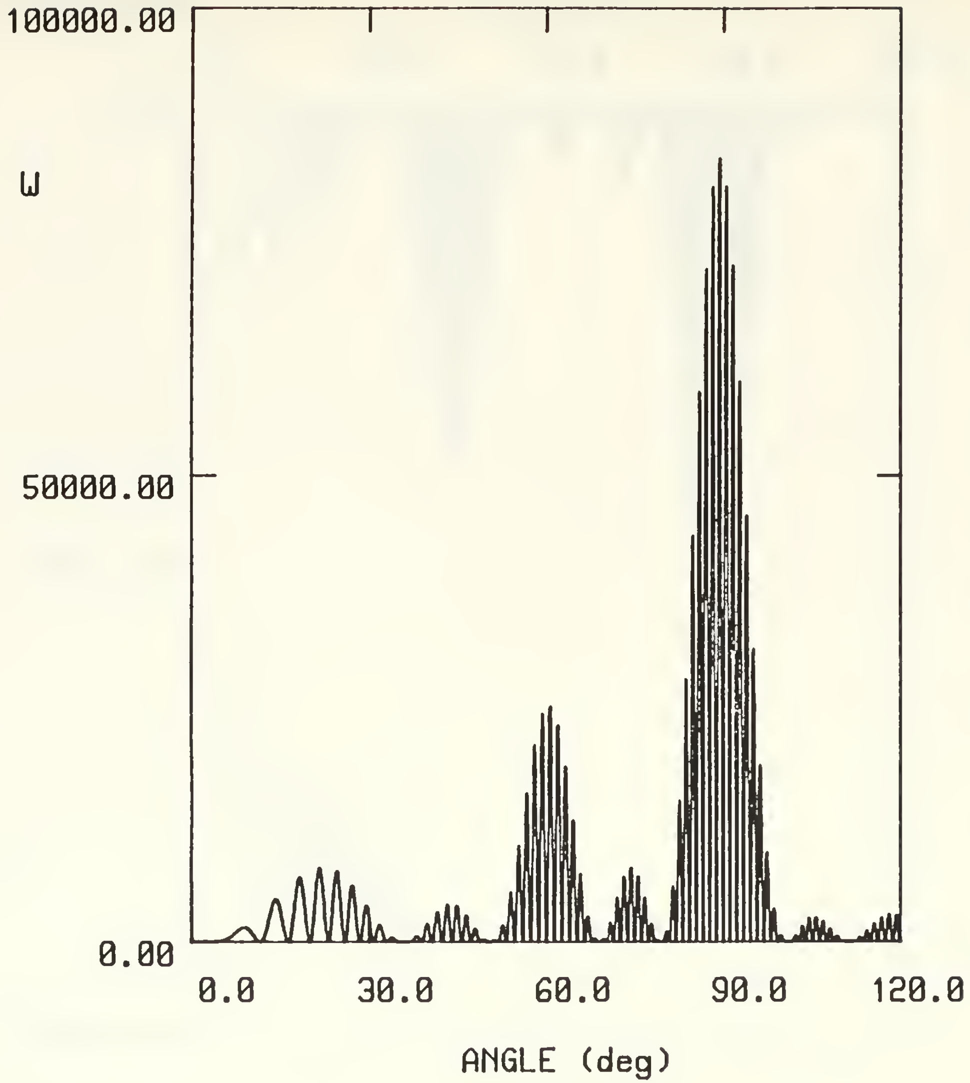

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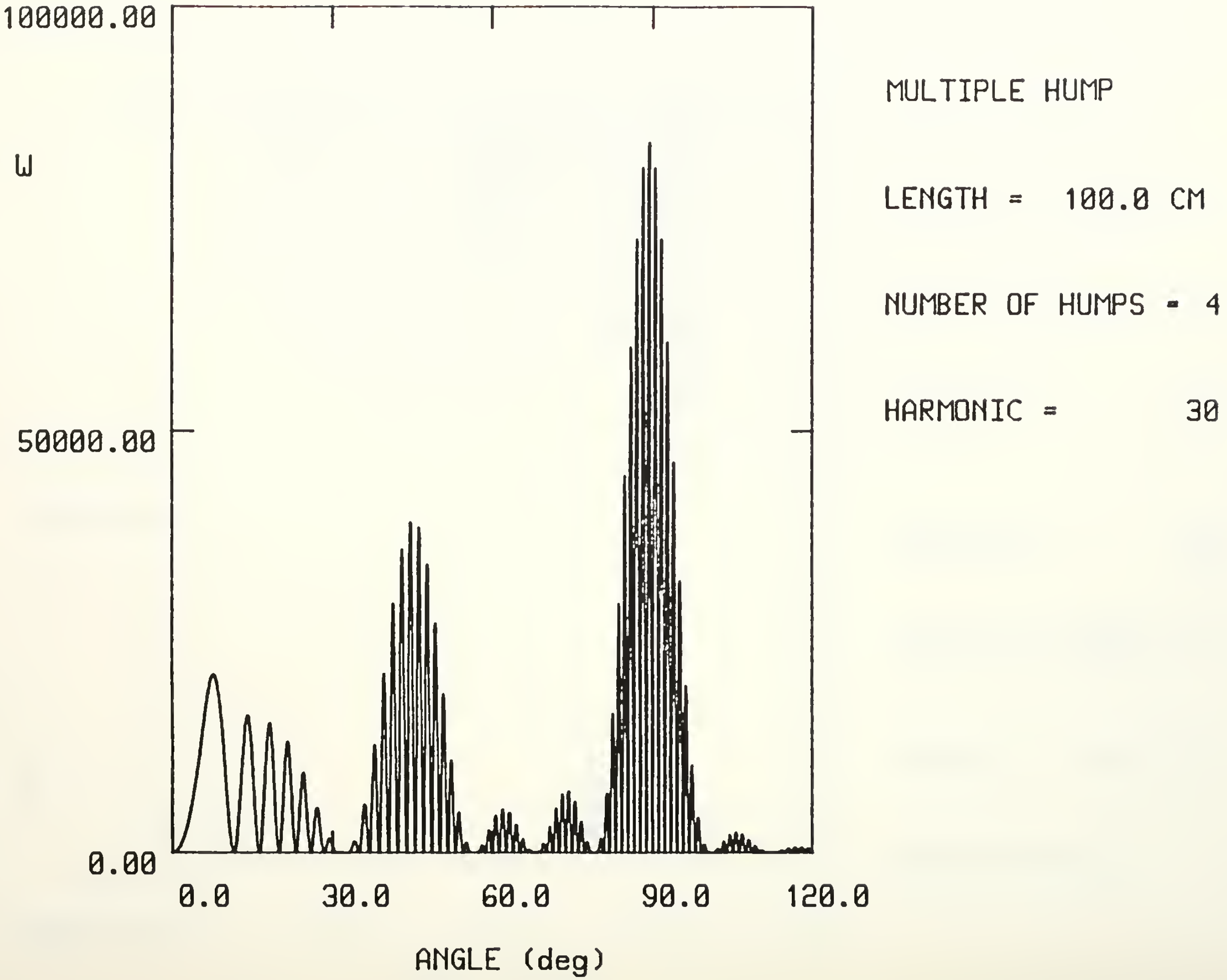
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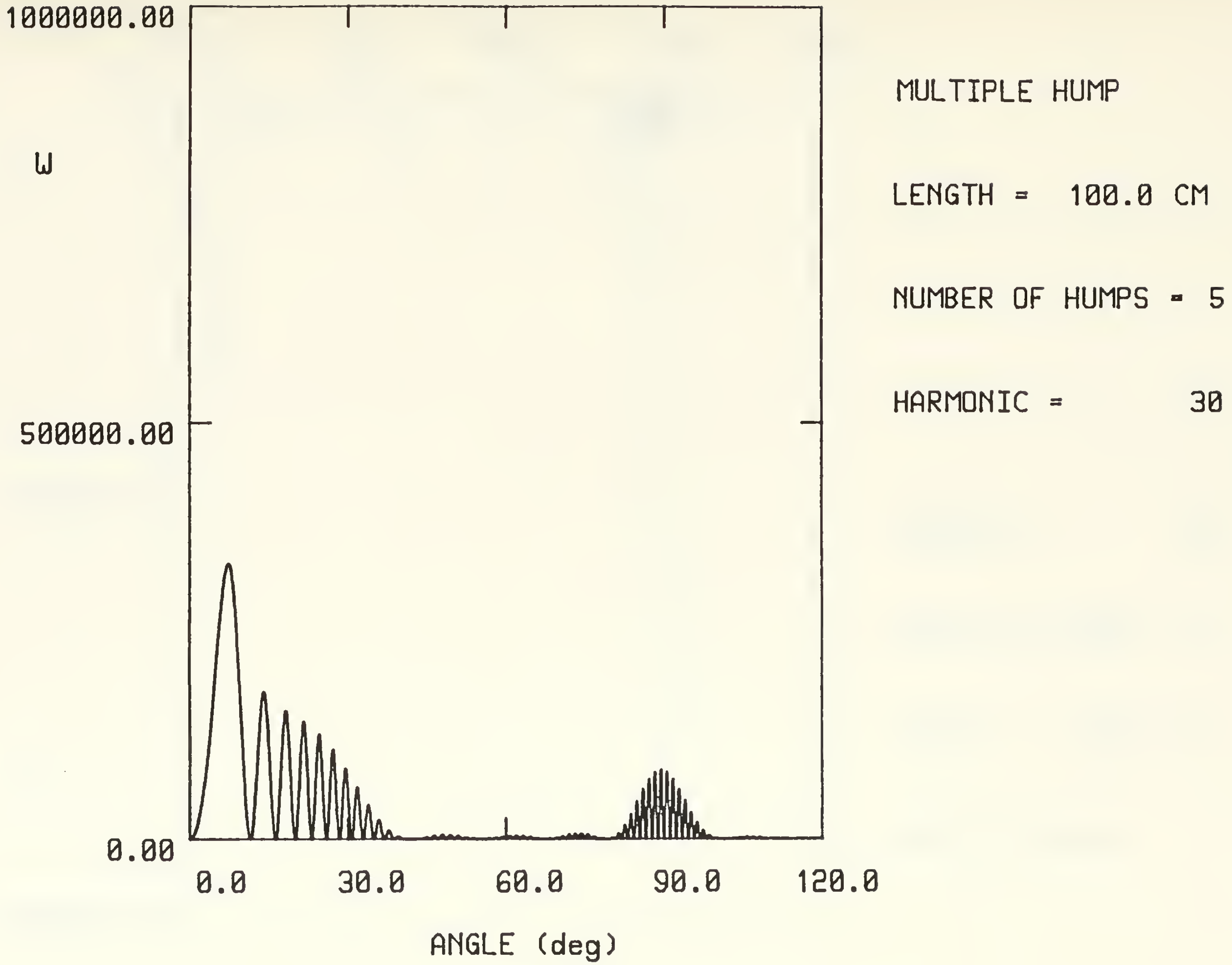
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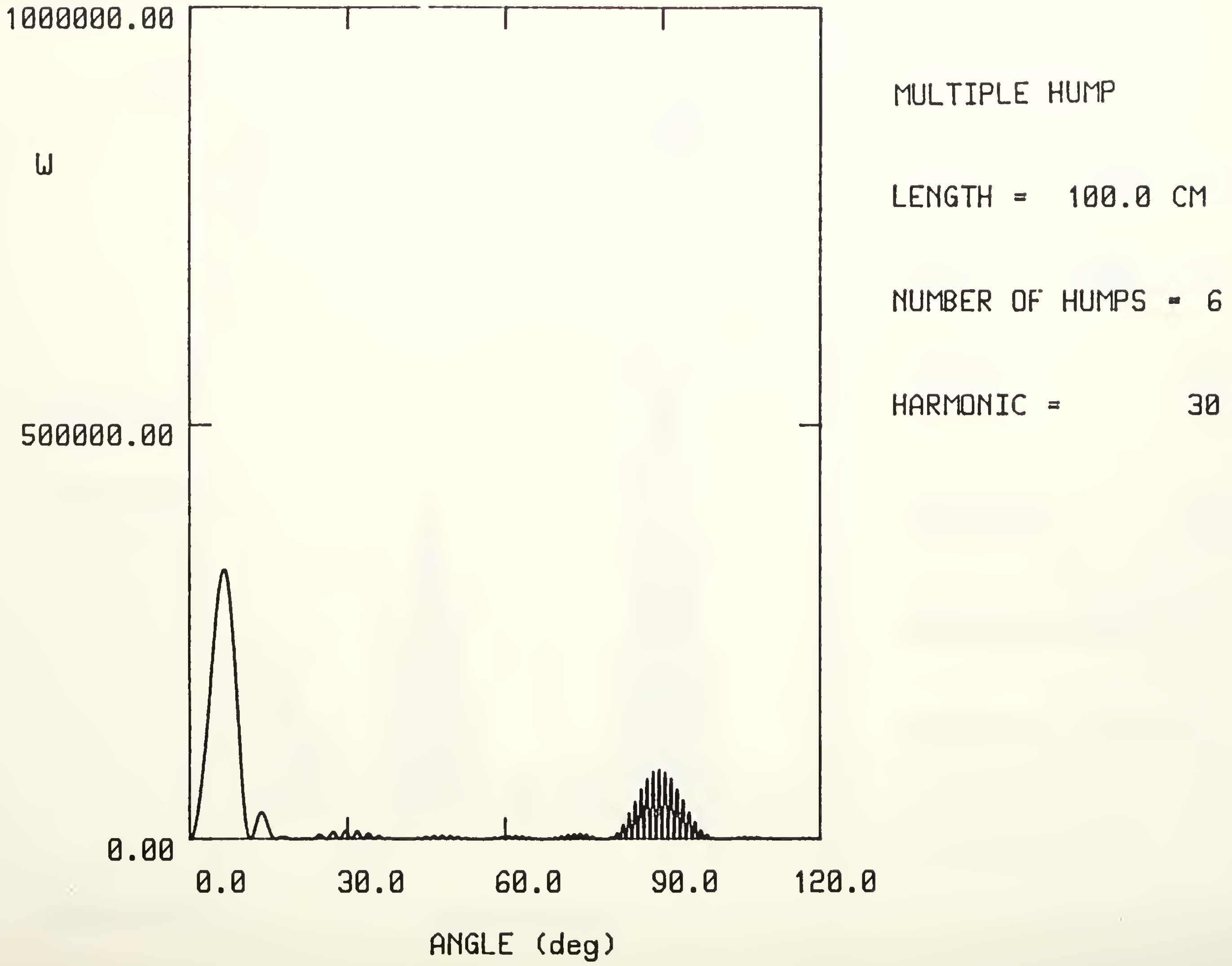
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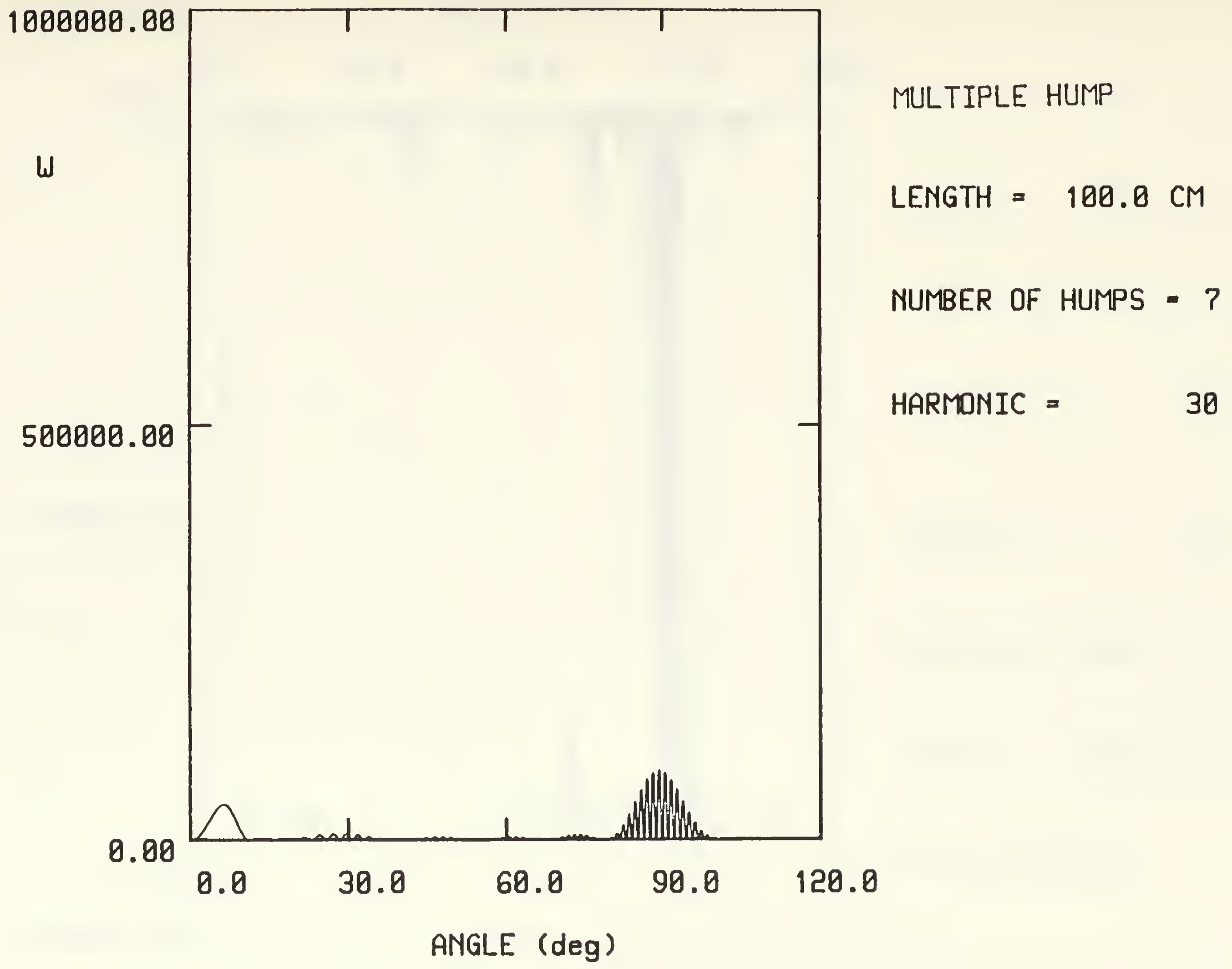
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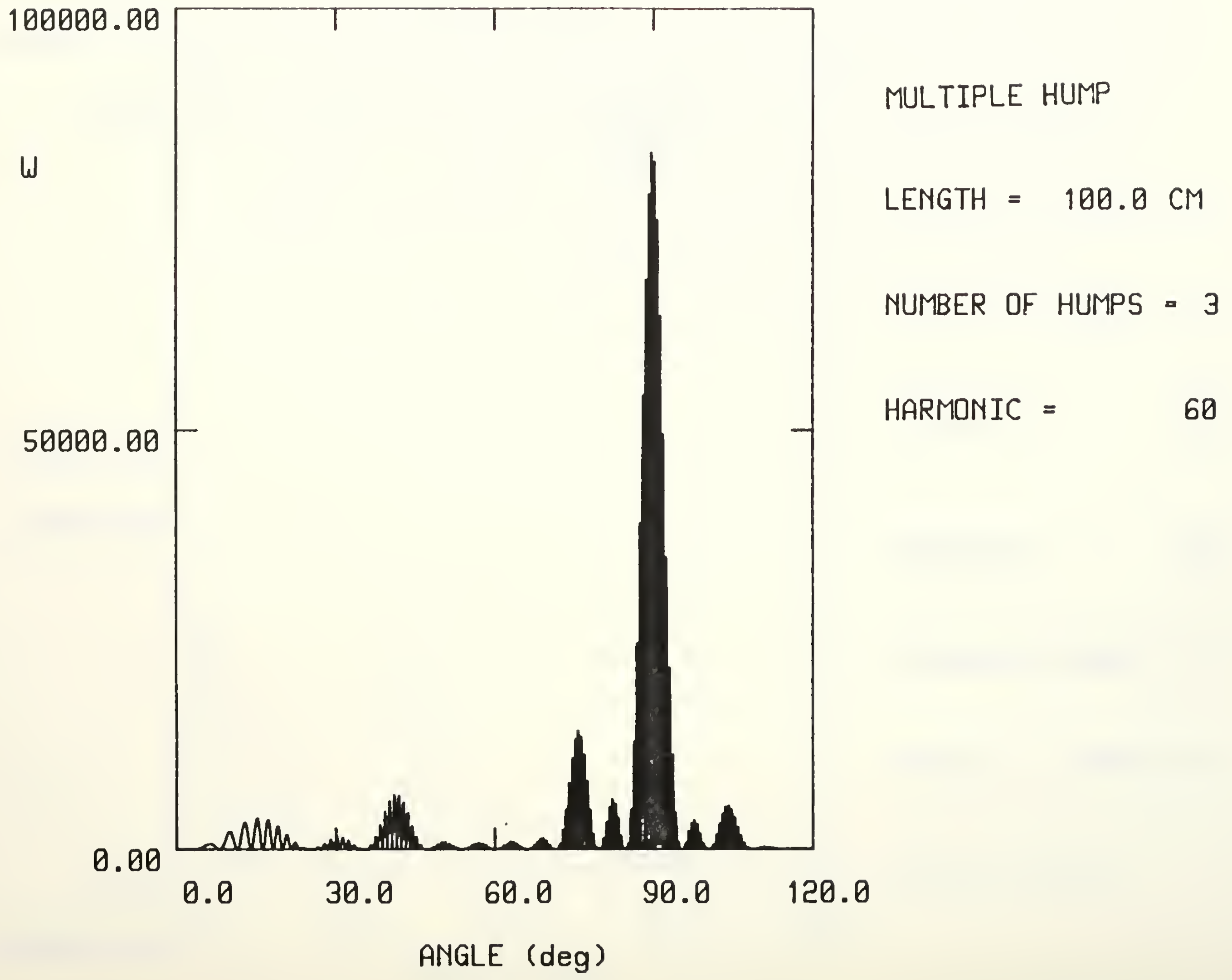
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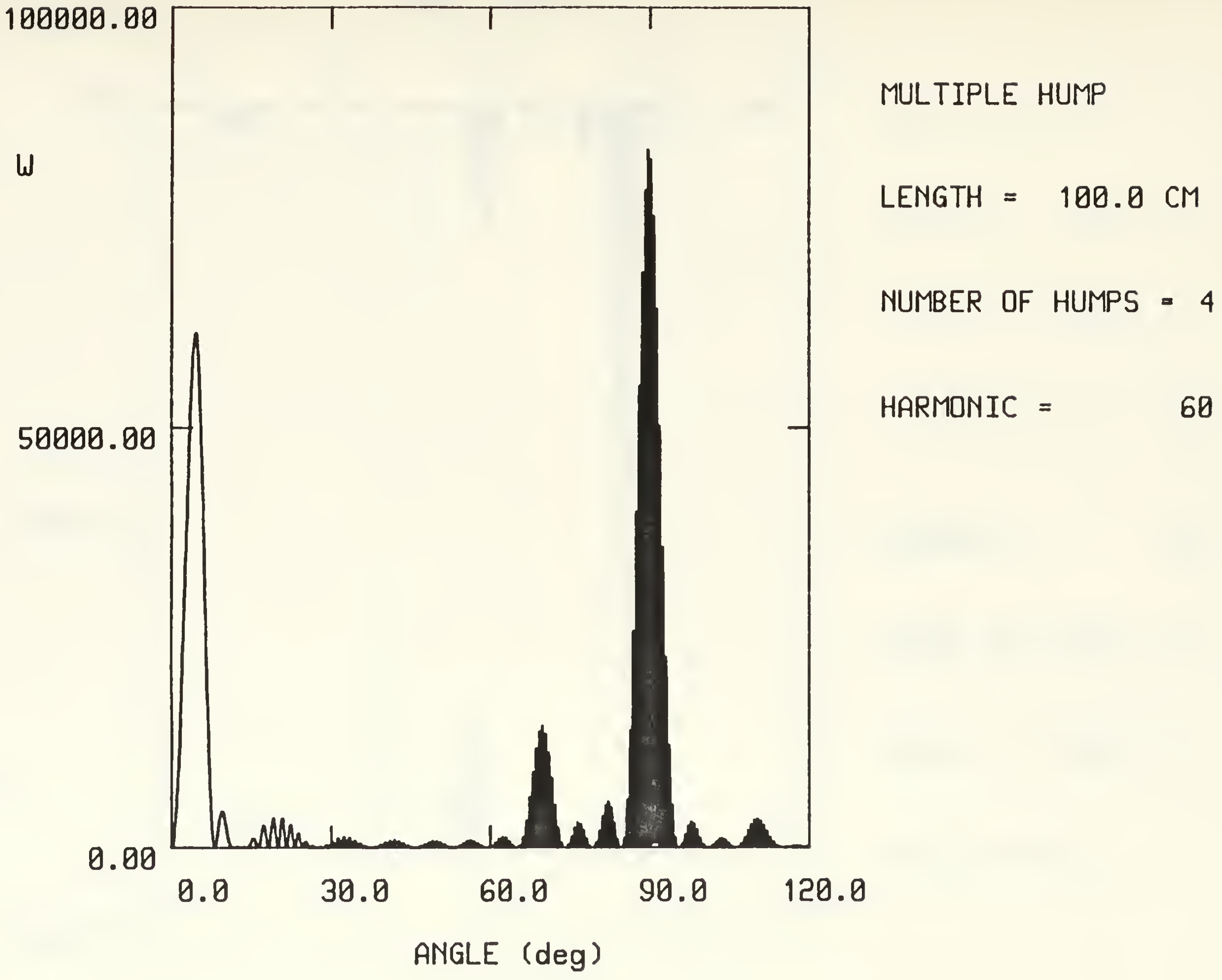
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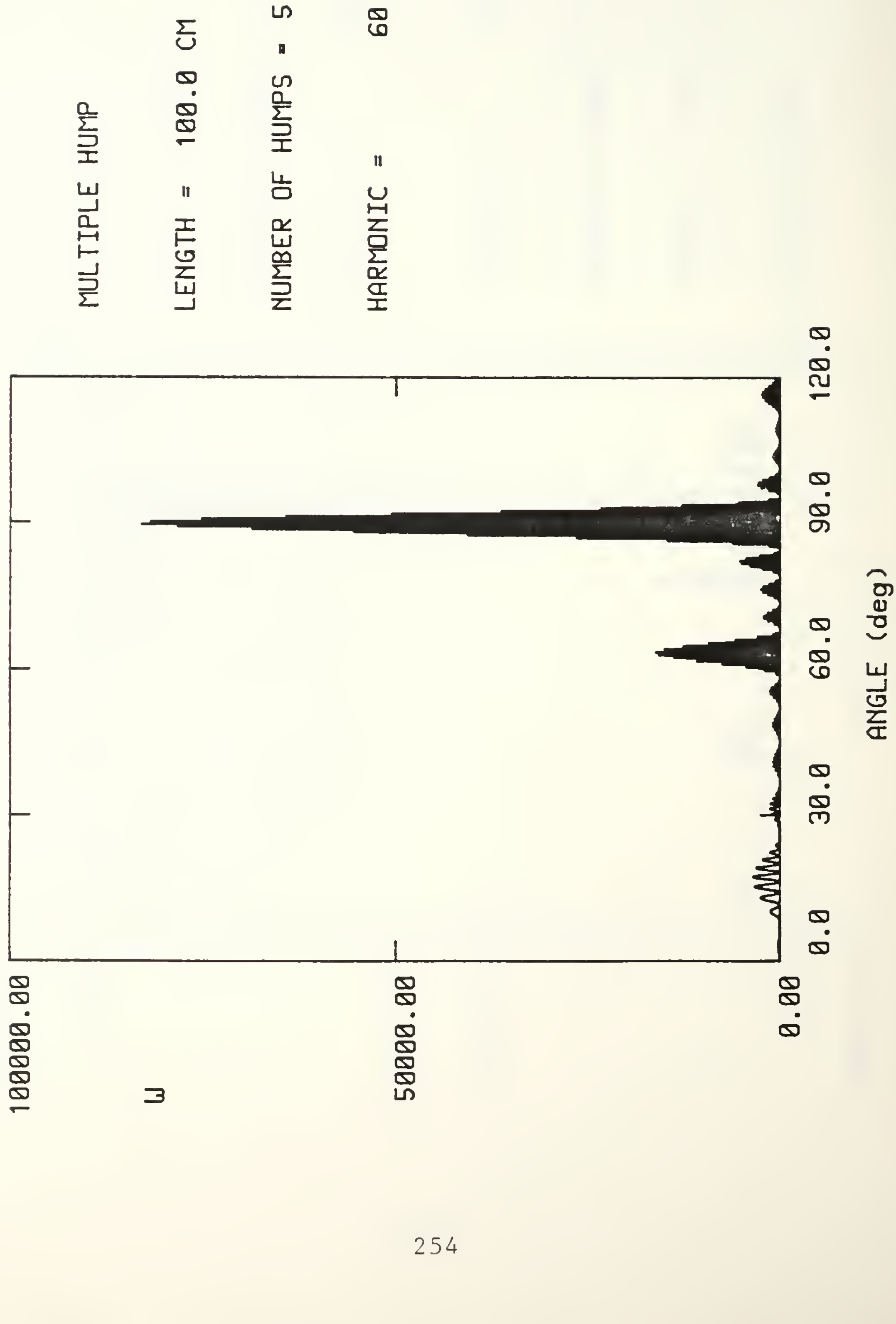
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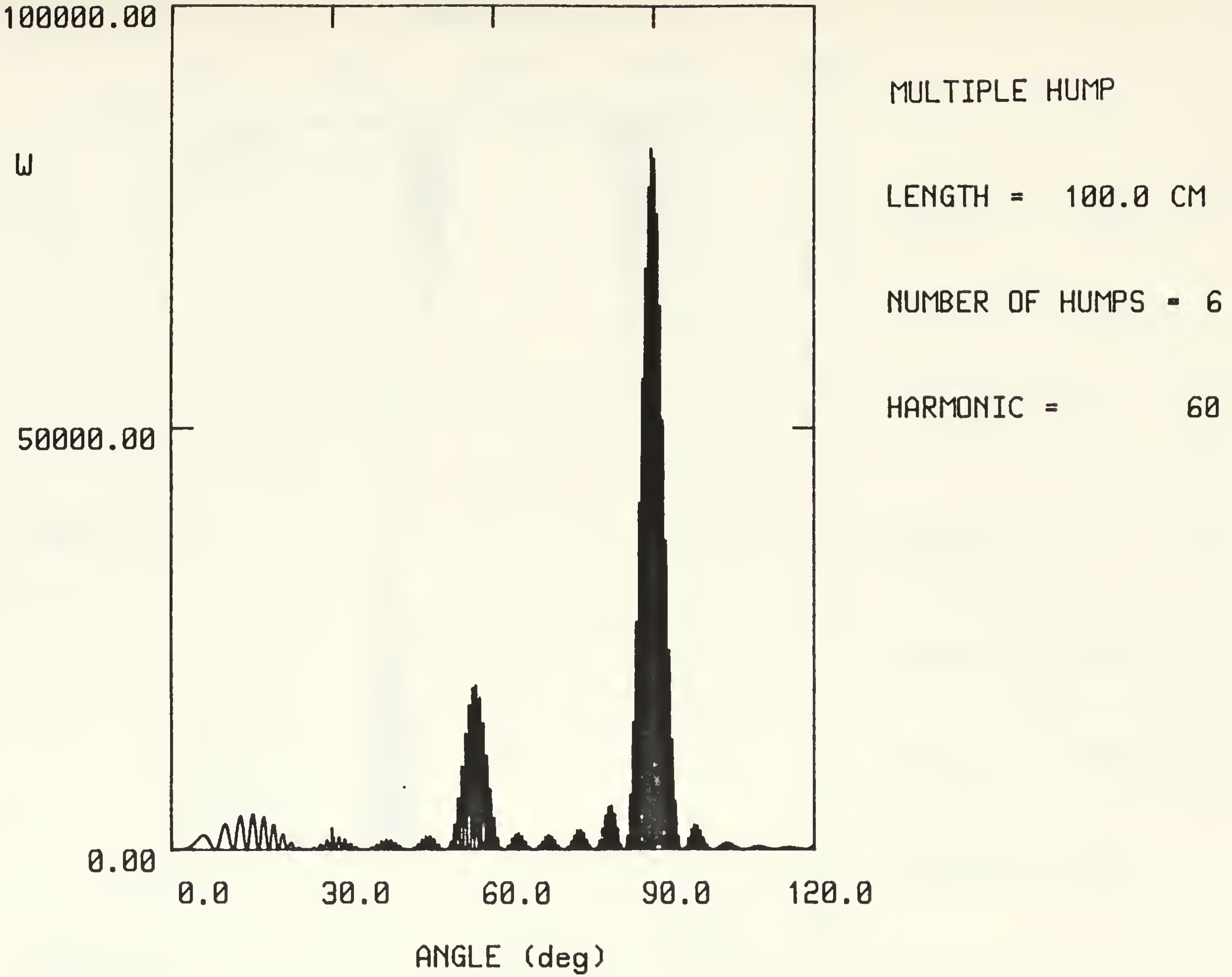
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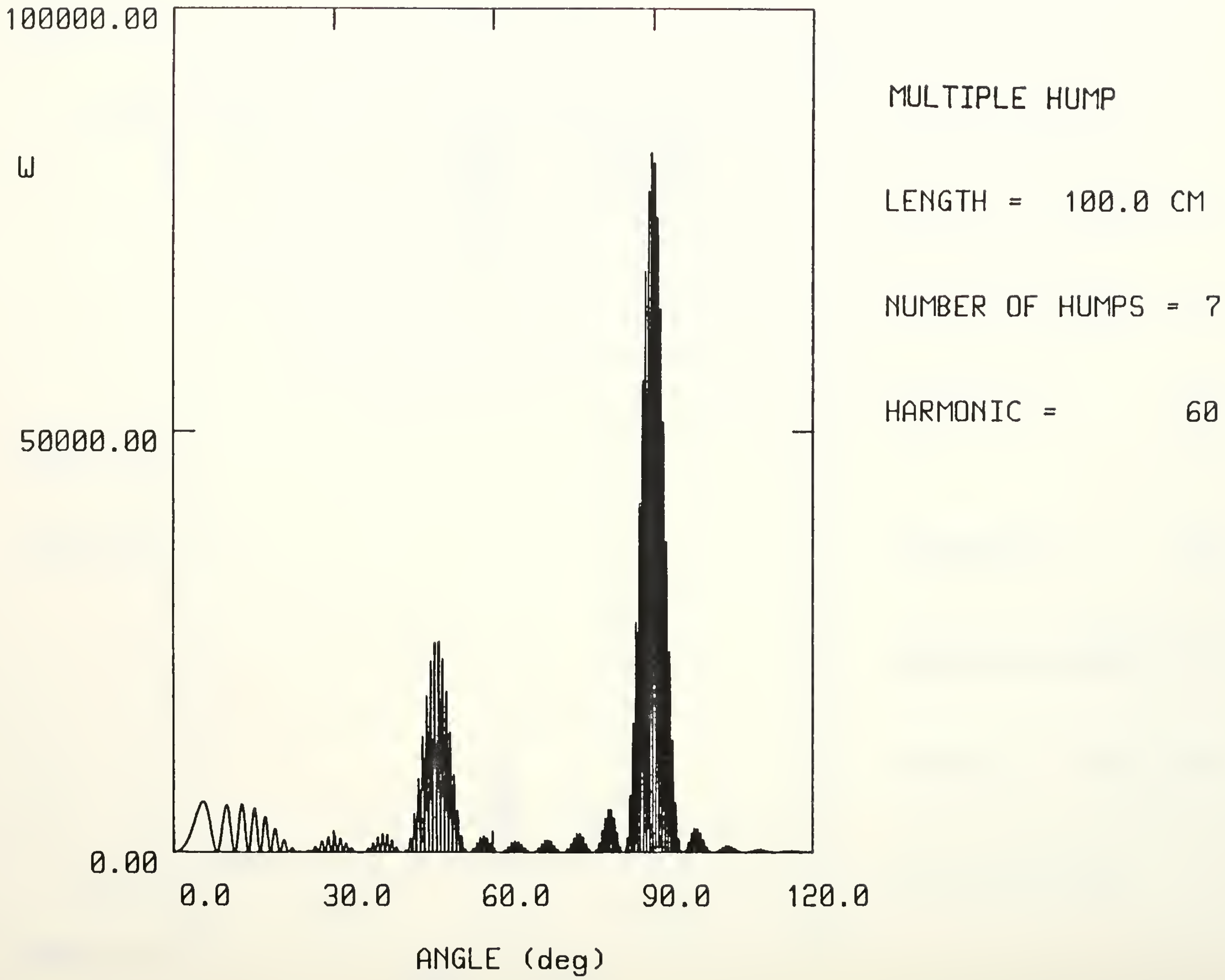
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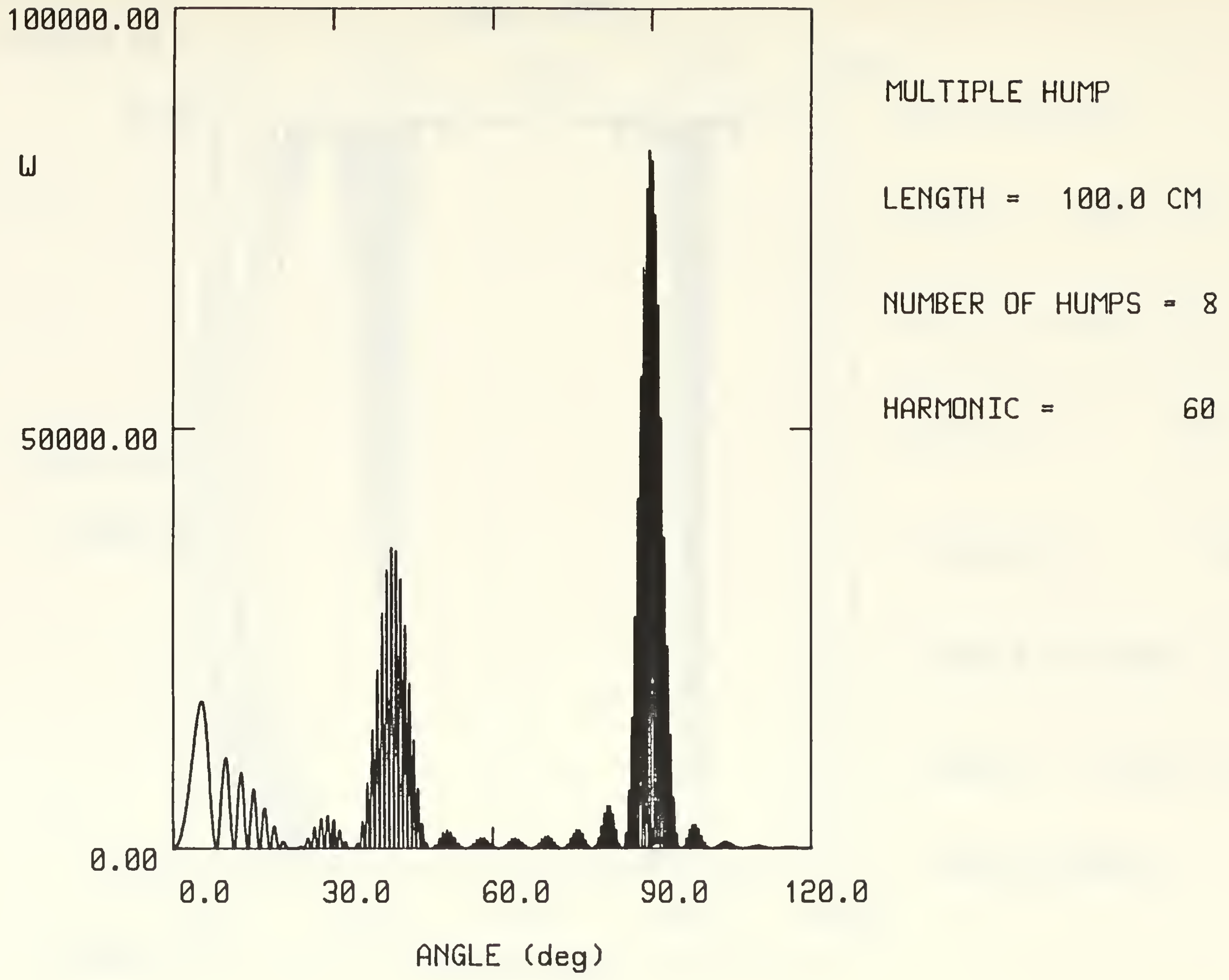
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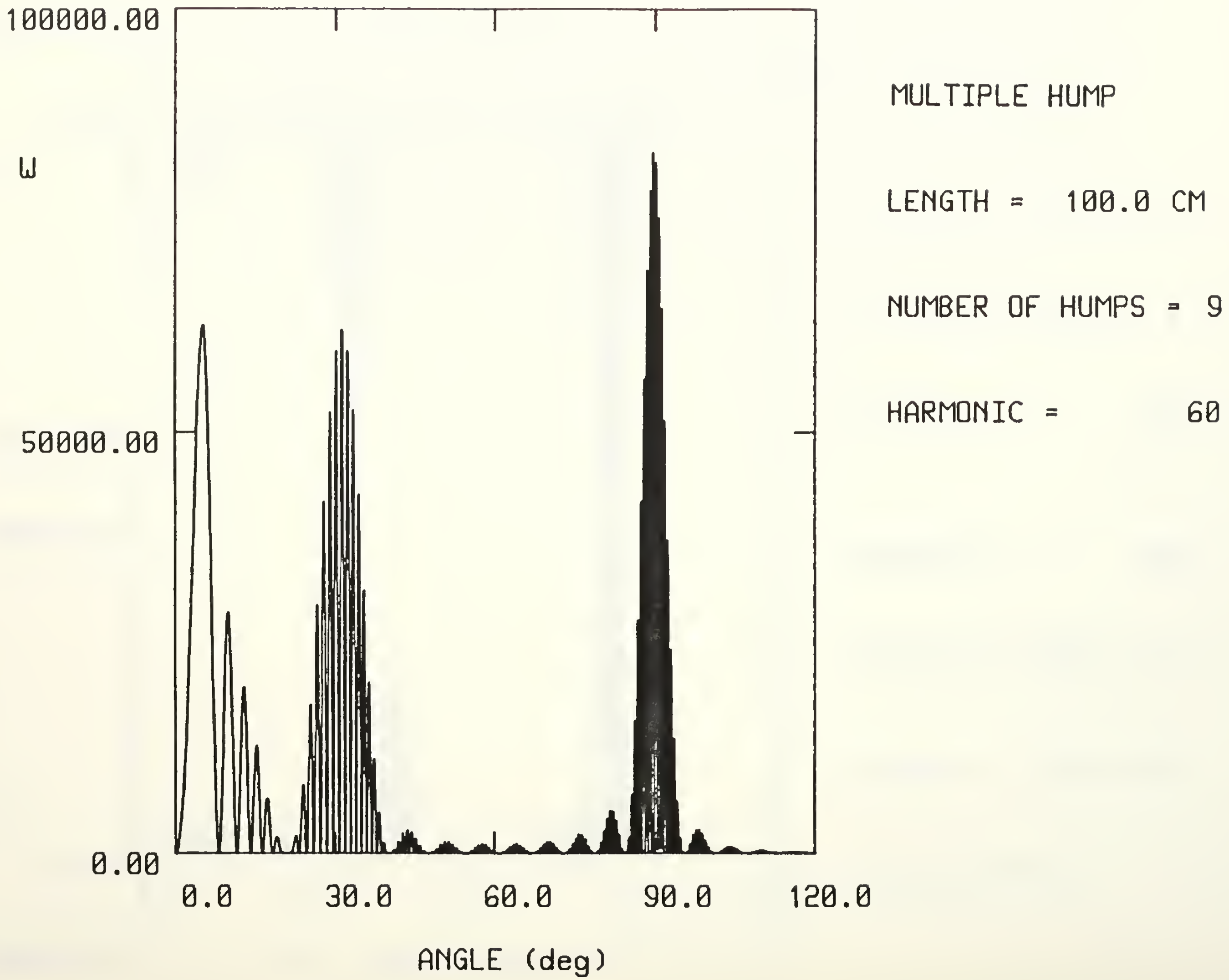
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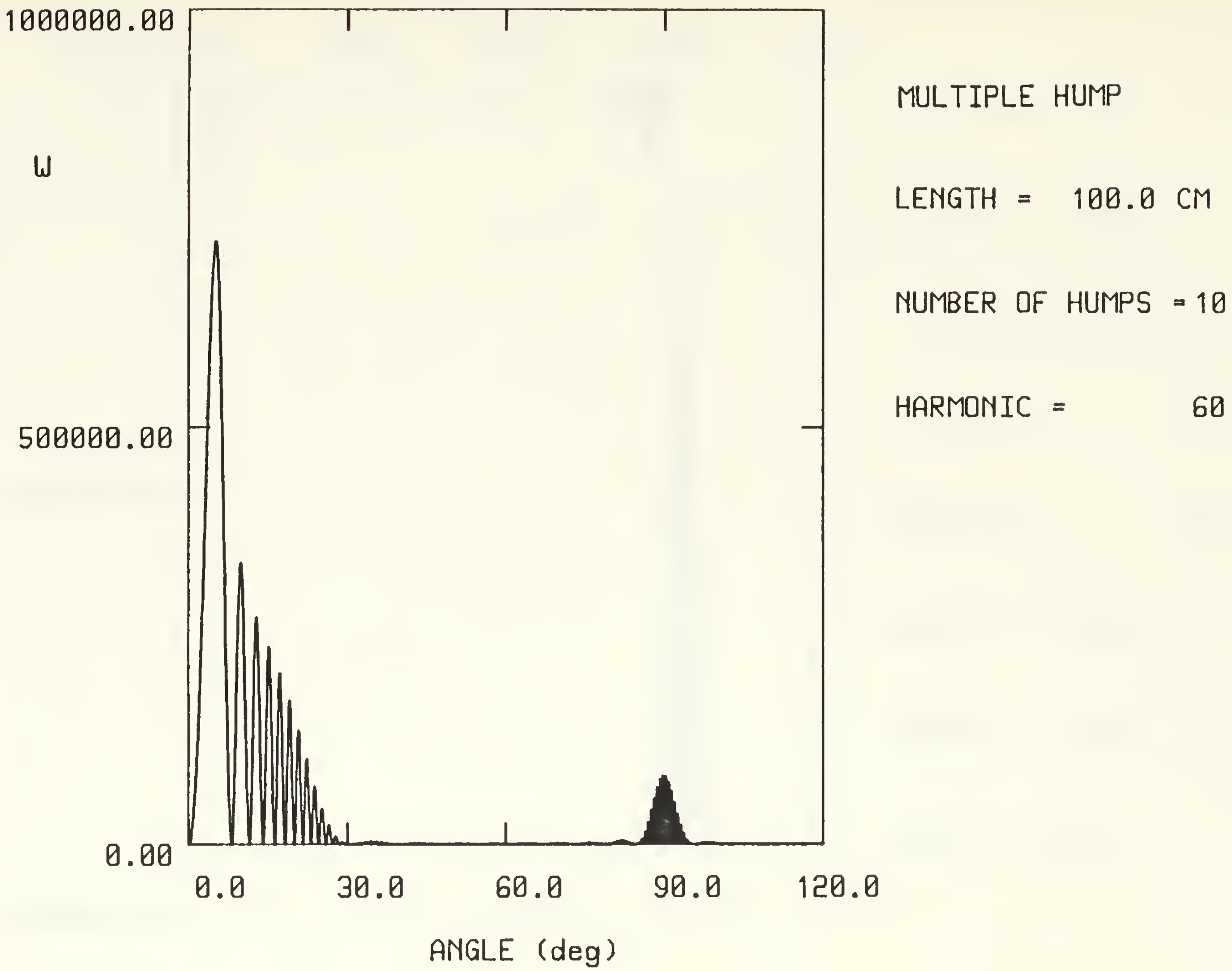
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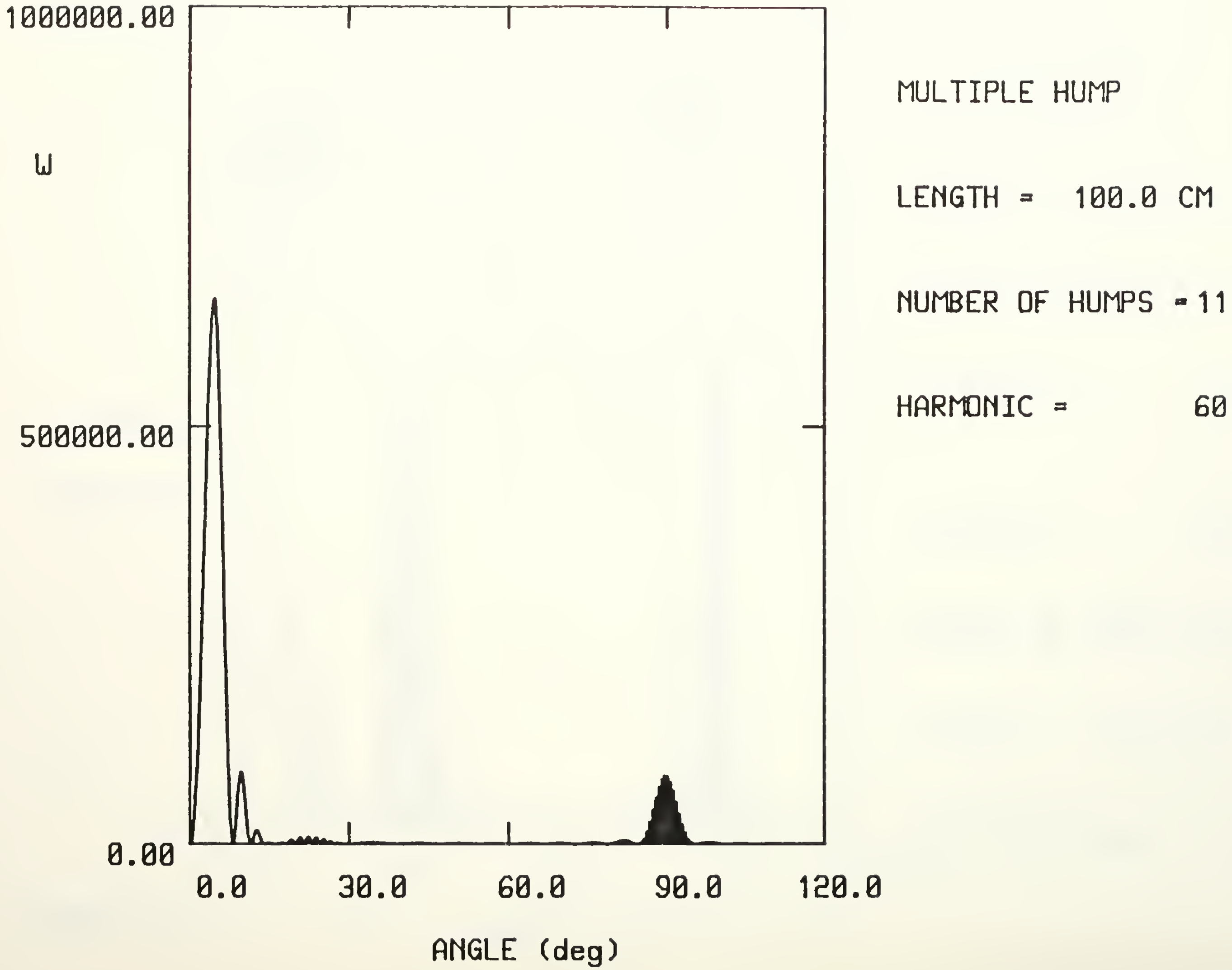
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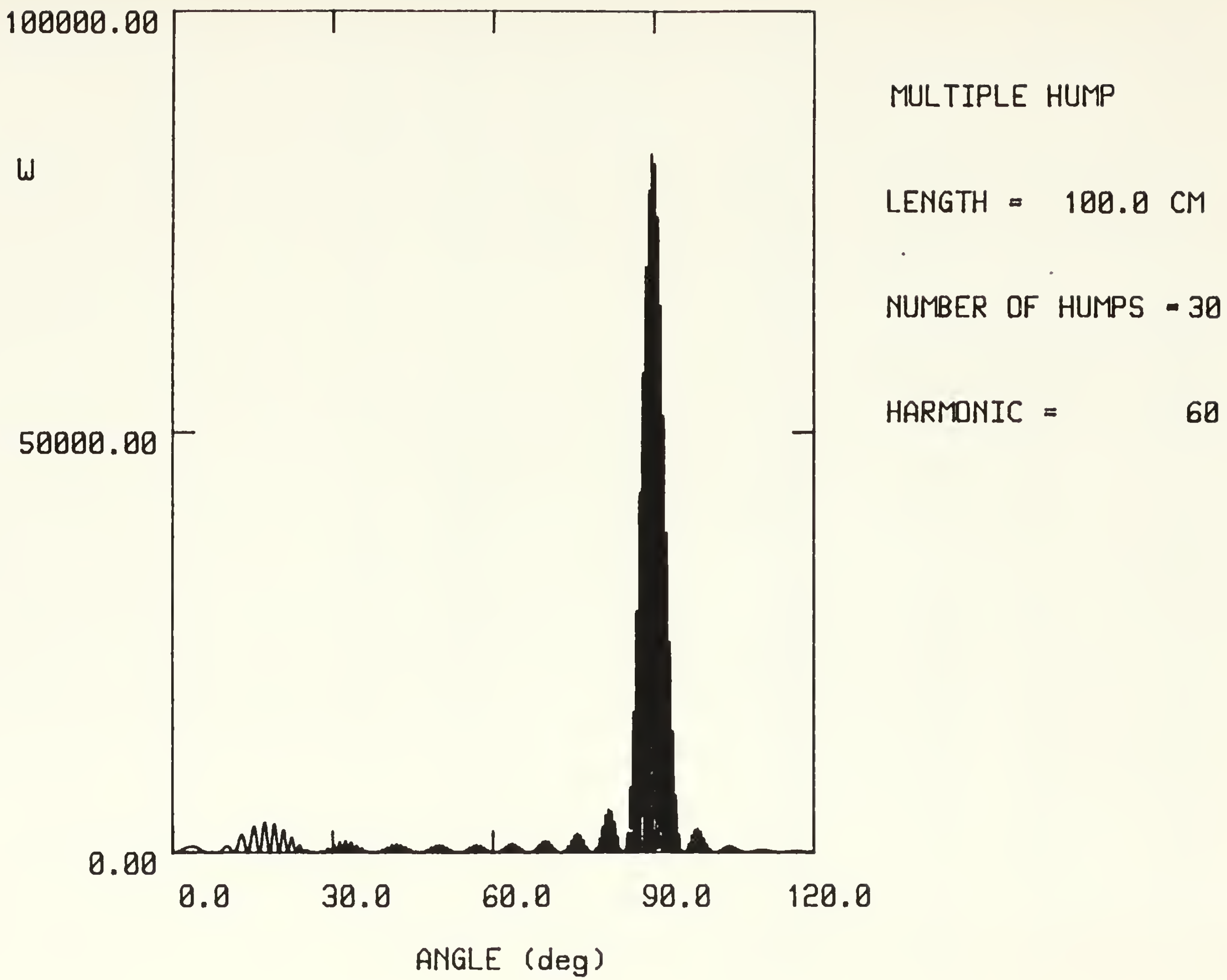
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LENGTH = 100.0 CM
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